Flexural Buckling Analysis of Thin Walled Lipped Channel Cross Section Beams with Variable Geometry

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ABSTRACT - The structural components of an aircraft consist mainly of thin plates stiffened by arrangements of ribs and stringers. Thin walled structure is a structure whose thickness is small compared to its other dimensions but which is capable of resisting bending in addition to membrane forces. Which is basic part of an aircraft structure, Thin plates (or thin sections or thin walled structures) under relatively small compressive loads are prone to buckle and so must be stiffened to prevent this. The determination of buckling loads for thin plates in isolation is relatively straight forward but when stiffened by ribs and stringers, the problem becomes complex and frequently relies on an empirical solution. The buckling of the thin plates is a phenomenon which could lead to destabilizing and failure of the aircraft structure; in this paper it is considered C cross section with variable geometry and length. The critical buckling loads have been studied for several combinations of the geometry parameters of the beam with the help of ANSYS and drown the result plots

KEYWORDS: Thin walled beams, Channel sections, buckling analysis, Finite element analysis

I. LITERATURE SURVEY

A great deal of attention has been focused on plates subjected to shear loading over the past decades. One main fact in design of such elements, which falls in the category of thin-walled structures, is their buckling behavior. Plate girders and recently shear walls are being widely used by structural engineers, as well as ship and aircraft designers. The role of stiffeners is proved to be vital in design of such structures to minimize their weight and cost.

Xiao-ting et al [1] presented an analytical model for predicting the lateral torsional buckling of thin walled channel section beams restrained by metal sheeting when subjected to an uplift load. And calculated the critical load from critical energy theory and showed that the critical buckling moment in the pure bending case is less than half of the critical moment, it is more effective to use the anti sag bars in the simply supported beams than in the fixed beams, the closer the loading point to the centre the lower the critical load. M.Ma et al [2] developed energy method for analyzing the lateral buckling behavior of the monosymmetric I beams subjected to distributed vertical load, with full allowance for distortion of web the method assumed that the flanges buckle as rigid the rectangular section beams, but the web distorts as an elastic plate during buckling, it is shown that the disparity between the distortional and classical critical load increases as h/l increases and that for short beams the classical method seriously over estimates the critical load. B. W. Schafer [3] worked on cold-formed thin-walled open cross-section steel columns and provided local, distortional, and flexural-torsional buckling. Experimental
and numerical studies indicated that post buckling strength in the distortional mode is less than in the local mode. In pin-ended lipped channel and zed columns, local and Euler interaction is well established. A direct strength method is proposed for column design. The method uses separate column curves for local buckling and distortional buckling with the slenderness and maximum capacity in each mode controlled by consideration of Euler equation. Attard Mario et al [4] investigated lateral-torsional buckling behavior of open-section thin-walled beams based on a geometrically nonlinear formulation, which considers the effects of shear deformations, also made Comparisons between the results based on fully nonlinear analysis and linearized buckling analysis in order to illustrate the effects of pre-buckling deformations as well as the shear deformations on the buckling load predictions. Ing. Antonin pistek,[5] analytical method for limit load capacity Calculation Of thin walled aircraft structures focused on description and Comparison of different methods for limit load Capacity calculation of thin walled aircraft Structures – considering all possible forms of Buckling and failures on nonlinear behavior of The structure under gradually increased Loading. Carine Louise Nilsen, et al [6] found that the behavior of thin-walled steel sections, including local buckling, distortional buckling, global buckling and shear buckling have been well understood and appropriate design methods existed. Foudil Mohria et al [7] derived analytical solutions Based on a non-linear stability model, for simply supported beam-column elements with bi-symmetric I sections under combined bending and axial forces. Jaehong Lee et al [8] explained lateral buckling of thin-walled composite beams with monosymmetric sections. A general geometrically nonlinear model for thin walled laminated composites with arbitrary open cross-section and general laminate stacking sequences is given by using systematic variational formulation based on the classical lamination theory. The load capacity of cold-formed thin-walled beams is usually restricted by their stability and post-buckling behaviour. Strength was considered by Cheng and Schafer [9], Trahair [10] Experimental investigations, stress and displacement distribution of cold-formed beams were shown by Paczos et al [11] Other examples of papers directly connected with the subject of this work are Biegus et al [12] Magnucki et al [13] Magnucki et al [14] Paczos, Jeyaragan et al [15] Lawrence W. Rehfield et al [16] design methodology for buckling of thin-walled laminated composite beams shoed buckling by pure bending and bending- torsion coupled modes can occur and that transition among modes are governed by structural parameters. Tomasz kubiak et al [17] presented analysis of local buckling of thin-walled beam-columns, taking account Global pre critical bending within the first order approximation. Marco et al [18] published a paper on buckling of thin-walled structures is presented using the 1D finite element based refined beam theory formulation Deepak et al [19] cold formed c and z sections because of their easy connections but they suffer from certain buckling modes. here the structural behavior of channel lipped beams due to lateral buckling and load carrying capacity is evaluated. Load Vs deflection curves are plotted in comparison with the experimental results attained through FEA modeling of the software as part of results. Brad ford [20] lateral-distortional buckling of steel I section members presented how the usual types of buckling of steel members assumed in design are lateral-torsional and local buckling modes. In lateral-torsional buckling, the cross-sections of the member translate and twist as rigid bodies. On the other hand, local buckling is characterized by localized distortions of the cross-section over a short wavelength in the absence of lateral translation.

II. INTRODUCTION

A Thin plate or a thin walled structure is a structure whose thickness is small compared with its other dimensions but which is capable of resisting bending in addition to membrane forces. Such a plate forms a basic part of an aircraft structure, being, for example, the area of stressed skin bounded by adjacent stringers and frames in a fuselage. The structural components of an aircraft consist mainly of thin plates stiffened by arrangements of ribs and stringers. Thin plates (or thin sections or thin walled structures) under relatively small compressive loads are prone to buckle and so must be stiffened to prevent this. The determination of buckling loads for thin plates in isolation is relatively straightforward but when stiffened by ribs and stringers, the problem becomes complex and frequently relies on an empirical solution. The buckling of the thin plates is a phenomenon which could lead to destabilizing and failure of the aircraft, hence we study the buckling phenomenon on thin plates or thin walled structures with the help of the finite element analysis software ANSYS.
The first significant contribution to the theory of the buckling of columns was made as early as 1744 by Euler. [22] His classical approach is still valid, and likely to remain so, for slender columns possessing a variety of end restraints. Our initial discussion is therefore a presentation of the Euler theory for the small elastic deflection of perfect columns. However, we investigate first the nature of buckling and the difference between theory and practice. It is common experience that if an increasing axial compressive load is applied to a slender column there is a value of the load at which the column will suddenly bow or buckle in some un- predeterminied direction. This load is patiently the buckling load of the column or something very close to the buckling load. Clearly this displacement implies a degree of asymmetry in the plane of the buckle caused by geometrical and/or material imperfections of the column and its load. However, in our theoretical stipulation of a perfect column in which the load is applied precisely along the perfectly straight centroidal axis, there is perfect symmetry so that, theoretically, there can be no sudden bowing or buckling. We therefore require a precise definition of buckling load which may be used in our analysis of the perfect column. Assume that it is in the displaced state of neutral equilibrium associated with buckling so that the compressive load \( P \) has attained the critical value \( P_{CR} \). Simple bending theory

\[
EI \frac{d^2 \nu}{dz^2} = -M \quad \text{or} \quad EI \frac{d^2 \nu}{dz^2} = -P_{CR} \nu
\]

so that the differential equation of bending of the column is

\[
\frac{d^2 \nu}{dz^2} + \frac{P_{CR}}{EI} \nu = 0
\]

The well-known solution [22]

\[ \nu = A \cos \mu z + B \sin \mu z \pi \]

Critical load

\[ P_{CR} = \frac{n^2 \pi^2 EI}{l^2} \]

Other values of \( P_{CR} \) corresponding to \( n=2, 3, \ldots \)

\[ P_{CR} = \frac{4 \pi^2 EI}{l^2}, \frac{9 \pi^2 EI}{l^2}, \ldots. \]

By using the energy methods can also be calculated the critical load. The total potential energy of the column in the neutral equilibrium of its buckled state is therefore

\[
U + V = \frac{EI}{2} \int_0^l \left( \frac{d^2 \nu}{dz^2} \right)^2 dz - \frac{P_{CR}}{2} \int_0^l \left( \frac{d\nu}{dz} \right)^2 dz
\]

And is capable, within the limits for which it is valid and if suitable values for the constant coefficients are chosen, of representing any continuous curve. We are therefore in a position to find \( P_{CR} \) exactly.

\[
U + V = \frac{\pi^2 EI}{4l^3} \sum_{n=1}^\infty \frac{4n^4 A_n^2}{l^2} - \frac{\pi^2 P_{CR}}{4l} \sum_{n=1}^\infty \frac{n^2 A_n^2}{l^2}
\]

In general form

\[ P_{CR} = \frac{n^2 \pi^2 EI}{l^2} \]
Where ‘I’ is moment of Inertia, ‘E’ is Young’s modulus of the material; ‘l’ is the effective length of the element.

C - Section thin walled beam flanges are behave similar to plate buckling, the following equations are valid for thin plate simply supported along edges.

\[ w = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} \]  \hspace{1cm} (10)

The total potential energy of the plate has a stationary value in the neutral equilibrium of its buckled state i.e. \( N_x = N_{x,CR} \).

\[ N_{x,CR} = \frac{k\Pi^2 D}{b^2} \]  \hspace{1cm} (12)

Where the plate buckling coefficient k is given by the minimum value of

\[ k = \left( \frac{mb}{a} + \frac{a}{mb} \right)^2 \]  \hspace{1cm} (13)

The critical stress of the plate is given by the equation

\[ \sigma_{CR} = \frac{k\Pi^2 E}{12(1-\nu^2)} \left( \frac{t}{b} \right)^2 \]  \hspace{1cm} (14)

Therefore the critical buckling load \( (P_{CR}) \) is \( = \) Critical stress \( (\sigma_{CR}) \) \( \times \) Cross sectional Area \( (A) \) \hspace{1cm} (15)

The local failure stress in longitudinally stiffened panels was determined by Gerard using a slightly modified form

\[ \frac{\sigma_f}{\sigma_{cy}} = \beta_g \left[ \frac{gt_{sk}t_{sl}}{A} \left( \frac{E}{\sigma_{cy}} \right)^{1/m} \right] \]  \hspace{1cm} (16)

Where ‘g’ is number of cuts + Flanges

III. MODELING

Considered lipped channel section with lip dimension ‘s’ Flange width ‘b’ web height ‘h’ and thickness ‘t’ , c is shear center , ‘o’ is centroid.

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IV. CLASSIFICATION OF CHANNELS:

Case 1: represents a channel cross section beam with $b = 50\text{mm}$, $h = 30\text{mm}$, $t = 0.8\text{mm}$, and $s = 0\text{mm}$.
Case 2: represents a Lipped channel cross section beam with $b = 40\text{mm}$, $h = 30\text{mm}$, $t = 0.8\text{mm}$, and $s = 10\text{mm}$.
Case 3: represents a Lipped channel cross section beam with $b = 35\text{mm}$, $h = 40\text{mm}$, $t = 0.8\text{mm}$, and $s = 10\text{mm}$.
Case 4: represents a Lipped channel cross section beam with $b = 30\text{mm}$, $h = 50\text{mm}$, $t = 0.8\text{mm}$, and $s = 10\text{mm}$.
Case 5: represents a Lipped channel cross section beam with $b = 25\text{mm}$, $h = 60\text{mm}$, $t = 0.8\text{mm}$, and $s = 10\text{mm}$.
Case 6: represents a Lipped channel cross section beam with $b = 30\text{mm}$, $h = 60\text{mm}$, $t = 0.8\text{mm}$, and $s = 5\text{mm}$.

V. ELASTIC BUCKLING OF THE BEAM

Analysis has been done using FEM the following parameters have been used: $72 \times 10^3 \text{N/mm}^2 =$ Young modulus ($E$); $0.3 =$ Poisson’s ratio ($\mu$); $30 \text{mm} =$ depth of a beam ($h$); $50 \text{mm} =$ Width of the flange; $150 \text{mm} =$ length of a beam ($L$) for model case

The critical load ($P_{cr}$)

$$ P_{cr} = \frac{\pi^2 E I}{4 L^2} \left( \frac{th^2 (6b + h)}{12} \right) $$

Where $I =$ Moment of Inertia.
By using the above equation and by ansys, the critical load obtained for thin walled channel section beam is $149.82 \times 10^3 \text{N/mm}^2$ and $147.16 \times 10^3 \text{N/mm}^2$ respectively.
Figure 2: (a) channel cross section beam with b = 50mm, h = 30mm, t = 0.8mm, and s = 0mm. (b) Lipped channel cross section beam with b = 40mm, h = 30mm, t = 0.8mm, and s = 10mm. (c) Lipped channel cross section beam with b = 35mm, h = 40mm, t = 0.8mm, and s = 10mm. (d) Lipped channel cross section beam with b = 30mm, h = 50mm, t = 0.8mm, and s = 10mm. (e) Lipped channel cross section beam with b = 25mm, h = 60mm, t = 0.8mm, and s = 10mm. (f) Lipped channel cross section beam with b = 30mm, h = 60mm, t = 0.8mm, and s = 5mm.

Figure 3: Von mises stresses of (a) channel cross section beam with b = 50mm, h = 30mm, t = 0.8mm, and s = 0mm. (b) Lipped channel cross section beam with b = 40mm, h = 30mm, t = 0.8mm, and s = 10mm. (c) Lipped channel cross section beam with b = 35mm, h = 40mm, t = 0.8mm, and s = 10mm. (d) Lipped channel cross section beam with b = 30mm, h = 50mm, t = 0.8mm, and s = 10mm. (e) Lipped channel cross section beam with b = 25mm, h = 60mm, t = 0.8mm, and s = 10mm. (f) Lipped channel cross section beam with b = 30mm, h = 60mm, t = 0.8mm, and s = 5mm.
Figure 4: First buckling mode shapes of (a) channel cross section beam with $b = 50\text{mm}$, $h = 30\text{mm}$, $t = 0.8\text{mm}$, and $s = 0\text{mm}$. (b) Lipped channel cross section beam with $b = 40\text{mm}$, $h = 30\text{mm}$, $t = 0.8\text{mm}$, and $s = 10\text{mm}$. (c) Lipped channel cross section beam with $b = 35\text{mm}$, $h = 40\text{mm}$, $t = 0.8\text{mm}$, and $s = 10\text{mm}$. (d) Lipped channel cross section beam with $b = 30\text{mm}$, $h = 50\text{mm}$, $t = 0.8\text{mm}$, and $s = 10\text{mm}$. (e) Lipped channel cross section beam with $b = 25\text{mm}$, $h = 60\text{mm}$, $t = 0.8\text{mm}$, and $s = 10\text{mm}$. (f) Lipped channel cross section beam with $b = 30\text{mm}$, $h = 60\text{mm}$, $t = 0.8\text{mm}$, and $s = 5\text{mm}$.

VI. RESULT AND DISCUSSION

The graphs for different C sections indicating the relationship between modes and critical buckling are given below:

*Length of the channel 1-150 mm: Bucking Analysis: Combined graph for all cases of C section*

Figure 5: Buckling Analysis of channel section beams (case 1 to case 6), the graph critical buckling $v_1$ mode shapes, case 5 observed that the more critical buckling values in first mode than other cases and case 2 more critical buckling values at fifth mode for the channel length $L = 150\text{mm}$

*Length 2-300 mm: Bucking Analysis: Combined graph for all cases of C sections:*
Figure 6: Buckling Analysis of channel section beams (case 1 to case 6), the graph critical buckling vs mode shapes, case 1 observed that the more critical buckling values in first mode than other cases and case 5 and case 2 more critical buckling values at fifth mode of the channel length L = 300mm.

Length 3–450 mm: Bucking Analysis: Combined graph for all cases of C sections:

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**Figure 7**: Buckling Analysis of channel section beams (case 1 to case 6), the graph critical buckling vs. mode shapes, case 4 observed that the more critical buckling values at first mode than other cases and case 5 more critical buckling values at fifth mode of the channel length L = 450mm

**Length4-600mm: Bucking Analysis: Combined graph for all cases of C sections:**

![Graph 1](image1)

**Figure 8**: Buckling Analysis of channel section beams (case 1 to case 6), the graph critical buckling vs. mode shapes, case 1 observed that the more critical buckling values in first mode than other cases and case 5 and case 6 more critical buckling values at fifth mode of the channel length L = 600mm

**Length5-750mm: Bucking Analysis: Combined graph for all cases of C sections:**

![Graph 2](image2)

**Figure 9**: Buckling Analysis of channel sections beams (case 1 to case 6), the graph critical buckling vs. mode shapes, case 5 observed that the more critical buckling values in first mode than other cases and case 2 more critical buckling values at fifth mode of the channel length L = 750mm
VII. CONCLUSIONS

Thin-walled C sections with different cross-sections and different lengths of extrusion have been considered for buckling analysis in this paper. It is observed that in buckling analysis, at length L = 150mm, at the first mode, the value of critical buckling is higher for case 5 whereas at the fifth mode, case 2 is giving higher value of critical buckling. At length L = 300mm, at the first mode, case 1 is giving higher value of critical buckling at the fifth mode, case 5 and case 2 are giving higher values of critical buckling. At length L = 450mm, at the first mode, case 4 is giving higher value of critical buckling...
whereas at the fifth mode, 5 is giving higher value of critical buckling. At length L = 600mm, at the first mode, case 1 is giving higher value of critical buckling whereas at the fifth mode, case 5 and case 6 are giving higher values of critical buckling. At length L = 750mm, at the first mode, case 5 is giving higher value of critical buckling whereas at the fifth mode, case 2 is giving higher value of critical buckling.

VIII. REFERENCES

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