Lucky Edge Labeling of Different Graphs

A. Aishwarya

Department of Mathematics, Bharath University, BIST, Chennai, India

ABSTRACT: Let G be a Simple Graph with Vertex set V(G) and Edge set E(G) respectively. Vertex set V(G) are labeled arbitrary by positive integers and let E(e) denote the edge label such that it is the sum of labels of vertices incident with edge e. The labeling is said to be lucky edge labeling if the edge set E(G) is a proper coloring of G, that is, if we have E(e_1) ≠ E(e_2) whenever e_1 and e_2 are adjacent edges. The least integer k for which a graph G has a lucky edge labeling from the set {1, 2, ..., k} is the lucky number of G denoted by \( \eta(G) \). A graph which admits lucky edge labeling is the lucky edge labeled graph. In this paper the lucky edge labeling of Bistar graph, Wheel graph and Fan graph are discussed.

KEYWORDS: Lucky edge labeling, Lucky number, Lucky edge labeled graph.

I. INTRODUCTION

All the graphs considered are undirected simple graphs. The labeling of graphs trace their origin being introduced by Rosa[1] in 1967. A graph labeling is an assignment of integers to the vertices or edges, or both, subject to certain conditions[5]. For the basics of graph theory concepts and notations refer to Harary[4] and Douglas B. West[2].

Definition 1.1: Let G be a Simple Graph with Vertex set V(G) and Edge set E(G) respectively. Vertex set V(G) are labeled arbitrary by positive integers and let E(e) denote the edge label such that it is the sum of labels of vertices incident with edge e. The labeling is said to be lucky edge labeling if the edge set E(G) is a proper coloring of G, that is, if we have E(e_1) ≠ E(e_2) whenever e_1 and e_2 are adjacent edges.[3]

Definition 1.2: The least integer k for which a graph G has a lucky edge labeling from the set {1, 2, ..., k} is the lucky number of G denoted by \( \eta(G) \).[4]

Definition 1.3: A graph which admits lucky edge labeling is the lucky edge labeled graph.

Definition 1.4: The Bistar graph \( B_{m,n} \) is the graph obtained from \( K_2 \) by joining m pendent edges to one end and n pendent edges to the other end of \( K_2 \).[6]

Definition 1.5: The wheel graph denoted by \( W_n \) is a graph with n vertices (\( n \geq 4 \)), formed by connecting a single vertex to all vertices of an \( (n-1) \)-cycle[6].

Definition 1.6: The Fan graph denoted by \( F_n \) can be constructed by joining n copies of the cycle graph \( C_3 \) with a common vertex. The Fan graph \( F_n \) is a planar undirected graph with \( 2n+1 \) vertices and \( 3n \) edges.

II. MAIN RESULTS

Theorem 1:- The lucky number of the Bistar graph \( B_{m,n} \) is given by
\[ \eta(B_{m,n}) = \begin{cases} m+2 & \text{if } m \geq n \\ n+2 & \text{if } n \geq m \end{cases} \]

**Proof:**

Let the vertices of \( K_2 \) be \( v_1 \) and \( v_2 \). Let the vertices joined to \( v_1 \) by the pendent edges be \( u_1, u_2, \ldots, u_m \) and let the vertices joined to \( v_2 \) by the pendent edges be \( w_1, w_2, \ldots, w_n \). Label the vertices \( v_1 \) and \( v_2 \) with 1. \[ \text{Label the vertices } u_i \text{ by the labels } i+1 (i = 1 \text{ to } m) \text{ and label the vertices } w_j \text{ by the labels } j+1 (j = 1 \text{ to } n). \]

This induces a lucky edge labeling for \( B_{m,n} \).

**Theorem 2:**

The lucky number of wheel graph \( W_n \) is \( \eta(W_n) = 2n - 1 \).

**Proof:**

Let the single vertex and the centre be \( v_1 \) and the other vertices of \((n-1)\) – cycle that are joined to the vertex \( v_1 \) be \( v_2, v_3, \ldots, v_n \). Label the vertices \( v_i \) by the label \( i \) (i = 1 to n). This induces a lucky edge labeling of \( W_n \) and \( \eta(W_n) = 2n - 1 \)[10].
Theorem 3:-

The lucky number of Fan graph $F_n$ is $\eta(F_n) = 4n+1$.

**Proof:**

Let the middle vertex of the join of all the $C_3$ be $v_1$ and let the remaining vertices be $v_2, v_3, \ldots, v_n$. Label the vertices $v_i$ with labels $i$ ($i = 1$ to $n$).[12] This induces a lucky edge labeling of the Fan graph $F_n$ with lucky number given by $\eta(F_n) = 4n+1$.

**REFERENCES**

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