Abel-Type Method of Summability in Non-Archimedean Fields

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ABSTRACT: The aim of this paper is to introduce Abel–type method of summability in complete, non-trivially valued, non-archimedean fields and prove some interesting properties.

KEYWORDS: Non-Archimedean, Abel–type method, Regular[1]

I. INTRODUCTION

Throughout this paper, K denotes complete, non-trivially valued, non-archimedean fields. Infinite matrices and sequences considered in the sequel have entries in K. A general reference for the study of summability methods in classical one is[4]. For analysis, in non-archimedean fields, see [5].

If \( A = (a_{n,k}) \), \( a_{n,k} \in K \), \( n,k = 0,1,2,... \) is an infinite matrix, then the A-transform of a sequence \( x = \{x_k\} \), \( x_k \in K \), \( k = 0,1,2,... \), is the sequence, where

\[
(Ax)_n = \sum_{k=0}^{n} a_{nk}x_k, \quad n = 0,1,2,...
\]

it being assumed that the series on the right converge.

In the sequel, the following definitions and theorems are needed.

Definition 1. Let \( \{x_k\} \) be a sequence in \( K \) and \( x \in K \). We say that \( \lim_{k \to \infty} x_k = x \) if for each \( \epsilon > 0 \), the set \( \{|x - x_k| \geq \epsilon\} \) is finite. In such a case, we say that \( x \) is the limit of \( \{x_k\} \).

Definition 2. Let \( A = (a_{n,k}) \), \( a_{n,k} \in K \), \( n,k = 0,1,2,... \) if \( \lim_{n \to \infty} (Ax)_n \) exists whenever \( \lim_{n \to \infty} x_k \) exists, then \( A \) is said to be convergence preserving[2]. Further, if \( \lim_{n \to \infty} (Ax)_n = s \) whenever \( \lim_{n \to \infty} x_k = s \), we say that \( A \) is regular.

Theorem 1. \( A = (a_{n,k}) \) is a regular if and only if

(i) \( \sup_{n,k} |a_{n,k}| < \infty \)

(ii) \( \lim_{n \to \infty} a_{n,k} = 0, \quad k = 0,1,... \)

(iii) \( \lim_{n \to \infty} \sum_{k=0}^{\infty} a_{n,k} = 1 \)

We introduce Abel–type method of summability in K and prove some interesting properties[3].
Definition 3. Let $q \in K$ be such that $|q| < 1$. The Abel-type method of order $q$ or the $[s, q]$ method is given by the matrix $\left(s_{n,k}^{(q)}\right)$ as follows [1].

If $q \neq 0$

$$s_{n,k}^{(q)} = \begin{cases} 
0 & k > n; \\
(n + k)C_k q^k (1 - q)^{n+1} & k \leq n.
\end{cases}$$

Remark 1. We note that $q \neq 0$, since $|q| < 1$.

II. MAIN RESULT

Theorem 2. The $[s, p]$ method is regular if $|q| < 1$.

Proof. Let $\sum_{k=0}^{\infty} s_{n,k}^{(q)} = \sum_{k=0}^{n} s_{n,k}^{(q)}$

$$= \sum_{k=0}^{n} (n + k)C_k q^k (1 - q)^{n+1}$$

$$= \sum_{k=0}^{n} (n + k)C_k q^k (1 - q)^{(n+k)-k} (1 - q)$$

$$= (1 - q) \sum_{k=0}^{n} (n + k)C_k q^k (1 - q)^{(n+k)-k}$$

$$= (1 - q) \sum_{k=0}^{n} (n + k)C_k q^k (1 - q)^{(n+k)-k}$$

$$= (1 - q)(1 - q + q)^{n+k}$$

$$= 1 \text{ for } |1 - q| = 1, |q| < 1$$

That is

$$\lim_{n \to \infty} \sum_{k=0}^{\infty} s_{n,k}^{(q)} = 1$$

Now, $s_{n,k}^{(q)} = 0$, if $k > n$ implies that

$$\lim_{n \to \infty} s_{n,k}^{(q)} = 0, k = 0, 1, 2, \ldots,$$

When $s_{n,k}^{(q)} \neq 0$,
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\[
\begin{align*}
  s_{n,k}^{(q)} &= \left( n + k \right) C_k q^k \left( 1 - q \right)^{n+1} \\
  &\leq \left( n + k \right) C_k q^k \left| 1 - q \right|^{n+1} \\
  &\leq \left( n + k \right) C_k \left| 1 - q \right|^{n+1}, \text{ since } |q| < 1 \\
  &= \left| n + k C_k \right|, \text{ since } |1 - q| = 1 \\
  &= \left| n + k C_k \right| \leq 1
\end{align*}
\]

So that \( \sup_{n,k} s_{n,k}^{(q)} < 1 \)

Thus the \( [S, q] \) method is regular.

**REFERRANCES**