Problems Based On Rational and Irrational Numbers in Real Number System

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ABSTRACT: In this paper, we define some problems based on rational and irrational numbers using real numbers.

KEYWORDS: rational number, irrational number.

I. INTRODUCTION

Several methods are used to introduce real numbers. One method starts with the positive integers 1, 2, 3, … as undefined concepts and uses them to build a largest system [4], the positive rational numbers (quotients of positive integers), their negatives , and zero. The rational numbers, in turn, are then used to construct the irrational numbers, real numbers like \( \sqrt{2} \) and \( \pi \) which are not rational [5]. The rational and irrational numbers together constitute the real-number system [3].

Definition 1.1. Quotients of integers \( a/b \) (where \( b \neq 0 \)) are called rational numbers.

For example, \( 1/2, -7/5, \) and \( 6 \) are rational numbers. The set of rational numbers, which we denote by \( Q \), contains \( Z \) as a subset.

Another example, if \( a \) and \( b \) are rational, their average \( (a + b)/2 \) is also rational and lies between \( a \) and \( b \) [6]. Therefore between any two rational numbers there are infinitely many rational numbers, which implies that if we are given a certain rational number we cannot speak of the “next largest” rational number [2].

Definition 1.2. Real numbers that are not rational are called irrational [1].

For example, the numbers \( \sqrt{2}, e, \pi \) and \( e^\pi \) are irrational [7].

Theorem 1.1. If \( n \) is a positive integer which is not a perfect square, then \( \sqrt{n} \) is irrational.

Proof. Suppose first that \( n \) contains no square factor > 1. We assume that \( \sqrt{n} \) is rational and obtain a contradiction [8].

Let \( \sqrt{n} = a/b \), where \( a \) and \( b \) are integers having no factor in common. Then \( nb^2 = a^2 \) and, since the left of this equation is a multiple of \( n \), so too is \( a^2 \). However, if \( a^2 \) is a multiple of \( n \), it must be a multiple of \( n \) since \( n \) has no square factors > 1 [8].

This means that \( a = cn \), where \( c \) is some integer. Then the equation \( nb^2 = a^2 \) becomes \( nb^2 = c^2 n^2 \) or \( b^2 = nc^2 \). The same argument shows that \( b \) must also be a multiple of \( n \). Thus \( a \) and \( b \) are both multiples of \( n \), which contradicts the fact that they have no factor in common. This completes the proof if \( n \) has no square factor > 1.

II. MAIN RESULTS

In this section, we prove some problems based on rational and irrational numbers using real number system [9].

Problem 2.1. Find the rational number whose decimal expansion is 0.334444444444....

Proof. Let \( x = \frac{3}{10} + \frac{3}{10^2} + \frac{4}{10^3} + \ldots + \frac{4}{10^n} + \ldots \) then \( x = \frac{33}{10^2} + \frac{4}{10^3} \left( 1 + \frac{1}{10} + \ldots + \frac{1}{10^n} + \ldots \right) \).
Problem 2.2. Prove that the decimal expansion of x will end in zeros (or in nines) if, and only if, x is a rational number whose denominator is of the form $2^n 5^m$, where m and n are nonnegative integers[10].

**Proof.** Suppose that $x = \frac{k}{2^n 5^m}$, if $n \geq m$, we have

\[
\frac{k}{2^n 5^m} = \frac{10^n}{10^n}
\]

So, the decimal expansion of x will end in zeros. Similarly for $m \geq n$.

Suppose that the decimal expansion of x will end in zeros (or in nines).

For case, $x = a_0, a_1 a_2, \ldots, a_n$. Then

\[
x = \sum_{k=0}^{n} \frac{10^{-k}}{2^k 5^k} a_k = \frac{10^{-n}}{2^k 5^k} a_k.
\]

For case, $x = a_0, a_1 a_2 \ldots a_n 999999 \ldots$. Then

\[
x = \sum_{k=0}^{n} \frac{10^{-k}}{2^k 5^k} a_k + \frac{9}{10^{n+1}} + \frac{9}{10^{n+2}} + \ldots.
\]

So, in both case, we prove that x is a rational number whose denominator is of the form $2^n 5^m$, where m and n are nonnegative integers

**REFERENCES**