Trilinearization of (2+1) Dimensional Long Dispersive Wave Equation Using PTA

P. Myvizhi, R. Velavan
Department of Physics, Bharath Institute of Higher Education and Research, Chennai, India
Department of Physics, Bharath University, Chennai, India

ABSTRACT: Using the methodology of Painleve Truncation Approach (PTA), an infinite integrable nonlinear dynamical system named long dispersive wave equation (2LDW) has been solved and higher order dromion solution is obtained from generalized periodic solution.

KEYWORDS: Painleve, trilinear, solitons

I. INTRODUCTION

The identification of a large number of arbitrary functions in the solutions of (2+1) dimensional integrable models have only added to the richness in the structure of them and hence construction of localized excitations in (2+1) dimensions continues to be challenging and rewarding contemporary problem.

In this equation we developed a very simple and straightforward procedure to generate a rather extended class of generic solutions of physical interest. In this 2LDW equation we utilize the local Laurent expansion of the general solution and truncate it at the constant level term (painleve truncation approach) and obtain solutions in terms of arbitrary functions. Through this procedure we generate various periodic and exponentially localized solutions. The novelty here is that the solution is generated through a simple procedure. But the solution obtained is rich in structure because of the arbitrary functions [9-14].

II. PAINLEVE TRUNCATION APPROACH

If the solution of a nonlinear differential equation is free from movable (integration constant/initial condition dependent) critical singular points (branch points and essential singularities) it is said to possess the painleve property. In this case the system is expected to be integrable.[16-19]

Painleve singularity structure analysis is used in which the solution of the given equation of motion of a dynamical system is analyzed for the presence or absence of movable critical singular points and it can be used as a powerful test to identify and isolate integrable infinite dimensional nonlinear dynamical system. It has been pointed out that there exists a large number of solitons possessing completely integrable infinite dimensional dynamical systems. [20,21]

Painleve truncation approach is a methodology in which we are going to prove that the (2+1) dimensional long dispersive wave equation is a completely integrable infinite dimensional nonlinear dynamical system.

The recent interest in the investigation of integrable models in (2+1) dimensions can be attributed to the identification of dromions in the Davey-Stewartson I equation [3]. These dromions which originate at the cross point of the intersection of two nonparallel ghost solitons decay exponentially in all directions and are driven by lower dimensional boundaries or velocity potentials (arbitrary functions) in the system. It must be mentioned that the presence of these so called boundaries enriches the structure of (2+1) dimensional integrable models leading to the formation of localized solutions like dromions, positions, etc. [5]. Thus a judicious harnessing of these lower dimensional arbitrary functions [9-14] of space and time may give rise to new spatio-temporal patterns in higher dimensions besides throwing more light on their integrability.
In this way, a simple procedure is used to solve the (2+1) dimensional system namely Painleve Truncation Approach (PTA) [5]. This approach converts the given evolution equation into a multi linear equation in terms of non characteristic manifold. This multi linear equation can be solved in terms of lower dimensional arbitrary functions of space and time. Using this approach one can generate various generalized periodic and localized solutions. Through PTA (2+1) dimensional long dispersive wave equation can be trilinearized. A four parameter special class of solutions involving three arbitrary functions in the variables naturally arises for this trilinear equation. Thus, there exists a possibility of obtaining more general solutions.

(2+1) DIMENSIONAL LONG DISPERSIVE WAVE EQUATION

Considering the long dispersive wave equation (2LDW) [15,16]

\[ \lambda q_t + q_{xx} - 2q_v = 0 \]  
\[ \lambda r_t - r_{xx} + 2r_v = 0 \]  
\[ (qr)_x = V_\eta \]

We now effect a local Laurent expansion in the neighbourhood of a noncharacteristic singular manifold \( \Phi (x, \eta, t) = 0, \Phi_x \neq 0, \Phi_\eta \neq 0, \Phi_t \neq 0. \)

Assuming the leading orders of eqn(1), the solution of equation (1) to have the form

\[ q = q_0 \Phi^\alpha \]  
\[ r = r_0 \Phi^\beta \]  
\[ V = v_0 \Phi^\gamma \]

where \( q_0, r_0, v_0 \) are analytic functions of \( \Phi (x, \eta, t) \) and \( \alpha, \beta, \gamma \) are integers to be determined.

With the condition of seed solution

\[ q_1 = r_1 = 0, V_2 = V_2(t) \]

Then collecting the coefficients of lower order dimensional terms and converting it into a multilinear equation such as trilinear equation is possible as follows.

\[ V_{10} = \Phi_{x\eta}^2 + (\Phi_x \Phi_{x\eta}/\Phi_\eta) - (\Phi_x \Phi_{x\eta} \Phi_{\eta}/\Phi_\eta^2)(\Phi_{xx\eta} \Phi_{\eta}^2 + \Phi_{x\eta} \Phi_{\eta} - \Phi_{x\eta} \Phi_{\eta} - \Phi_{xx\eta} \Phi_{\eta}^2) = 0 \]

Equation (4) indicates trilinear form of the equation.

III. CONCLUSION

In this paper, I have investigated the (2+1) dimensional long dispersive wave equation by using the Painleve Truncation Approach (PTA) method. In particular, I have shown that the equation is trilinearizable. More general solutions can be generated from the trilinear equation which remains, to be investigated. It will enrich the existence of lower dimensional arbitrary functions, which will help us to construct novel localized solutions such as dromions.

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REFERENCES