On Pseudo Regular and Irregular Intuitionistic Fuzzy Graphs

Dr. N.R. Santhi Maheswari¹, Dr. C. Sekar²
Department of Mathematics, G. Venkataswamy Naidu College, Kovilpatti, Tamil Nadu, India. ¹
P.G. Extension Centre, Manonmaniam Sundaranar University, Nagercoil, Tamil Nadu, India. ²

ABSTRACT: In this paper, pseudo regular intuitionistic fuzzy graphs, pseudo irregular intuitionistic fuzzy graphs, highly pseudo irregular intuitionistic fuzzy graphs, neighbourly pseudo irregular intuitionistic fuzzy graphs, strongly pseudo irregular intuitionistic fuzzy graphs are introduced and some of their properties are discussed.

KEYWORDS: Pseudo degree in a fuzzy graph, Pseudo regular fuzzy graph, pseudo irregular fuzzy graph, highly pseudo irregular fuzzy graph, neighbourly pseudo irregular fuzzy graphs, strongly pseudo irregular fuzzy graph, intuitionistic fuzzy graph.

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I. INTRODUCTION

In this paper, we consider only finite, simple, connected graphs. We denote the vertex set and the edge set of a graph G by V(G) and E(G) respectively. The degree of a vertex v is the number of edges incident at v, and it is denoted by d(v). A graph G is regular if all its vertices have the same degree. The 2-degree of v [4] is the sum of the degrees of the vertices adjacent to v and it is denoted by t(v). We call \( \frac{d(v)}{\sqrt{n}} \), the average degree of v. A graph is called pseudo-regular if every vertex of G has equal average degree [3].

In 1965, Lofti A. Zadeh [16] introduced the concept of fuzzy subset of a set as method of representing the phenomena of uncertainty in real life situation. K.T.Attanassov [1] introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. K.T.Atanassov added a new component (which determines the degree of non-membership) in the definition of fuzzy set. The fuzzy sets give the degree of membership of an element in a given set (and the non-membership degree equals one minus the degree of membership), while intuitionistic fuzzy sets give both a degree of membership and a degree of non-membership which are more-or-less independent from each other, the only requirement is that the sum of these two degrees is not greater than one. Intuitionistic fuzzy sets have been applied in a wide variety of fields including computer science, engineering, mathematics, medicine, chemistry and economics [1, 2].


N.R.Santhi Maheswari and C.Sekar introduced (m, (c₁, c₂))-regular intuitionistic fuzzy graphs. N.R.Santhi Maheswari and C.Sekar introduced pseudo regular fuzzy graphs and discussed its properties [9]. N.R.Santhi Maheswari and M.Sudha introduced pseudo irregular fuzzy graphs and highly pseudo irregular fuzzy graphs and discussed their properties [9]. N.R.Santhi Maheswari and M. Rajeswari introduced strongly pseudo irregular fuzzy graphs and
discussed its properties [9]. N.R. Santhi Maheswari and S. Ravinarayanan introduced neighbourly pseudo irregular fuzzy graphs and discussed its properties [9].

These motivate us to introduce pseudo degree, total pseudo degree of a vertex in intuitionistic fuzzy graph and also we introduce pseudo regular intuitionistic fuzzy graphs, pseudo irregular intuitionistic fuzzy graphs, neighbourly pseudo irregular intuitionistic fuzzy graphs, highly pseudo irregular intuitionistic fuzzy graphs, strongly pseudo irregular intuitionistic fuzzy graphs and discussed some of their properties.

II. PRELIMINARIES

We present some known definitions and results for ready reference to go through the work presented in this paper.

**Definition 2.1.** A Fuzzy graph denoted by \( G : (\sigma, \mu) \) on the graph \( G^{\sigma} : (V,E) \) is a pair of functions (\( \sigma, \mu \)) where \( \sigma : V \rightarrow [0,1] \) is a fuzzy subset of a set \( V \) and \( \mu : VXV \rightarrow [0,1] \) is a symmetric fuzzy relation on \( \sigma \) such that for all \( u,v \) in \( V \) the relation \( \mu(u,v) = \sigma(u) \land \sigma(v) \) is satisfied[7].

**Definition 2.2.** Let \( G : (\sigma, \mu) \) be a fuzzy graph on \( G^{\sigma} : (V,E) \). The 2-degree of a vertex \( v \) in \( G \) is defined as the sum of degrees of the vertices adjacent to \( v \) and is denoted by \( t_{G}(v) \). That is, \( t_{G}(v) = \sum_{\gamma_{i}(v) \leq \mu_{i}(v)} \) where \( d_{G}(v) \) is the degree of the vertex \( u \) which is adjacent with the vertex \( v \).[13].

**Definition 2.3.** A pseudo (average) degree of a vertex \( v \) in fuzzy graph \( G \) is denoted by \( d_{G}(v) \) and is defined by \( d_{G}(v) = \frac{d}{d_{G}(v)} \mu_{G}(v) \), where \( d_{G}(v) \) is the number of edges incident at \( v \). The total pseudo degree of a vertex \( v \) in \( G \) is denoted by \( t_{G}(v) \) and is defined as \( t_{G}(v) = d_{G}(v) + \sigma(v) \) for all \( v \) in \( G \)[13].

**Definition 2.4.** Let \( G : (\sigma, \mu) \) be a fuzzy graph on \( G^{\sigma} : (V,E) \). If \( d_{G}(v) = k \), for all \( v \) in \( V \); then \( G \) is called \( k \)-pseudo regular fuzzy graph [13].

**Definition 2.5.** A fuzzy graph \( G \) is said to be pseudo irregular fuzzy graph if there exists a vertex which is adjacent to the vertices with distinct pseudo degrees [14].

**Definition 2.6.** A fuzzy graph \( G \) is said to be highly pseudo irregular fuzzy graph if each vertex is adjacent to the vertices having distinct pseudo degrees [14].

**Definition 2.7.** A fuzzy graph \( G \) is said to be a neighbourly pseudo irregular fuzzy graph if every pair of adjacent vertices of \( G \) have distinct pseudo degrees.

**Definition 2.8.** A fuzzy graph \( G \) is said to be a strongly pseudo irregular fuzzy graph if every pair of vertices of \( G \) have distinct pseudo degrees.

**Definition 2.9.** An intuitionistic fuzzy graph with underlying set \( V \) is defined to be a pair \( G = (V,E) \) where (i) \( V = \{v_{1}, v_{2}, v_{3},...,v_{n}\} \) such that \( \mu_{i} : V \rightarrow [0,1] \) and \( \gamma_{i} : V \rightarrow [0,1] \) denote the degree of membership and non-membership of the element \( v_{i} \) in \( V \), \((i=1,2,3,...,n)\), such that \( 0 \leq \mu_{i}(v_{i}) + \gamma_{i}(v_{i}) \leq 1 \)

(ii) \( E \subseteq V \), where \( \mu_{2} : VXV \rightarrow [0,1] \) and \( \gamma_{2} : VXV \rightarrow [0,1] \) are such that \( \mu_{2}(v_{i}, v_{j}) \leq min \{\mu_{i}(v_{i}), \mu_{j}(v_{j})\} \) and \( \gamma_{2}(v_{i}, v_{j}) \leq \mu_{i}(v_{i}) + \gamma_{i}(v_{i}) \) and \( 0 \leq \mu_{2}(v_{i}, v_{j}) + \gamma_{2}(v_{i}, v_{j}) \leq 1 \), for every \( (v_{i}, v_{j}) \in E \), \((i,j=1,2,...,n)\) [7].

**Definition 2.10.** Let \( G = (V,E) \) be an intuitionistic fuzzy graph on \( G^{\sigma} : (V,E) \). Then the degree of a vertex \( v \in G \) is defined by \( d(v) = (d_{G}(v), d_{G}(v)) \), where \( d_{G}(v) = \sum_{\gamma(v) \leq \mu(v)} \) and \( d_{G}(v) = \sum_{\gamma(v) \leq \mu(v)} \), for every \( v \in E \), \((i,j=1,2,...,n)\) [7].

**Definition 2.11.** Let \( G = (V,E) \) be an intuitionistic fuzzy graph on \( G^{\sigma} : (V,E) \). Then the total degree of a vertex \( v \in G \) is defined by \( td(v) = (td_{G}(v), td_{G}(v)) \), where \( td_{G}(v) = d_{G}(v) + \mu(v) \) and \( td_{G}(v) = d_{G}(v) + \gamma(v) \) [7].

III. PSEUDO DEGREE OF A VERTEX IN INTUITIONISTIC FUZZY GRAPH

**Definition 3.1.** Let \( G = (V,E) \) be an intuitionistic fuzzy graph on \( G^{\sigma} : (V,E) \). Then 2-degree of a vertex \( v \in G \) is defined by \( t(v) = (t_{G}(v), t_{G}(v)) \), where \( t_{G}(v) = \sum_{\gamma(v) \leq \mu(v)} \) and \( t_{G}(v) = \sum_{\gamma(v) \leq \mu(v)} \) the vertex \( u \) is the adjacent to the vertex \( v \).

**Definition 3.2.** Let \( G = (V,E) \) be an intuitionistic fuzzy graph on \( G^{\sigma} : (V,E) \). Then pseudo (average) degree of a vertex \( v \) in intuitionistic fuzzy graph \( G \) is defined by \( d_{G}(v) = (d_{G}(v), d_{G}(v)) \), where \( d_{G}(v) = \frac{t_{G}(v)}{d_{G}(v)} \), where \( d_{G}(v) \) is the number of edges incident at \( v \) and \( d_{G}(v) \) and is defined by \( d_{G}(v) = \frac{t_{G}(v)}{d_{G}(v)} \), where \( d_{G}(v) \) is the number of edges incident at \( v \).
Definition 3.3 Let $G = (V, E)$ be an intuitionistic fuzzy graph on $G(V, E)$. The total pseudo degree of a vertex $v$ in intuitionistic fuzzy graph $G$ is defined by $t_{d_v} (v) = (t_{d_{u,v}} (v), t_{d_{v,t}} (v))$, where $t_{d_{u,v}} (v) = d_{u,v} (v) + \mu_1 (v)$, for all $v$ in $G$ and $t_{d_{v,t}} (v) = d_{v,t} (v) + \gamma_1 (v)$, for all $v$ in $G$.

IV PSEUDO REGULAR INTUITIONISTIC FUZZY GRAPHS

Definition 4.1. Let $G = (V, E)$ be an intuitionistic fuzzy graph on $G(V, E)$. If $d_v (v) = (f_1, f_2)$, for all $v$ in $V$, then $G$ is called $(f_1, f_2)$ - pseudo regular intuitionistic fuzzy graph.

Definition 4.2. Let $G = (V, E)$ be an intuitionistic fuzzy graph on $G(V, E)$. If $td_v (v) = (k_1, k_2)$, for all $v$ in $V$, then $G$ is called $(k_1, k_2)$ - totally pseudo regular intuitionistic fuzzy graph.

Example 4.3. Consider an intuitionistic fuzzy graph on $G(V, E)$.

(iv) is proved. Hence (iv) and (ii) are equivalent.

\[
\begin{align*}
      d_{d_1}(u) &= 0.4; d_{d_2}(u) = 0.6 \\
      d_{d_1}(u) &= (0.4, 0.6), d_{d_2}(v) = (0.4, 0.6), d_{d_2}(w) = (0.4, 0.6), d_{d_2}(x) = (0.4, 0.6), d_{d_2}(y) = (0.4, 0.6). \\
      \text{So, all vertices have same pseudo degree. Hence } G \text{ is (0.4,0.6)-regular intuitionistic fuzzy graph. Also, } td_u(u) = (0.8, 1.1), td_v(v) = (0.8, 1.1), td_w(w) = (0.8, 1.1), td_x(x) = (0.8, 1.1), td_y(y) = (0.8, 1.1), \\ td_z(z) &= (0.8, 1.1). \text{ So, all vertices have same total pseudo degree. Hence } G \text{ is totally (0.8,1.1)-regular intuitionistic fuzzy graph.}
\end{align*}
\]

Remark 4.4. A pseudo regular intuitionistic fuzzy graph need not be a totally pseudo regular intuitionistic fuzzy graph.

Remark 4.5. A totally pseudo regular intuitionistic fuzzy graph need not be a pseudo regular intuitionistic fuzzy graph.

Theorem 4.6. Let $G : (V, E)$ be an intuitionistic fuzzy graph on $G^*(V, E)$. Then $A(v) = (\mu_1(v), \gamma_1(v))$, for all $v \in V$ is a constant function if and only if the following are equivalent.

(i) $G$ is a pseudo regular intuitionistic fuzzy graph.

(ii) $G$ is a totally pseudo regular intuitionistic fuzzy graph.

Proof. Assume that $A(v) = (\mu_1(v), \gamma_1(v))$ is a constant function. Let $A(v) = (\mu_1(v), \gamma_1(v)) = (c_1, c_2)$, for all $v \in V$. Suppose $G$ is a pseudo regular intuitionistic fuzzy graph. Then $d_v(u) = (f_1, f_2)$, for all $u \in V$. Now, $td_u(u) = d_v(u) + A(u) = (f_1, f_2) = (f_1, f_2) + (\gamma_1(v))$, for all $u \in V$. Hence $G$ is a totally pseudo regular intuitionistic fuzzy graph. Thus (i) $\implies$ (ii) is proved.

Suppose $G$ is a totally pseudo regular intuitionistic fuzzy graph. Then $td_u(u) = (k_1, k_2)$, for all $u \in V$ $\implies$ $d_v(u) + A(u) = (k_1, k_2)$, for all $u \in V$ $\implies$ $d_v(u) + (c_1, c_2) = (k_1, k_2)$, for all $u \in V$ $\implies$ $d_v(u) = (k_1 - c_1, k_2 - c_2)$, for all $u \in V$. Hence $G$ is a pseudo regular intuitionistic fuzzy graph. Thus (ii) $\implies$ (i) is proved. Hence (i) and (ii) are equivalent.
Conversely, suppose (i) and (ii) are equivalent. Let \( G \) be a pseudo regular intuitionistic fuzzy graph and a totally pseudo regular intuitionistic fuzzy graph. Then \( d_\epsilon(u) = (f_1, f_2) \) and \( t_d(u) = (k_1, k_2) \), for all \( u \in V \). Now \( t_d(u) = (k_1, k_2) \), for all \( u \in V \) \( \implies d_\epsilon(u) + A(u) = (k_1, k_2) \), for all \( u \in V \) \( \implies A(u) = (k_1 - f_1, k_2 - f_2) \), for all \( u \in V \). Hence \( A \) is a constant function.

**Theorem 4.7.** Let \( G : (A, B) \) be a intuitionistic fuzzy graph on \( G(V, E) \). If \( G \) is both pseudo regular and totally pseudo regular intuitionistic fuzzy graph then \( A \) is a constant function.

**Proof.** Assume that \( G \) is both pseudo regular and totally pseudo regular intuitionistic fuzzy graph. Then \( d_\epsilon(u) = (f_1, f_2) \) and \( t_d(u) = (k_1, k_2) \), for all \( u \in V \). Now, \( t_d(u) = (k_1, k_2) \implies d_\epsilon(u) + A(u) = (k_1, k_2) \implies (f_1, f_2) + A(u) = (k_1, k_2) \implies A(u) = (k_1 - f_1, k_2 - f_2) \) = constant. Hence \( A \) is a constant function.

**Theorem 4.8.** Let \( G : (A, B) \) be a intuitionistic fuzzy graph on \( G^*(V, E) \), a cycle of length \( n \). If \( B \) is a constant function, then \( G \) is a pseudo regular intuitionistic fuzzy graph.

**Proof.** If \( B \) is a constant function say \( B(uv) = (\mu_2(uv), \nu_2(uv)) = (c_1, c_2) \), for all \( u \in E \). Then \( t_d(u) = (2c_1, 2c_2) \), for all \( u \in E \). Hence \( G \) is a \( (2c_1, 2c_2) \)-pseudo regular intuitionistic fuzzy graph.

**Remark 4.9.** Converse of the above theorem 4.8 need not be true. Converse of the above theorem 4.8 need not be true.

**Result 4.10.** If \( v \) is a pendant vertex, then pseudo degree of \( v \) in intuitionistic fuzzy graph is the degree of the vertex in intuitionistic fuzzy graph which is adjacent with \( v \). (or) If \( v \) is a pendant vertex, then \( d_\epsilon(v) = d_\epsilon(u) \), where \( u \) is the vertex adjacent with \( v \).

**Theorem 4.11.** If \( G : (A, B) \) be a regular intuitionistic fuzzy graph on \( G^*(V, E) \), an \( r \)-regular graph, then \( d_\epsilon(v) = d_\epsilon(u) \), for all \( v \in G \).

**Proof.** Let \( G \) be a \( (k_1, k_2) \)-regular intuitionistic fuzzy graph on \( G(V, E) \), an \( r \)-regular graph. Then \( d_\epsilon(v) = (k_1, k_2) \), for all \( v \in G \) and \( d_\epsilon(v) = r \), for all \( v \in G \). So, \( t_{d_\epsilon}(v) = \sum d_{\epsilon(i)}(v) \), where each \( v_i \) for \( i = 1,2, \ldots r \) is adjacent with vertex \( v \implies t_{d_\epsilon}(v) = \sum d_{\epsilon(i)}(v) = r \). Also, \( d_\epsilon(v) = \sum d_{\epsilon(i)}(v) \implies d_\epsilon(v) = r(k_1 - k_2) \implies d_\epsilon(v) = r(k_1 - k_2) \implies d_\epsilon(v) = d_\epsilon(u) \).

**Theorem 4.12.** Let \( G : (A, B) \) be a intuitionistic fuzzy graph on \( G^*(V, E) \), an \( r \)-regular graph. Then \( G \) is a pseudo regular intuitionistic fuzzy graph if \( G \) is a regular intuitionistic fuzzy graph.

**Proof.** Let \( G \) be a \( (k_1, k_2) \)-regular intuitionistic fuzzy graph on \( G^*(V, E) \), an \( r \)-regular graph. \( \implies d_\epsilon(v) = d_\epsilon(u) \), for all \( v \in G \). \( \implies d_\epsilon(v) = (k_1, k_2) \), for all \( v \in G \) \( \implies \) all the vertices have same pseudo degree \( (k_1, k_2) \). Hence \( G \) is \( (k_1, k_2) \)-pseudo regular intuitionistic fuzzy graph.

V. PSEUDO IRREGULAR INTUITIONISTIC FUZZY GRAPH

**Definition 5.1.** Let \( G : (V, E) \) be an intuitionistic fuzzy graph on \( G^*(V, E) \). Then \( G \) is said to be a pseudo irregular intuitionistic fuzzy graph if there exits a vertex which is adjacent to the vertices with distinct pseudo degrees

**Definition 5.2.** Let \( G : (V, E) \) be an intuitionistic fuzzy graph on \( G^*(V, E) \).

Then \( G \) is said to be a pseudo totally irregular intuitionistic fuzzy graph if there exits a vertex which is adjacent to the vertices with distinct total pseudo degrees.

**Remark 5.3.** A pseudo irregular intuitionistic fuzzy graph need not be a pseudo totally irregular intuitionistic fuzzy graph.

**Remark 5.4.** A pseudo totally irregular intuitionistic fuzzy graph need not be a pseudo irregular intuitionistic fuzzy graph.

**Theorem 5.5.** Let \( G : (A, B) \) be a intuitionistic fuzzy graph on \( G^*(V, E) \). If \( A \) is a constant function , then the following conditions are equivalent

(i) \( G \) is a pseudo irregular intuitionistic fuzzy graph

(ii) \( G \) is a pseudo totally irregular intuitionistic fuzzy graph.

**Proof.** Assume that \( A(v) = (\mu_2(v), \nu_2(v)) \) is a constant function. Let \( A(v) = (c_1, c_2) \), for all \( v \in V \). Suppose \( G \) is a pseudo irregular intuitionistic fuzzy graph. Then at least one vertex of \( G \) which is adjacent to the vertices with distinct pseudo degree. Let \( u_1 \) and \( u_2 \) be the adjacent vertices of \( u_3 \) with distinct pseudo degrees \( (k_1, k_2) \) and \( (f_1, f_2) \) respectively. Then \( (k_1, k_2) \neq (f_1, f_2) \). Suppose \( G \) is not a pseudo totally irregular intuitionistic fuzzy graph. Then every vertex of \( G \) which is adjacent to the vertices with same pseudo total degree \( \implies t_{d_\epsilon}(u_1) = t_{d_\epsilon}(u_2) \implies d_\epsilon(u_1) + A(u_1) = d_\epsilon(u_2) + A(u_2) \implies (k_1, k_2) + (c_1, c_2) = (f_1, f_2) + (c_1, c_2) \implies (k_1 - f_1, k_2 - f_2) = 0 \implies \)
\( (k_1, k_2) = (f_1, f_2) \), which is a contradiction to \( (k_1, k_2) \neq (f_1, f_2) \). Hence \( G \) is pseudo totally irregular intuitionistic fuzzy graph. Thus (i) \( \implies \) (ii) is proved. Now, suppose \( G \) is a pseudo totally irregular intuitionistic fuzzy graph. Then atleast one vertex of \( G \) which is adjacent to the vertices with distinct pseudo total degree. Let \( u_1 \) and \( u_2 \) be the adjacent vertices of \( u_3 \) with distinct pseudo total degrees \( (g_1, g_2) \) and \( (h_1, h_2) \) respectively. Now, \( td_a(u_1) \neq td_a(u_2) \implies d_a(u_1) + A(u_1) \neq d_a(u_2) + A(u_2) \implies (g_1, g_2) \neq (h_1, h_2) \). Hence \( G \) is pseudo irregular intuitionistic fuzzy graph. Thus (ii) \( \implies \) (i) is proved. Hence (i) and (ii) are equivalent.

VI. HIGHLY PSEUDO IRREGULAR INTUITIONISTIC FUZZY GRAPH

\textbf{Definition 6.1.} \textit{Let } \( G = (V, E) \text{ be an intuitionistic fuzzy graph on } G^*(V, E) \text{. Then } G \text{ is said to be a highly pseudo irregular intuitionistic fuzzy graph if each vertex of } G \text{ is adjacent to the vertices with distinct pseudo degrees.}

\textbf{Definition 6.2.} \textit{Let } \( G = (V, E) \text{ be an intuitionistic fuzzy graph on } G^*(V, E) \text{. Then } G \text{ is said to be a highly pseudo totally irregular intuitionistic fuzzy graph if each vertex of } G \text{ is adjacent to the vertices with distinct total pseudo degrees.}

\textbf{Example 6.3.} \textit{Consider an intuitionistic fuzzy graph } \( G = (V, E) \text{ on } G^*(V,E) \).

\begin{figure}[h]
\centering
\includegraphics[width=0.7\textwidth]{example_graph.png}
\caption{Example Graph}
\end{figure}

\( d_{ap}(u) = 0.4; \ d_{ap}(v) = 0.6 \).
\( d_a(u) = (0.4, 0.6), \ d_a(v) = (0.45, 0.65), \ d_a(w) = (0.35, 0.6), \ d_a(x) = (0.4, 0.65), \ d_a(y) = (0.45, 0.7), \ d_a(z) = (0.35, 0.6) \). It is noted that each vertex in \( G \) is adjacent to the vertices with distinct pseudo degrees. Hence \( G \) is highly pseudo irregular intuitionistic fuzzy graph.

\textbf{Remark 6.3.} A highly pseudo irregular intuitionistic fuzzy graph need not be a highly pseudo totally irregular intuitionistic fuzzy graph.

\textbf{Remark 6.4.} A highly pseudo totally irregular intuitionistic fuzzy graph need not be a highly pseudo irregular intuitionistic fuzzy graph.

\textbf{Theorem 6.5.} \textit{Let } \( G = (A, B) \text{ be an intuitionistic fuzzy graph on } G^*(V, E) \text{. If } A \text{ is a constant function, then the following conditions are equivalent.}

(i) \( G \) is a highly pseudo irregular intuitionistic fuzzy graph.
(ii) \( G \) is a highly pseudo totally irregular intuitionistic fuzzy graph.

\textbf{Proof.} Proof is similar to the theorem 5.5.

\textbf{Remark 6.6.} The converse of the above theorem need not be true.

\textbf{Theorem 6.7.} Every highly pseudo irregular intuitionistic fuzzy graph is pseudo irregular intuitionistic fuzzy graph.

\textbf{Proof.} Let \( G \) be a pseudo highly irregular intuitionistic fuzzy graph. Then every vertex of \( G \) is adjacent to the vertices with distinct pseudo degrees \( \implies \) there is a vertex which is adjacent to the vertices with distinct pseudo degrees. Hence the graph \( G \) is pseudo irregular intuitionistic fuzzy graph.

\textbf{Remark 6.8.} The converse of the above theorem need not be true.
VII. NEIGHBOURLY PSEUDO IRREGULAR INTUITIONISTIC FUZZY GRAPH

Definition 7.1. Let $G = (V, E)$ be an intuitionistic fuzzy graph on $G^*(V, E)$. Then $G$ is said to be a neighbourly pseudo irregular intuitionistic fuzzy graph if adjacent vertices of $G$ have distinct pseudo degrees.

Definition 7.2. Let $G = (V, E)$ be an intuitionistic fuzzy graph on $G^*(V, E)$. Then $G$ is said to be a neighbourly pseudo totally irregular intuitionistic fuzzy graph if adjacent vertices of $G$ have distinct total pseudo degrees.

Example 7.3. Consider an intuitionistic fuzzy graph $G = (V, E)$ on $G(V, E)$ given in example 6.3. It is noted that adjacent vertices of $G$ have distinct pseudo degrees. Hence $G$ is neighbourly pseudo irregular intuitionistic fuzzy graph.

Remark 7.4. A neighbourly pseudo irregular intuitionistic fuzzy graph need not be a neighbourly pseudo totally irregular intuitionistic fuzzy graph.

Remark 7.5. A neighbourly pseudo totally irregular bipolar fuzzy need not be a neighbourly pseudo irregular intuitionistic fuzzy graph.

Theorem 7.6. Let $G : (A, B)$ be a intuitionistic fuzzy graph on $G^*(V, E)$ and $A$ is a constant function. Then the following are equivalent.

(i) $G$ is a neighbourly pseudo irregular intuitionistic fuzzy graph.

(ii) $G$ is a neighbourly pseudo totally irregular intuitionistic fuzzy graph.

Proof. Let $G : (A, B)$ be a connected intuitionistic fuzzy graph on $G^*(V, E)$ and $A$ is a constant function. Then $A(u) = (c_1, c_2)$ for all $u$ in $V$, where $c_1$ and $c_2$ are constant. Assume that $G$ is neighbourly pseudo irregular intuitionistic fuzzy graph. Then every pair of adjacent vertices in $G$ have distinct pseudo degrees. Let $u_1$ and $u_2$ be any two adjacent vertices with distinct pseudo degrees $(k_1, k_2)$ and $(f_1, f_2)$ respectively. Then $(k_1, k_2) \neq (f_1, f_2)$. Suppose $G$ is not a neighbourly pseudo totally irregular intuitionistic fuzzy graph. Then at least one pair of adjacent vertices in $G$ with same total pseudo degree $(tda(u_1) = tda(u_2)) \Rightarrow d(a(u_1) = A(u_1) = d(a(u_2) + A(u_2)) \Rightarrow (k_1, k_2) + (c_1, c_2) \Rightarrow (f_1, f_2)$, which is a contradiction. Hence $G$ is neighbourly pseudo totally irregular intuitionistic fuzzy graph. Thus (i) $\Rightarrow$ (ii) is proved.

Remark 7.7. If every pair of adjacent vertices in $G$ have distinct total pseudo degrees. Now, suppose $G$ is a neighbourly pseudo totally irregular intuitionistic fuzzy graph. Then every pair of adjacent vertices in $G$ have distinct total pseudo degrees. Let $u_1$ and $u_2$ be any pair of adjacent vertices in $G$ with distinct total pseudo degrees. Now, $tda(u_1) \neq tda(u_2) \Rightarrow d(a(u_1) \neq d(a(u_2) + A(u_2)) \Rightarrow d(a(u_1) \neq d(a(u_2) + (c_1, c_2)) \Rightarrow d(a(u_1) \neq d(a(u_2)).$ Hence $G$ is neighbourly pseudo irregular intuitionistic fuzzy graph. Thus (ii) $\Rightarrow$ (i) is proved.

VIII. STRONGLY PSEUDO IRREGULAR INTUITIONISTIC FUZZY GRAPH

Definition 8.1. Let $G = (V, E)$ be an intuitionistic fuzzy graph on $G^*(V, E)$. Then $G$ is said to be a strongly pseudo irregular intuitionistic fuzzy graph if every pair of vertices of $G$ have distinct pseudo degrees.

Definition 8.2. Let $G = (V, E)$ be an intuitionistic fuzzy graph on $G^*(V, E)$. Then $G$ is said to be a strongly pseudo totally irregular intuitionistic fuzzy graph if every pair of vertices of $G$ have distinct total pseudo degrees.

Example 8.3. For example consider $G = (V, E)$ be an intuitionistic fuzzy graph such that $G(V, E)$, a path on 6 vertices.
Remark 8.5. A strongly pseudo totally irregular intuitionistic fuzzy need not be a strongly pseudo irregular intuitionistic fuzzy graph.

IX. CONCLUSION

We have introduced pseudo regular intuitionistic fuzzy graphs, pseudo irregular intuitionistic fuzzy graphs, highly pseudo irregular intuitionistic fuzzy graphs, neighbourly pseudo irregular intuitionistic fuzzy graphs, strongly pseudo regular intuitionistic fuzzy graphs in this paper. We can discuss these regularity and irregularity in interval valued fuzzy graphs.

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REFERENCES


BIOGRAPHY

Dr.N.R.Santhi Maheswari is Head and Associate professor in Department of Mathematics, G.Venkatasswamy Naidu College, Kovilpatti,Tamil Nadu,India. Her area of research is regularity and irregularity concept in graph theory. She is doing one minor research project in fuzzy graph theory. She acted as a board of study member in Sadhakathullah appa college, Tirunelveli. She is a member in professional body ADMA. She published more than 35 papers in various esteemed peer reviewed international journals.