

A Biomechanical Approach for the Study of Two Phase Blood Flow through Venules during Tuberculosis

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ABSTRACT: A mathematical model for blood flow through veinules is discussed in this paper. Blood has been represented by a non Newtonian fluid obeying Herschel-Bulkley fluid. The variation of blood flow fluctuation and blood pressure during tuberculosis has been analyzed by numerical techniques. The role of hematocrit explicit in determination of pressure drop.

KEYWORDS: Hematocrit, Blood Flow, Herschel Bulkley non-Newtonian model, circulatory system.

I. INTRODUCTION

The lungs are pyramid shaped paired organs that are connected to the trachea by the right and left bronchi; on the inferior surface. The lungs are bordered by the diaphragm. The lungs are enclosed by the pleurae, which are attached to the mediastinum. The right lung is shorter and wider than the left lung and the left lung occupies a smaller volume than the right. Each lung is composed of smaller unit called lobes fissures separate these lobes from each other the right lung consist of three lobes: the superior, middle and inferior lobes. The left lung consists of two lobes: the superior and inferior lobes [1]. Tuberculosis is caused by infection with the bacterium mycobacterium tuberculosis. It is primarily transmitted by the respiratory route; individuals with active disease may infect others if the airborne particles they produce when they cough, talk or sing are inhaled by others. Once infected, an individual's enters a period of latency during which he exhibits no symptoms and is not infectious to others [2]. Lungs receive the whole amount of blood that is pumped out from right ventricle. Output of blood per minute is same in both right and left ventricle. It is about 5 liter. Thus, the lungs accommodate amount of blood, which is equal to amount of blood accommodated by all other part of body [3].

II. BASIC BIO FLUID EQUATION FOR TWO PHASE BLOOD FLOW

Blood is mixed fluid. Mainly there are two phases in blood. The first phase is plasma, while the other is that of blood cells are enclosed with semi permeable membrane whose density is greater than that of plasma. These blood cells are uniformly distributed in plasma. Thus blood can be considered as a homogenous mixture of two phases [4].

Equation of continuity: The flow of blood is affected by the presence of blood cells. This effect is directly proportional to the volume occupied by blood cells [5]. The mass ratio of blood cells to plasma r is defined by-

$$r = \frac{X\rho_c}{(1-X)\rho_p} \dots\dots\dots(1)$$

Where ρ_c = density of blood cells

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ρ_p = density of blood plasma.

X = Volume portion covered by blood cells.

$1 - X$ = Volume portion covered by plasma.

The both phase of blood that means blood cells and plasma move with the common velocity. Hence equation of continuity for two phases according to the principle of conservation of mass defined as follows-

$$\frac{\partial(X\rho_c)}{\partial t} + (X\rho_c v^i)_{,j} = 0 \dots\dots\dots (2)$$

$$\frac{\partial(1-X)\rho_p}{\partial z} + ((1-X)\rho_p v^i)_{,j} = 0 \dots\dots\dots (3)$$

Where v is the common velocity of two phase blood cells and plasma.

If we define the uniform density of the blood ρ_m as follow-

$$\frac{1+r}{\rho_m} = \frac{r}{\rho_c} + \frac{1}{\rho_p} \dots\dots\dots (4)$$

Above equation can be combined together as follows,

$$\frac{\partial\rho_m}{\partial t} + (\rho_m v^i)_{,j} = 0. \dots\dots\dots (5)$$

III. EQUATION OF MOTION FOR TWO PHASE BLOOD FLOW

The hydro dynamical pressure p between the two phases of blood can be supposed to uniform because the both phases are always in equilibrium state in blood [6]. Taking viscosity coefficient of blood cells to be η_c and applying the principle of conservation of momentum, we get the equation of motion for the phase of blood cells as follows:

$$X\rho_c \frac{\partial v^i}{\partial t} + (X\rho_c v^i)_{,j} = -Xp_{,j} g^{ij} + X\eta_c (g^{jk} v_{,k}^i)_{,j} \dots\dots\dots (6)$$

The equation of motion for plasma will be as follows:

$$(1-X)\rho_p \frac{\partial v^i}{\partial t} + \{(1-X)\rho_p v^i\}_{,j} = -(1-X)p_{,j} g^{ij} + (1-X)\eta_c (g^{jk} v_{,k}^i)_{,j} \dots\dots\dots (7)$$

From above equations, we have

$$\rho_m \frac{\partial v^i}{\partial t} + (\rho_m v^i)_{,j} = -p_{,j} + \eta_m (g^{jk} v_{,k}^i)_{,j} \dots\dots\dots (8)$$

Where $\eta_m = X\eta_c + (1-X)\eta_p$ is the viscosity coefficient of blood as a mixture of two phases.

IV. MATHEMATICAL MODELING

As the velocity of blood flow decreases, the viscosity of blood increases. The velocity of blood decreases successively. The Herschel Bulkley law holds good on the two phase blood flow through veinules and whose constitutive equation is as follows:

$$T' = \eta_m e^n + T_p (T' > T_p) \text{ and } e = 0 (T' < T_p), \text{ where } T_p \text{ is the yield stress.}$$

When strain rate $e = 0 (T' < T_p)$ a core region is formed which flows just like a plug. Let the radius of the plug be r_p , The stress acting on the surface of plug will be T_p . Equating the forces acting on the plug, we get,

$$P\pi r_p^2 = T_p 2\pi r_p$$

$$\text{Implies that } r_p = 2 \frac{T_p}{P} \dots\dots\dots (9)$$

η_p = viscosity of plasma layer

The constitutive equation of the blood vessel is

$$T' = \eta_m e^n + T_p$$

$$T' - T_p = \eta_m e^n = T_e$$

Where,

η_m = viscosity of core layer

η_c = viscosity of blood cells

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$$\rho_m = X\rho_c + (1 - X)\rho_p$$

The blood flow in capillary is symmetric w. r t. axis.

Hence,

$$v_\theta = 0, v_z, v_r \text{ and } p \text{ do not depend upon } \theta$$

Since the one component of velocity which is along axis is effective

We have,

$$v_\theta = 0, v_z = v, v_r = 0$$

Since flow is steady,

$$\frac{\partial p}{\partial t} = \frac{\partial v_r}{\partial t} = \frac{\partial v_\theta}{\partial t} = \frac{\partial v_z}{\partial t} = 0$$

$$\frac{\partial v_z}{\partial z} = 0$$

$$v_z = v(r)$$

rComponent

$$\rho_m(0) = \frac{\partial \rho}{\partial r} + \eta_m(0)$$

$$\frac{\partial p}{\partial r} = 0$$

$$p = p(z)$$

θComponent

$$\rho_m(0) = 0 + \eta_m(0)$$

$$0 = 0$$

z Component

$$\rho_m v \frac{\partial v}{\partial t} = -\frac{\partial p}{\partial z} + \eta_m \left[\frac{1}{r} \frac{\partial}{\partial r} \left\{ r \frac{\partial v_z}{\partial r} \right\} + \frac{\partial v}{\partial z} \right] \dots \dots \dots (10)$$

And pressure p depends on z

$$\text{Pressure gradient } \frac{dp}{dz} = p, \quad r \left(\frac{dv}{dz} \right)^n = -\frac{pr^2}{2\eta_m} + A$$

Using boundary condition- at $r = 0, v = v_0$ then $A = 0$

$$-\frac{dv}{dr} = \left(\frac{pr}{2\eta_m} \right)^{\frac{1}{n}}$$

$$-\frac{dv}{dr} = \left(\frac{pr - pr_p}{2\eta_m} \right)^{\frac{1}{n}}$$

$$\frac{dv}{dr} = -\left(\frac{p}{2\eta_m} \right)^{\frac{1}{n}} (r - r_p)^{\frac{1}{n}} \dots \dots \dots (11)$$

Integrating equation (11) under no slip boundary condition $V = 0$ at $r = R$ we get

$$v = \left(\frac{p}{2\eta_m} \right)^{\frac{1}{n}} \frac{n}{n+1} [R - r_p]^{\frac{1}{n}+1} - (r - r_p)^{\frac{1}{n}+1} \dots \dots \dots (12)$$

This is the formula for velocity of blood flow in veinules.

Putting $r = r_p$ to get the velocity v_p of plug flow -

$$v_p = \frac{n}{n+1} \left(\frac{p}{2\eta_m} \right)^{\frac{1}{n}} (R - r_p)^{\frac{1}{n}+1} \dots \dots \dots (13)$$

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V. RESULT & DISCUSSION (BIO-PHYSICAL INTERPRETATION)

The flow flux of two phased blood flow in venules is given by the following formula

$$\begin{aligned}
 Q &= \int_0^{r_p} 2\pi r v_p + \int_{r_p}^R 2\pi r v dr \\
 &= \int_0^{r_p} 2\pi r \frac{n}{n+1} \left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}} (R-r_p)^{\frac{1}{n}+1} dr + \int_0^{r_p} 2\pi r \frac{n}{n+1} \left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}} \left[(R-r_p)^{\frac{1}{n}+1} - (R-r_p)^{\frac{1}{n}+1} \right] dr \\
 &= \frac{2\pi n}{(n+1)} \left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}} (R-r_p)^{\frac{1}{n}+1} \left[\frac{r^2}{2} \right]_0^{r_p} + \frac{2\pi n}{(n+1)} \left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}} \left[\frac{r^2}{2} (R-r_p)^{\frac{1}{n}+1} - \frac{r(r-r_p)^{\frac{1}{n}}}{\frac{1}{n}+1} + \frac{(r-r_p)^{\frac{1}{n}+3}}{\left(\frac{1}{n}+1\right)\left(\frac{1}{n}+3\right)} \right]_{r_p}^R \\
 &= \frac{\pi n}{(n+1)} \left(\frac{P}{2\eta_m}\right)^{\frac{1}{n}} R^{\frac{1}{n}+3} \left[\frac{r_p^2}{r^2} \left(1 - \frac{r_p^2}{R}\right)^{\frac{1}{n}+1} + \left(1 + \frac{r_p}{R}\right)^{\frac{1}{n}+1} \left(1 - \frac{r_p}{R}\right)^{\frac{1}{n}+2} - \frac{2\left(1 - \frac{r_p}{R}\right)^{\frac{1}{n}+2}}{\left(\frac{1}{n}+2\right)} + \frac{2\left(1 - \frac{r_p}{R}\right)^{\frac{1}{n}+3}}{\left(\frac{1}{n}+2\right)\left(\frac{1}{n}+3\right)} \right] \dots \dots \dots (14)
 \end{aligned}$$

Substituting the value of r_p and R in equation (14), we get

$$Q = \pi \left(\frac{2P}{6\eta_m}\right)^{\frac{1}{n}} \left(\frac{2}{27}\right) \left[\frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1} \right]$$

$$\frac{27 \times Q}{2\pi} = \left(\frac{P}{3\eta_m}\right)^{\frac{1}{n}} \left[\frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1} \right] \dots \dots \dots (15)$$

Let $A = \left[\frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1} \right] \Rightarrow \frac{P}{3\eta_m} = \left(\frac{27 \times Q}{2\pi A}\right)^n \Rightarrow P = \left(\frac{27 \times Q}{2\pi A}\right)^n \cdot 3\eta_m$

$$\begin{aligned}
 P &= -\frac{dp}{dz} \\
 \int_{P_f}^{P_i} dP &= - \int_{Z_f}^{Z_i} \left(\frac{27 \times Q}{2\pi A}\right)^n \cdot 3\eta_m dz
 \end{aligned}$$

$$P_f - P_i = \left(\frac{27 \times Q}{2\pi A}\right)^n \cdot 3\eta_m \cdot (Z_f - Z_i) \dots \dots \dots (16)$$

Now,

$$\frac{27 \times Q}{2\pi A} = \left(\frac{P_f - P_i}{(Z_f - Z_i) 3\eta_m}\right)^{1/n}$$

$$Q = \frac{2\pi A}{27} \left(\frac{p_f - p_i}{3\eta_m(z_f - z_i)}\right)^{\frac{1}{n}} \dots \dots \dots (17)$$

Where $p_f - p_i$ = pressure drop and $z_f - z_i$ = length of venule.

Patient name: -Vinita

Date	HB	Blood pressure	Hematocrit	BP(in Pascal)
15/04/2015	8.9	0.025188679	150/90	-3106.413333
05/07/2015	8.7	0.024622642	130/70	-2514.715556
25/09/2015	8.6	0.024339623	110/70	-2366.791111
02/11/2015	8.3	0.023490566	120/70	-2440.75333
30/01/2016	8.0	0.022641509	140/90	-3032.451111

Table 1

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By above table we have, $H = 0.023490566$ and $\Delta P = 2440.75333$ Pascal second.

$$\eta_m = 0.0271, \eta_p = 0.0013 \quad [7]$$

Using relation $\eta_m = \eta_c X + \eta_p (1 - X)$

$$0.0271 = \eta_c (0.00023490566) + 0.0013(0.999765094)$$

$$\eta_c = 109.8326255 \text{ Pascal second}$$

Again putting these values in above relation, we get

$$\eta_m = 1.098326255H + 0.001299695$$

The flow flux of blood flow in veinules:

$$Q = \pi \left(\frac{2p}{6\eta_m} \right)^{\frac{1}{n}} \frac{2}{27} \left[\frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1} \right]$$

$$(Z_f - Z_i) = 0.0015 [8]$$

$$\eta_m = 0.0271, \quad \Delta P = 2440.75333,$$

$$Q = 0.0071 \quad [9]$$

Solve by numerical methods (Trial and Error method) we find out the value of n

$$\frac{27 \times 0.0071}{6.28} = \left[\frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1} \right] \left(\frac{2440.75333}{0.0015 \times 0.0813} \right)^{1/n}$$

$$0.03045669 = \left[\frac{26n^3 + 33n^2 + 9n}{6n^3 + 11n^2 + 6n + 1} \right] (20014377.45)^{1/n}$$

$$n = -3.26578$$

Now ΔP is-

$$\Delta P = \left(\frac{27 \times 0.00708333}{6.28 \times 5.240278376} \right)^{-3.26578} \times 3\eta_m \times (0.0015)$$

$$\Delta P = 90079.22017 (1.098326255H + 0.001299695)$$

$$\Delta P = 98936.37252H + 117.0754781$$

Date	Hematocrit(g/dl)	Pressure drop(Pascal)
15/04/2015	0.025188679	2609.152007
05/07/2015	0.024622642	2553.150359
25/09/2015	0.024339623	2525.149486
02/11/2015	0.023490566	2441.146867
30/01/2016	0.022641509	2357.144247

Table 2

Table 2 shows change in blood pressure drop with increase in hemoglobin. The graph of above table is given in figure-1.

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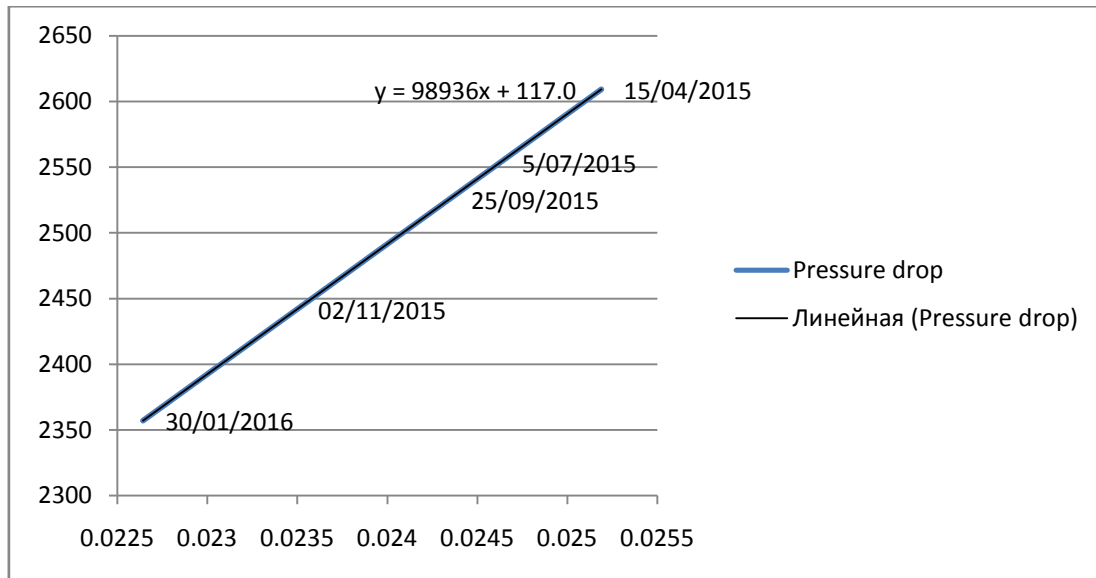


Figure-1

VI. CONCLUSION

1. The two phase model is $\eta_m = X\eta_c + (1-X)\eta_p$ where $X = \frac{H}{100}$ and H is Hematocrit which is 3 times of Hemoglobin. Explained the role of Hematocrit in determination of blood pressure drop.
2. The above graph shows that when hematocrit increases then pressure drop increased with slope 98936.

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