

| e-ISSN: 2319-8753, p-ISSN: 2320-6710| <u>www.ijirset.com</u> | Impact Factor: 7.512|

|| Volume 9, Issue 6, June 2020 ||

Investigation of Rake Contact and Friction Behavior in Turning Operation

K.Srinivasa Rao¹, G.Bala Murali Krishna²

Associate Professor, Department of Mechanical Engineering, Malla Reddy Engineering College (A), Secunderabad,

Telangana, India¹

Research Scholar, Mechanical Engineering, Andhra University, Visakhapatnam, India²

ABSTRACT: The friction between the specimen and the tool bit is one of the key objects in machining research. It involves three deformation zones. The primary shear zone, i.e. the shear plane, is responsible for the chip formation whereas in the secondary shear zone on the rake face the work piece and tool are in a complex state of contact. The third region is responsible for the ploughing and flank contact. Although there are numerous models proposed involving, analytical, semi analytical, and numerical (mostly FEM based methods, two important inputs for these models are the material model parameters and the friction coefficient between the tool and the work piece material. In this thesis, the friction behaviour in metal cutting operations is analysed using a thermo mechanical cutting process model that represents the contact on the rake face by sticking and sliding regions. The relationship between the sliding and the overall, apparent, friction coefficients are analysed quantitatively. The sliding friction coefficient is identified for different work piece—tool couples using cutting and non-cutting tests. In addition, the effect of the total, sticking and sliding contact lengths on the cutting mechanics is investigated. The effects of cutting conditions on the friction coefficients are analysed

KEYWORDS: work piece, ploughing, sliding friction, FEM.

I.INTRODUCTION

Machining process such as turning, milling, boring and drilling are among others the most important process for discrete part manufacturing. Researchers have been studying machining processes for more than a century to gain better understanding and develop more advanced manufacturing technology. The study of turning has lasted more than a century, but it still attracts a large amount of research effort. This is because turning is not only most frequently used machining operation in the modern manufacturing industry, but also because it is typical single-point machining operation. Other machining operations, such as milling, drilling and boring are multiple-point machining operation that can be investigated based on the combinations of single-point machining operation. Thus, the study of turning can contribute greatly to the knowledge of metal cutting principles and machining practice.

II.TURNING PROCESS

Turning is the process whereby a single point cutting tool is parallel to the surface. It can be done manually, in a traditional form of lathe, which frequently requires continuous supervision by the operator, or by using a computer controlled and automated lathe which does not. This type of machine tool is referred to as having computer numerical control, better known as CNC, and is commonly used with many other types of machine tool besides the lathe.

Although now quite rare, early lathes could even be used to produce complex geometric figures, even the platonic solids although until the advent of CNC it had become unusual to use one for this purpose for the last three quarters of the twentieth century. It is said that the lathe is the only machine tool that can reproduce itself.

When turning, a piece of material (wood, metal, plastic even stone) is rotated and a cutting tool is traversed along two axes of motion to produce precise diameters and depths. Turning can be either on the outside of the cylinder or on the inside (also known as boring) to produce tubular components to various geometries.

The turning processes are typically carried out on a lathe, considered to be the oldest machine tools, and can be of four different types such as straight turning, taper turning, profiling g or external grooving. These types of turning processes can produce various shapes of materials such as straight, conical, curved, or grooved work piece. In general, turning uses simple single-point cutting tools. Each group of work piece materials has an optimum set of tools angles which have been developed through the years. The bits of waste metal from turning operations are known as chips.

Turning produces solids of revolution which can be tightly tolerance because of the specialized nature of the operation.



| e-ISSN: 2319-8753, p-ISSN: 2320-6710| <u>www.ijirset.com</u> | Impact Factor: 7.512|

|| Volume 9, Issue 6, June 2020 ||

Turning is performed on a machine called a lathe in which the tool is stationary and the part is rotated. The figure below illustrates an engine lathe. Lathes are designed solely for turning operations, so that precise control of the cutting results in tight tolerances. The work piece is mounted on the chuck, which rotates relative to the stationary tool. Turning refers to cutting as shown below:



The term "facing" is used to describe removal of material from the flat end of a cylindrical part, as shown below. Facing is often used to improve the finish of surfaces that have been parted.



III.STANDARD TOOL POST TOOL

The tool inserted in the tool holder is shown below:



IV.SINGLE-POINT CUTTING TOOL VARIETY

There are many types of cutting tools for different operations. Below is shown a few of the variety, here shown with a tool holder adapter that fits into a larger tool post fixture.



| e-ISSN: 2319-8753, p-ISSN: 2320-6710| <u>www.ijirset.com</u> | Impact Factor: 7.512|

|| Volume 9, Issue 6, June 2020 ||



V.PARTING TOOL

The illustration below shows how a parting tool is fixture and used. Parting is important at the end of a turning process in order to separate the part from the raw material. Parting must be carried out slowly and carefully since the tool is quite long and is prone to chattering. Parting is not very accurate, and a finishing cut must often be undertaken after parting if the parted surface is to be accurate.

VI.FRICTION CHARACTERISTICS AT CHIP- TOOL INTERFACE

Friction in metal cutting is difficult to determine. The contact regions and the friction parameters between the chip and the tool are influenced by factors such as cutting speed, feed rate, rake angle, etc., mainly because of the very high normal pressure at the surface. The prevalent conditions at the tool-chip interface constrain the use of the empirical values of the coefficient of friction found from ordinary sliding test conditions.

While numerous works have been reported to quantitatively explain the variable and high values of friction at the rake face, none has provided reliable quantitative predictive models that are devoid of experimental testing. In conventional machining at low cutting speeds, the friction mechanism is mostly effective at the tool flank face. However, in high speed machining, a tremendous increase in the chip velocity, the chip-tool friction contact and the temperatures are encountered at the tool rake face. As a result, the increasing sliding velocity and frictional stress cause significant wear on the tool rake face. Therefore, the rate of the tool wear heavily depends on the frictional conditions in high speed machining.

VII.METHODOLOGY

The friction due to the contact between the work piece and the cutting tool is one of the key subjects in machining research. It is well known that cutting involves three deformation zones. The primary shear zone, i.e. the shear plane, is responsible for the chip formation whereas in the secondary shear zone on the rake face the work piece and tool are in a complex state of contact. The third region, on the other hand, is responsible for sloughing and flank contact. Numerous models have been proposed involving, analytical within and thick shear zone approaches, semi analytical, and numerical (mostly FEM based) methods. Two important inputs for these models are the material model parameters and the friction coefficient between the tool and the work piece material. These two inputs can be considered to be independent of the cutting mechanics as they are related to the mechanical and physical properties of the materials. Identification of both properties is very critical for accurate modeling of the machining processes. The focus of this paper is on the friction characteristics in metal cutting operations. Being a common topic in mechanics, friction has been extensively studied in basic sciences. However, machining researchers have also paid special attention to friction due to its importance in cutting processes. The early studies on the subject concluded that there is a direct relationship between the shear angle and the friction. Using minimum energy principle for the continuous type chips, Merchant concluded a relationship between the shear angle and the rake face friction. Oxley, on the other hand, included the strain hardening effect in the slip-line model. As another approach, the semi-analytical method known as the mechanics of cutting relates the apparent friction coefficient to the rake angle, feed rate and the cutting speed, and uses them in force predictions. However, this approach may take longer testing time since high number of tests must be carried out depending on the ranges. Similar mechanistic models do not provide much insight about the friction behavior of the workpiece and tool couples. In FEM modeling the researchers mostly focused on the material behavior



| e-ISSN: 2319-8753, p-ISSN: 2320-6710| <u>www.ijirset.com</u> | Impact Factor: 7.512|

|| Volume 9, Issue 6, June 2020 ||

and chip formation and lacks the importance of the friction conditions on the rake face. Lee and Shaffer obtained a similar relationship by applying slip-line field theory to the orthogonal cutting. The solutions presented in these studies have assumptions which do not accurately represent the friction behavior of the process. Based on the experimental observations, however, it has been well accepted that the overall (apparent) friction coefficient on the rake face decreases with the increasing rake angle. On the other hand, the effects of other parameters such as cutting speed or feed rate were not known that well. Eventually, Zorev approached the problem by observing the normal pressure and shear stress distributions on the rake face, and proposed distribution forms for them. Basically, Zorev proposed that the material exiting the primary shear zone reaches the rake face with such a high normal pressure that there is a sticking (Coulomb) friction away from the tool tip on the rake face. This behavior is also verified by numerous researches in later studies mostly by split-tool experiments measuring the normal pressure and shear stress distributions on the rake face.

In a later study, Fang concludes that the tool-chip friction decreases with increasing absolute value of negative rake angle and with increasing cutting speed, by applying slip-line field analysis. Friction between two contacting bodies has several depen- dencies such as the material pair, temperature, pressure, and speed depending on the application ranges. For instance, in a recent study, Philippon et al. conducted several experiments in an original test setup in order to investigate the sliding friction behavior at high sliding velocities, and concluded that the sliding coefficient of friction strongly depends on the speed and the pressure. In a different study Moufki et al. proposed an orthogonal cutting model which relates the sliding friction coefficient to the mean temperature on the rake face. A recent study proposed an analytical orthogonal cutting model, which considers the sticking and sliding friction regions on the rake face and uses Johnson-Cook (JC) constitutive model. The JC constitutive relation is relatively simple, one dimensional model that accounts for the effects of strain, strain rate, and thermal softening on flow stress utilizing von Mises yield criterion. It describes the material hardening behavior based on the well-known power-law function. Also, this empirical relation is relatively simple to calibrate for a given material. It is relatively easy to implement into computer codes, inexpensive to use, and produces reasonably accurate predictions for a range of materials if the loading conditions do not exceed those used during the parameter identification tests. In the model, the sliding friction coefficient is related to the friction speed, and calibrated using a small number of orthogonal cutting tests, where the JC parameters are also calibrated for the given cutting speed range. The proposed model has both calibration and prediction capabilities. Accurate representation of contact behavior on the rake face is critical for the thorough understanding and modeling of the metal cutting operations. In this regard, quantitative analysis of the friction behavior in metal cutting is important for better understanding of the nature of the process.

The identification of the sliding friction coefficient between the work piece-tool couple and the relation of the sliding friction coefficient to the apparent one is critical for process modeling. The contact lengths which are basically the physical representations of the friction behavior on the rake face, must also be modeled and analyzed. Based on these, the objective of this study is to further investigate the friction behavior in metal cutting opera- tions. The focus of the study in this paper is on the mechanical behavior rather than micro-structural investigation. For this purpose, the rake contact model presented in is used throughout this paper.

VIII.THERMO MECHANICAL DUAL-ZONE MODEL

The cutting model which is used in this study is briefly presented. In this model, the contact between the chip and the tool on the rake face is represented by a dual-zone approach. Basically, the contact is divided in to the sticking and a sliding friction region, which was originally proposed by Zorev (see Fig). In the first region, the contact condition is plastic due to the high normal pressure exerted on the tool, whereas in this cond region the contact is elastic which can be represented by the sliding friction. There are two different friction coefficients that are defined on the rake contact. The apparent friction coefficient ma is due to the total cutting forces acting on the rake face. The sliding friction coefficient m, on the other hand, is only due to the forces acting on the sliding region on the rake face. The normal pressure distribution on the rake face is needed for the formulation of forces. The following distribution is selected as it is used and verified by several studies

$$P(x) = P_0 \left(1 - \frac{x}{\ell_c} \right)^{\zeta}$$

where ℓ_{c} is the total contact



| e-ISSN: 2319-8753, p-ISSN: 2320-6710| <u>www.ijirset.com</u> | Impact Factor: 7.512|

|| Volume 9, Issue 6, June 2020 ||

Where x the distance on the rake face from the tool tip, and ζ an exponential constant which represents the distribution of the pressure, and is selected as 3 in the current study based on the analysis of the split-tool test results. that the shear stress on the rake face is equal to the shear yield stress of the material along the sticking region with length In addition, the shear stress in the sliding region is equal to the product of the sliding friction coefficient and the normal stress(P), according to the Coulomb friction law. Therefore, the mathematical representation of the shear stress distribution on the rake face can be defined as follows:

$$\tau = \begin{cases} \tau_1 & x \le \ell_p \\ \mu P & \ell_p \le x \le \ell_0 \end{cases}$$

The three important outputs of the model is the total contact length, the sticking length, and the relationship between the sliding and sticking friction coefficients

$$\begin{split} \ell_{c} &= f \frac{\zeta + 2}{2} \frac{\sin(\phi_{n} + \lambda_{a} - \alpha_{n})}{\sin \phi_{n} \cos \lambda_{a} \cos \eta_{c}} \\ \ell_{p} &= \ell_{c} \left(1 - \left(\frac{\tau_{1}}{P_{0}\mu}\right)^{\frac{1}{\zeta}} \right) \\ \mu_{a} &= \tan(\lambda_{a}) = \frac{\tau_{1}}{P_{0}} \left(1 + \varsigma \left(1 - \left(\frac{\tau_{1}}{P_{0}\mu}\right)^{1/\varsigma} \right) \right) \\ \text{where} \\ \frac{\tau_{1}}{P_{0}} &= \frac{\ell_{c}}{f(\zeta + 1)} \frac{\sin \phi_{n} \cos \eta_{c} \cos(\phi_{n} + \lambda_{a} - \alpha_{n})}{\cos \eta_{s} \cos \lambda_{a}} \end{split}$$

In the above equations, f is the uncut chip thickness, ϕ_n the normal shear angle, the inclination angle, η_s the chip flow angle, The proposed contact model can be used with any primary shear zone model provided that the shear stress at the exit of the shear zone is calculated accurately. For the analysis and predictions conducted in this study, the thermo mechanical primary shear zone model proposed by is used. The main assumption in modelling the primary shear zone is that the shear plane has a constant thickness, and that no plastic deformation occurs before and after the shear plane up to the sticking region on the rake face. In addition, it is assumed that the material behaviour can be represented by the JC constitutive model. The material entering the primary shear zone sustains an initial shear stress value, and by the exit of the primary shear zone the shear stress evolves to τ_1 which is different from the initial value when inertia effects are important. Assuming a constant thickness of the shear zone and a uniform pressure distribution, the stress at the entrance of the shear band can be calculated iteratively. From the equations of motion for a steady state solution and a continuous type chip, the shear stress at the exit of the shear plane τ_1 can be calculated and used in the rake contact analysis as presented above. Other outputs of the model are the cutting forces, the shear stress, the normal pressure distribution on the rake face, and the shear angle.

IX.IDENTIFICATION OF SLIDING FRICTION COEFFICIENT

In the foregoing analysis, there are two important inputs for the model: the material model parameters and the friction coefficient. Although the main focus of this study is on the friction behavior, choosing the correct material model parameters is another subject. However, it can be briefly stated that the dual-zone model can also be used to calibrate the material model coefficients. As for the friction, it should be noted that two friction coefficients are defined: the apparent and the sliding. If one of them is known the other can be calculated by using Eq. The dual-zone model can be used to calibrate these two inputs as well. The orthogonal tube cutting tests are used for calibration purposes as discussed in the following section. In addition, non-cutting friction tests were also conducted to compare with the sliding friction coefficients identified from the cutting tests, which is also discussed in the next section.



| e-ISSN: 2319-8753, p-ISSN: 2320-6710| <u>www.ijirset.com</u> | Impact Factor: 7.512|

|| Volume 9, Issue 6, June 2020 ||

X.IDENTIFICATION OF FRICTION FROM ORTHOGONAL CUTTING TESTS

The proposed dual-zone model can be used to calculate the sliding friction coefficient between the tool and the work piece material if the apparent friction coefficient is provided together with the cutting parameters and the material model coefficients. The calibration of the sliding friction coefficient between the tool and the work piece material is done by orthogonal tube cutting tests. The tests are conducted on a lathe, where the orthogonal cutting conditions are satisfied by selecting inserts without inclination angles. Work piece geometry is a tube in order to avoid the nose radius contact with the material. The test setup also involves a table type dynamometer and a DAQ system in order to collect the cutting force data. After each experiment the cut chip thickness is measured to identify the shear angle. The tests were conducted at different cutting speeds and feed rates. By the mechanics of cutting approach the apparent friction coefficient on the rake face between the tool and the work piece is calculated as follows:

$$\mu_a = \tan(\alpha_n + \tan^{-1}(F_f/F_t))$$

Where F_f and F_t are the measured feed and tangential(cutting) forces, respectively. The sliding friction coefficient μ is calculated by the above equation and a function relates to the chip velocity on the sliding region is obtained by data fitting techniques. The chip velocity along the sliding contact length is assumed to be uniform and is calculated as

$$V_{chip} = V \frac{\sin \phi}{\cos(\phi - \alpha)}$$

Where V is the cutting speed. Although the relationship between the apparent and the sliding friction coefficients is given by Eq,

The following mathematical representations for the friction coefficients can be deduced:

$$\mu = A/(A+B)$$
$$\mu_a = (A+C)/(A+B+C+D)$$

Where A, B, C, and D are the areas under the curves. Since the area difference between the normal and shear stress under the sticking region is higher than the one in the sliding region, the value of μ_0 must always be smaller than μ . This situation is always true for cases where the sliding friction is smaller than 1.

XI.IDENTIFICATION OF FRICTION FROM NON- CUTTING TESTS

In order to obtain the sliding coefficient of friction between the work piece and the cutting tool materials independently from the cutting tests and the model, a non- cutting friction test setup is prepared. This modified pinon- disc setup is built on a manual lathe, and involves a dynamometer in order to measure the normal and the frictional forces, and a DAQ setup in order to collect the data. The contact between the tool and work piece is realized by moving the tool with a fine slider in order to make the initial contact smoother. The sliding friction speed is controlled by the rotational speed of the work material and the radial position of the carbide rod with respect to the center of rotation.

The sliding coefficient of friction is calculated using the mean values of the friction (F_{friction}) and normal (F_{normal}) forces

μ

Note again that the non-cutting friction tests are performed in order to compare the results with the identified friction coefficients from the cutting model, and they are not needed in regular identification procedure.



The non-cutting friction test setup,



| e-ISSN: 2319-8753, p-ISSN: 2320-6710| <u>www.ijirset.com</u> | Impact Factor: 7.512|

|| Volume 9, Issue 6, June 2020 ||

XII.EXPERIMENTAL PHOTOS





XIII.RESULT ANALYSIS

The experimental results and the analysis of the friction behavior for work piece material Mild Steel and cutting tool materials Uncoated Carbide and Coated Carbide are presented.

CUTTING TOOL MATERIALS – Uncoated Carbide and Coated Carbide WORK PIECE MATERIAL – Mild Steel FEED – 0.16 mm/rev CUTTING SPEED – 300m/min DEPTH OF CUT – 2mm

SPEE D	CHIP	RAKE	ANGLE OF	WIDT	CHIP
(rpm)	THICKNE	ANGL	INCLINATI	H OF	FLOW
	SS (mm)	E (deg)	ON (deg)	CUT	ANGL
		_	_	(mm)	E (deg)
500	1	6	90	100	45
1000	1.5	7	90	95	45
1260	2	8	90	90	45
SPEE D	CHIP	RAKE	ANGLE OF	WIDT	CHIP
(rpm)	THICKNE	ANGL	INCLINATI	H OF	FLOW
	SS (mm)	E (deg)	ON (deg)	CUT	ANGL
		_	_	(mm)	E (deg)
500	1	6	80	85	40
1000	1.5	7	80	80	40
1260	2	8	80	75	40
SPEE D	CHIP	RAKE	ANGLE OF	WIDT	CHIP
(rpm)	THICKNE	ANGL	INCLINATI	H OF	FLOW
	SS (mm)	E (deg)	ON (deg)	CUT	ANGL
		_	_	(mm)	E (deg)
500	1	6	70	70	35
1000	1.5	7	70	65	35

EN8 – CARBIDE TOOL BIT



| e-ISSN: 2319-8753, p-ISSN: 2320-6710| <u>www.ijirset.com</u> | Impact Factor: 7.512|

|| Volume 9, Issue 6, June 2020 ||

SPEE D	CHIP	RAKE	ANGLE OF	WIDT	CHIP FLOW
(rpm)	THICKNES	ANGL	INCLINATI	H OF	ANGL
-	S (mm)	E	ON (deg)	CUT	E (deg)
		(deg)		(mm)	
500	1	6	90	100	45
1000	2	7	90	95	45
1260	2.5	8	90	90	45
SPEE D	CHIP	RAKE	ANGLE OF	WIDT	CHIP FLOW
(rpm)	THICKNES	ANGL	INCLINATI	H OF	ANGL
-	S (mm)	E	ON (deg)	CUT	E (deg)
		(deg)	-	(mm)	-
500	1	6	80	85	40
1000	1.5	7	80	80	40
1260	2	8	80	75	40
SPEE D	CHIP	RAKE	ANGLE OF	WIDT	CHIP FLOW
(rpm)	THICKNES	ANGL	INCLINATI	H OF	ANGL
-	S (mm)	E	ON (deg)	CUT	E (deg)
		(deg)		(mm)	
500	1	6	70	70	35
1000	1.5	7	70	65	35
1260	2	8	70	60	35

ALUMINUM – CARBIDE TOOL BIT

SPEED (rpm)	CHIP THICKNES S (mm)	RAKE ANGL E (deg)	ANGLE OF INCLINATIO N (deg)	WIDT H OF CUT (mm)	CHIP FLOW ANGL E (deg)
500	0.5	6	90	100	45
1000	0.5	7	90	95	45
1260	0.5	8	90	90	45
SPEED (rpm)	CHIP THICKNESS (mm)	RAKE ANGLE (deg)	ANGLE OF INCLINATION (deg)	WIDTH OF CUT (mm)	CHIP FLOW ANGLE (deg)
500	0.5	6	80	85	40
1000	0.5	7	80	80	40
1260	0.5	8	80	75	40
SPEED (rpm)	CHIP THICKNESS (mm)	RAKE ANGLE (deg)	ANGLE OF INCLINATION (deg)	WIDTH OF CUT (mm)	CHIP FLOW ANGLE (deg)
500	0.5	6	70	70	35
1000	0.5	7	70	65	35
1260	0.5	8	70	60	35

EN19 – CARBIDE TOOL BIT

RESULTS TABLE

Material	DISPLACEMENT(mm)	STRESS(N/mm ²)	STRAIN
EN8	0.016116	15.4107	0.784E- 04
EN19	0.015254	15.5903	0.744E- 04
ALUMINUM	0.04658	15.3214	0.226E-3

XIV.CONCLUSION

The friction due to the contact between the workpiece and the tool is one of the key subjects in machining research. It is well known that cutting involves three deformation zones. The primary shear zone, i.e. the shear plane, is responsible for the chip formation whereas in the secondary shear zone on the rake face the work piece and tool are



| e-ISSN: 2319-8753, p-ISSN: 2320-6710| <u>www.ijirset.com</u> | Impact Factor: 7.512|

|| Volume 9, Issue 6, June 2020 ||

in a complex state of contact. The third region, on the other hand is responsible for the ploughing and flank contact. Although there are numerous models proposed involving, analytical, semi analytical, and numerical (mostly FEM based methods, two important inputs for these models are the material model parameters and the friction coefficient between the tool and the work piece material

REFERENCES

- [1] Chen, Y.Z., "Interaction between compressive force and vibration frequency for a varying cross-section cantilever under action on generalized follower force," Journal of Sound and Vibration, 259, 991-999, (2003).
- [2] W.Y. Poon, C.F. Ng, Y.Y. Lee, Dynamic stability of a curved beam under sinusoidal loading, Proceedings of Institution of Mechanical Engineers, Part G: Journal of Aerospace Engineering 216 (2002) 209–217.
- [3] S.H. Chen, Y.K. Cheung, H.X. Xing, Nonlinear vibration of plane structures by finite element and incremental harmonic balance method, Nonlinear Dynamics 26 (2001) 87–104.
- [4] P. Ribeiro, E. Manoach, The effect of temperature on the large amplitude vibrations of curved beams, Journal of Sound and Vibration 285 (2005) 1093–1107.
- [5] Y.Y. Lee, W.Y. Poon, C.F. Ng, Anti- symmetric mode vibration of a curved beam subject to auto parametric excitation, Journal of Sound and Vibration 290 (2006)
- [6] J.S. Chen, C.H. Yang, Experiment and theory on the nonlinear vibration of a shallow arch under harmonic excitation at the end, ASME Journal of Applied Mechanics 74 (2007) 1061–1070.
- [7] P. Mata, S. Oller, A.H. Barbat, Dynamic analysis of beam structures considering geometric and constitutive nonlinearity, Computer Methods in Applied Mechanics and Engineering 197 (2008) 857–878.
- [8] P. Ribeiro, Non-linear forced vibrations of thin/thick beams and plates by the finite element and shooting methods, Computers and Structures 82 (2004) 1413–1423.
- [9] P. Ribeiro, Forced large amplitude periodic vibrations of cylindrical shallow shells, Finite Elements in Analysis and Design 44 (2008) 657–674.
- [10] J.T. Katsikadelis, The analog equation method. A boundary-only integral equation method for nonlinear static and dynamic problems in general bodies, Appl. Mech. 27 (2002) 13–38
- [11] I. Sheinman, Dynamic large displacement analysis of curved beams involving shear deformation, International Journal of Solids and Structures 16 (1980) 1037–1049.
 [12] U.Lee, H.Oh, Dynamics of an axially moving viscoelastic beam subject to axial tension,InternationalJournalofSolidsandStructures42 (2005)2381–2398.