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Effects of Air-Mucus Interface through a Human Trachea with Mild Constriction of Aerosols

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ABSTRACT: A two-layer fluid model is proposed to study the transport of mucus in the trachea and air motion by considering mucus as a viscoelastic fluid. The effect of air flow due to inspiration and expiration is considered by prescribing shear stress at the mucus-air interface. The resistance to flow has been computed for different constriction length and for different constriction height. Shear stress distribution along the axial distance has been computed for different constriction height. It has been shown that, axial velocity profile, resistance flow, wall shear stress and concentration increases as the size of the constricted trachea increases.

KEYWORDS: Aerosols, Constriction, Air-Mucus interface,Diffusion, viscoelastic fluid.

I.INTRODUCTION

Mucus covers the ciliated epithelium of the airways, which includes the nose, trachea, sinuses and the proximal bronchioles. The mucus continuously moves upwards towards the upper end of the trachea. The regular airflow reversals are obliviously very important in contributing to the particle deposition on the surface of the mucus layer, which is one of the main functions of the mucus.

To understand this mechanism there have been several experimental investigations related to rheological properties of mucus as well as about factors causing mucus transport due to cilia beating (see King *et al.* 1974, Chen and Dulfano 1976, King and Macklem 1977, King 1980b, Winet and Blake 1980, Puchelle *et al.* 1983 and Winet 1987).

The importance of airflow on mucus transport was also investigated by Blake (1975) by considering two layers steady state Newtonian fluid model. King *et al.* (1993) have also proposed a planar non-symmetrical two layer fluid laminar flow model to study mucus transport in the respiratory tract due to cilia beating and air motion by considering mucus as a viscoelastic fluid. In recent years several experiments related to two layer flow in tubes under externally applied pressure gradient simulating mucus transport in airways due to forced expiration of cough have been conducted by several authors (see Clarke *et al.* 1970, Clarke 1973, Scherer and Burtz 1978, Scherer 1981 and Chong *et al.* 1986 and Shukla *et al.*1999).Conditions for the mucus transport by two layer gas-liquid flow in the airways in relation to airflow velocity, mucus layer thickness and rheological properties of mucus were investigated by Chong *et al.* (1986). Their work was mainly concerned with straight tube much attention has not been given to the study of the constriction on mucus layer.

To achieve these objectives, this paper is organized as follows. The required basic equations and the relative boundary equations are obtained following the assumptions of fully developed unidirectional steady flow, where all the quantities except the pressure are functions of *r* only.

In this paper, we study the following aspects of the flow of air and mucus region in the constricted trachea

1.Effects of axial velocity profile, flow to resistance, wall shear stress, concentration and diffusivity coefficient of the constricted trachea.

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2. Effects of ratio of flow rate air region to mucus region with respect to the interface α and thickness δ_n for different values of the ratio of the viscosity of mucus region to air region for fixed length and slip parameter.
The study of constricted trachea on mucus region is the main objective of this paper.

II. MATHEMATICAL FORMULATION

We consider the laminar and steady flow of the air and mucus region of the trachea which is idealized by symmetrical constricted cylindrical tube as shown in Figure 1. which is represented by

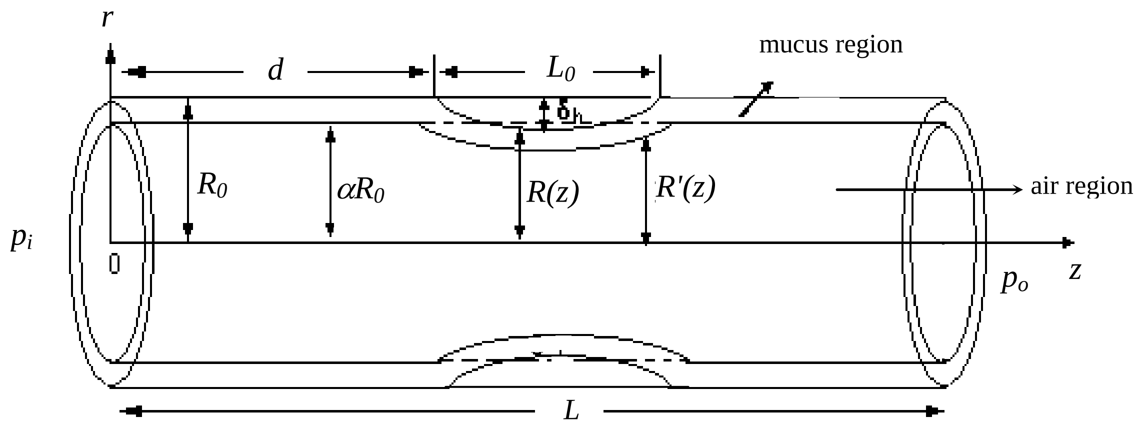

Figure 1 Geometry of constricted trachea with mucus region

$$R(z)=\frac{R'(z)}{R_0}$$

$$=\begin{cases}1-\dfrac{\delta_h}{2R_0}\left[1+\cos\dfrac{2\pi}{L_0}\left(z-d-\dfrac{L_0}{2}\right)\right]; & if\ d\le z\le d+L_0\\ 1 \qquad ; & Otherwise\end{cases} \tag{1}$$

$$R'(z)=\frac{\overline{R'(z)}}{R_0}=\begin{cases}\alpha-\dfrac{\alpha\delta_h}{2R_0}\left[1+\cos\dfrac{2\pi}{L_0}\left(z-d-\dfrac{L_0}{2}\right)\right]; & d\le z\le d+L_0\\ \alpha \qquad ; & Otherwise\end{cases} \tag{2}$$

Where $R(z)$ is the radius of the tube with constricted trachea, R_0 is the radius of the trachea (R_0 = 0.98cm), L is the length of the trachea (L=12.0cm) and L_0 is the length of the constricted trachea. δ_h is the thickness of the deposition on the trachea ($\delta_h << R_0$), $R'(z)$ represents the geometry of the interface. Where α represents the ratio of the central layer radius to the tube radius in the unconstructed region, where $R'(z) = \alpha R(z)$. The physical contribution of the constriction and mucus region of trachea is shown in Fig. 1.The governing equation of the air and mucus region in the trachea assuming fully developed unidirectional steady flow where the flows are driven by the same pressure gradient are given as follows

Region : I Air region $[0 \le r \le R'(z)]$

$$0=-\frac{dp}{dz}+\frac{\mu_a}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_a}{\partial r}\right) \tag{3}$$

Region: II Mucus region $[R'(z) \le r \le R(z)]$

$$0=-\frac{dp}{dz}+\frac{\mu_m}{r}\frac{\partial}{\partial r}\left(r\frac{\partial u_m}{\partial r}\right) \tag{4}$$

The concentration C_a of air region following Taylor (1953) satisfies the diffusion equation

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$$(u_a - \bar{u}_a)\frac{\partial C_a}{\partial z} = D_a\left[\frac{\partial^2 C_a}{\partial r^2} + \frac{1}{r}\frac{\partial C_a}{\partial r}\right] + \frac{\partial^2 C_a}{\partial z^2} \tag{5}$$

The concentration C_m of the mucus region following Taylor (1953) satisfies the diffusion equation

$$(u_m - \bar{u}_m)\frac{\partial C_m}{\partial z} = D_m\left[\frac{\partial^2 C_m}{\partial r^2} + \frac{1}{r}\frac{\partial C_m}{\partial r}\right] + \frac{\partial^2 C_m}{\partial z^2} \tag{6}$$

Following Taylor (1953), we assume that the longitudinal diffusion is much less than the transverse diffusion that is

$$\frac{\partial^2 C_a}{\partial z^2} << \frac{\partial^2 C_a}{\partial r^2} + \frac{1}{r}\frac{\partial C_a}{\partial r}$$, and

$$\frac{\partial^2 C_m}{\partial z^2} << \frac{\partial^2 C_m}{\partial r^2} + \frac{1}{r}\frac{\partial C_m}{\partial r}$$

Then equations (5) and (6) take the form

$$(u_a - \bar{u}_a)\frac{\partial C_a}{\partial z} = D_a\left[\frac{\partial^2 C_a}{\partial r^2} + \frac{1}{r}\frac{\partial C_a}{\partial r}\right] \tag{7}$$

$$(u_m - \bar{u}_m)\frac{\partial C_m}{\partial z} = D_m\left[\frac{\partial^2 C_m}{\partial r^2} + \frac{1}{r}\frac{\partial C_m}{\partial r}\right] \tag{8}$$

Where p is the pressure of air, u_a and u_m are the velocities of air in air and mucus region respectively. μ_a and μ_m are the viscosities in air and mucus region respectively. C_a, C_m and D_a and D_m are respectively the concentrations and Taylor diffusivity diffusion coefficient in air and mucus region. β is the slip coefficient due to roughness in the mucus region ($\beta = 0.00 – 0.10$ gm^{-1} cm^2 sec) (see Shukla *et al.* 1999)
To solve the above system of equations (3), (4), (7) and (8), we use the following boundary conditions.
The symmetry condition

$$\frac{\partial u_a}{\partial r} = 0 \qquad \text{at } r = 0 \tag{9}$$

Interface condition

$$u_a = u_m \qquad \text{at } r = R'(z) \tag{10}$$

Continuity of the velocity and wall shear stress due to roughness is

$$\mu_a \frac{\partial u_a}{\partial r} = \mu_m \frac{\partial u_m}{\partial r} \qquad \text{at } r = R'(z) \tag{11}$$

Slip on the boundary

$$\mu_m = -\beta\,\tau_m \qquad \text{at } r = R(z) \tag{12}$$

$$\frac{\partial C_a}{\partial r} = 0 \qquad \text{at } r = 0 \tag{13}$$

$$C_a = \lambda C_m \qquad \text{at } r = R'(z) \tag{14}$$

$$D_m \frac{\partial C_m}{\partial r} = D_a \frac{\partial C_a}{\partial r} \qquad \text{at } r = R'(z) \tag{15}$$

$$C_a = C_0 \qquad \text{at } r = R(z) \tag{16}$$

$$p^* = p_i \qquad \text{at } z = 0 \tag{17}$$

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$$p^* = p_0 \qquad \text{at } z = L \tag{18}$$

III. SOLUTIONS

The solution of equation (3) using boundary conditions (9) and (10), in the air region is

$$u_a = \left(-\frac{dp}{dz}\right)\frac{R(z)}{4\mu_m}\left\{\left[1-\left(\frac{r}{R(z)}\right)^2 + \mu'\left(\alpha^2-\left(\frac{r}{R(z)}\right)^2\right)\right]R(z)+2\beta\mu m\right\} \quad \text{where} \quad 0<r<R'(z) \tag{19}$$

The average velocity of air region is defined by

$$\bar{u}_a = \frac{2}{R'^2(z)}\int_0^{R'(z)} ru_a dr = \left(\frac{-dp}{dz}\right)\frac{R(z)}{8\mu_m}\left\{\left[2+(\mu'-1)\alpha^2\right]R(z)+4\beta\mu_m\right\} \tag{20}$$

$$v_a = u_a - \bar{u}_a = \left(\frac{-dp}{dz}\right)\frac{R^2(z)}{8\mu_m}\left\{\left[\alpha^2 - 2\left(\frac{r}{R(z)}\right)^2\right](\mu'+1)\right\} \tag{21}$$

Solving equation (4) with boundary conditions (11) and (12) the axial velocity of mucus region is

$$u_m = \left(-\frac{dp}{dz}\right)\frac{R(z)}{4\mu_m}\left[\left(1-\left(\frac{r}{R(z)}\right)^2\right)R(z)+2\beta\mu_m\right] \tag{22}$$

where $R'(z) < r < R(z)$ and $\mu' = \dfrac{\mu_m}{\mu_a}$

The average velocity of mucus region is defined by

$$\bar{u}_m = \frac{2}{[R(z)-R'(z)]^2}\int_{R'(z)}^{R(z)} ru_m dr$$

$$= \left(-\frac{dp}{dz}\right)\frac{R(z)}{8\mu_m}\left[(1-\alpha^2)R(z)+4\beta\mu_m\right] \tag{23}$$

$$v_m = u_m - \bar{u}_m = \left(\frac{-dp}{dz}\right)\frac{R^2(z)}{8\mu_m}\left[1+\alpha^2-2\left(\frac{r}{R(z)}\right)^2\right] \tag{24}$$

The total flow rate Q has been obtained by adding the flow rate Q_a and Q_m where

$$Q_a = \int_0^{R'(z)} 2\pi r u_a dr \quad Q_a = \frac{\pi R'^2(z)\, R(z)}{8\mu_m}\left(\frac{-dp}{dz}\right)\left(\left[2+(\mu'-1)\alpha^2\right]R(z)+4\beta\mu_m\right) \tag{25}$$

$$Q_m = \int_{R'(z)}^{R(z)} 2\pi r u_m dr$$

$$Q_m = \frac{\pi R^3(z)}{8\mu m}\left(\frac{-dp}{dz}\right)\left\{\left[1+\alpha^4-2\alpha^2\right]R(z)+4\beta\mu_m(1-\alpha^2)\right\} \tag{26}$$

$$Q = Q_a + Q_m = \left(-\frac{dp}{dz}\right)\frac{\pi R^3(z)}{8\mu_m}\left[R(z)(\mu'\alpha^4+1)+4\beta\mu_m\right] \tag{27}$$

Equation (27) has been solved for the pressure gradient

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$$-\frac{dp}{dz}=\frac{8Q\mu_m}{\pi R^3(z)}\left[\frac{1}{R(z)\left(\mu'\alpha^4+1\right)\ +4\beta\mu_m}\right]$$

The ratio of flow rate Q_a of air region to flow rate Q_m of mucus region using equations (25) and (26) is

$$\frac{Q_a}{Q_m}=\alpha^2\left[\frac{R(z)\left[2+(\mu'-1)\alpha^2\right]+4\beta\mu_m}{R(z)\left(1+\alpha^4-2\alpha^2\right)+4\beta\mu_m\left(1-\alpha^2\right)}\right] \tag{28}$$

Integrating the above equation using boundary conditions (17) and (18), we have

$$p_i-p_0=\frac{8Q\mu_m}{\pi}\int_0^L\frac{dz}{R^3(z)\left[\left(\mu'\alpha^4+1\right)R(z)+4\beta\mu_m\right]} \tag{29}$$

Resistance to flow which is of physiological and clinical importance has been obtained as

$$\lambda=\frac{p_i-p_0}{Q}$$

$$=\frac{8\mu_m L}{\pi}\left\{\left(1-\frac{L_0}{L}\right)\left(\frac{1}{\mu'\alpha^4+1+4\beta\mu_m}\right)+\int_d^{d+L_0}\frac{dz}{R^3(z)\left[\left(\mu'\alpha^4+1\right)R(z)+4\beta\mu_m\right]}\right\} \tag{30}$$

The velocity in the constricted region is the sum of u_a and u_m and is given by

$$u=u_a+u_m$$

$$=\left(-\frac{dp}{dz}\right)\frac{R(z)}{4\mu_m}\left[\left\{2-2\left(\frac{r}{R(z)}\right)^2+\mu'\left(\alpha^2-\left(\frac{r}{R(z)}\right)^2\right)\right\}R(z)+4\beta\mu_m\right] \tag{31}$$

The wall shear stress is given by

$$\tau=\left[-\mu\frac{\partial u}{\partial r}\right]_{r=R(z)}$$

$$=\frac{4\mu_a Q(\mu'+2)}{\pi\left[(\mu'\alpha^4+1)R^3(z)+4\beta\mu_m R^2(z)\right]} \tag{32}$$

The expression for the concentration C_a of air region in the trachea is obtained by solving equation (7) using the boundary conditions (13) and (14) in the form

$$C_a=\frac{R^2(z)}{64\mu_m D_a}\left(-\frac{dp}{dz}\right)\frac{\partial C_a}{\partial z}\left\{(\mu'+1)\left[2\alpha^2r^2-\frac{r^4}{R^2(z)}-\alpha^2R'^2(z)\right]\right.$$
$$\left.+\ \lambda\left[\left(3R^2(z)-2R'^2(z)\right)+\alpha^2\left[R'^2(z)-2R^2(z)\right]+R'^2(z)\log\alpha\right]\right\}+\lambda C_0 \tag{33}$$

where $0<r<R'(z)$.

The expression for the concentration C_m of mucus region in the trachea is obtained by solving equation (8) using the boundary conditions (15) and (16) in the form

$$C_m=\frac{R^2(z)}{64\mu_m D_m}\left(-\frac{dp}{dz}\right)\frac{\partial C_m}{\partial z}\left\{4R'^2(z)\left[\log r-\log R(z)\right]+R^2(z)(3-2\alpha^2)-\frac{r^4}{R^2(z)}-2r^2(1-\alpha^2)+C_0\right\} \tag{34}$$

where $R'(z)<r<R(z)$

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The volumetric rate of air region of the constricted trachea of unit breadth, following Taylor (1953) is

$$M_a = \int_0^{R'(z)} 2\pi r v_a C_a dr$$

On integration we get

$$M_a = \frac{-\pi(\mu'+1)\alpha^8 R^8(z)}{1024 D_a \mu_m^2} \left(\frac{dp}{dz}\right)^2 \frac{\partial C_a}{\partial z} \qquad (35)$$

Following Taylor (1953) we assume that the variations of C_a with r are small compared with those in the longitudinal direction and if C_{am} is the mean concentration of solute over a section then $\frac{\partial C_a}{\partial z}$ is indistinguishable from $\frac{\partial C_{am}}{\partial z}$ so that we can replace $\frac{\partial C_a}{\partial z}$ in (35) by $\frac{\partial C_{am}}{\partial z}$ and obtain

$$M_a = \frac{-\pi \bar{u}_a^2 (\mu'+1)^2 \alpha^8 R^6(z)}{16 D_a \left\{\left[2+\alpha^2(\mu'-1)\right] R(z) + 4\beta\mu_m\right\}^2} \left(\frac{\partial C_{am}}{\partial z}\right) \qquad (36)$$

$$M_a = \frac{-\pi(\mu'+1)\alpha^8 R^8(z)}{1024 D_a \mu_m^2} \left(\frac{dp}{dz}\right)^2 \left(\frac{\partial C_{am}}{\partial z}\right) \qquad (37)$$

Equation (37) shows that C_{am} is dispersed relative to a tube which moves with the mean velocity of air region $\bar{u}_a$ exactly as though it has been diffused by a process which obeys the same law as molecular diffusion but with a relative diffusion coefficient $(D_a)_0$ is called Taylor diffusion coefficient in the air region,

$$M_a = -(D_a)_0 (F_a)_0 \frac{\partial C_{am}}{\partial z} \qquad (38)$$

where

$$(D_a)_0 = \frac{\bar{u}_a^2}{D_a} (F_a)_0 \qquad (39)$$

$$(F_a)_0 = \frac{\pi(\mu'+1)^2 \alpha^8 R^6(z)}{16\left[\left(2+\alpha^2(\mu'-1)\right)R(z) + 4\beta\mu_m\right]^2} \qquad (40)$$

Also from equation (36)

$$M_a = -(D_a)_1 (F_a)_1 \frac{\partial C_{am}}{\partial z} \qquad (41)$$

Where

$$(D_a)_1 = \frac{1}{D_a}\left(\frac{dp}{dz}\right)^2 (F_a)_1 \qquad (42)$$

$$(F_a)_1 = \frac{\pi(\mu'+1)^2 \alpha^8 R^8(z)}{1024 \mu_m^2} \qquad (43)$$

The fact that no material is lost in the process is expressed by the continuity equation for C_{am} namely

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$$\frac{\partial M_a}{\partial z} = -\frac{\partial C_{am}}{\partial t} \tag{44}$$

where $\frac{\partial}{\partial t}$ represents differentiation with respect to time at point where z is constant. Equations (38) and (41) using (44) becomes

$$\frac{\partial C_{am}}{\partial t} = (D_a)_0 \frac{\partial^2 C_{am}}{\partial z^2} \tag{45}$$

$$\frac{\partial C_{am}}{\partial t} = (D_a)_1 \frac{\partial^2 C_{am}}{\partial z^2} \tag{46}$$

Equations (45) and (46) are the governing longitudinal dispersion where $(D_a)_0$ and $(D_a)_1$ are Taylor diffusivity coefficients of air region given by equation (39) and (42).

The volumetric rate of mucus region of the constricted trachea of unit breadth following Taylor (1953) is

$$M_m = \int_{R'(z)}^{R(z)} 2\pi r v_m C_m dr$$

On integration, we get

$$M_m = \frac{-\pi R^8(z)}{4608 D_m \mu_m^2} \left(\frac{dp}{dz}\right)^2 \frac{\partial C_m}{\partial z} \left[\frac{72}{\alpha} + 57\alpha^2 - 18\alpha^3 + 63\alpha^4 + 40\alpha^5 - 24\alpha^5 \log\alpha - 27\alpha^6 - 18\alpha^9\right] \tag{47}$$

Following Taylor (1953), we assume that the variations of C_m with r are small compared with those in the longitudinal direction and if C_{m_m} is the mean concentration of solute over a section then $\frac{\partial C_m}{\partial z}$ is indistinguishable from $\frac{\partial C_{mm}}{\partial z}$,

so that we can replace $\frac{\partial C_m}{\partial z}$ in (47) by $\frac{\partial C_{mm}}{\partial z}$ and obtain

$$M_m = \frac{-\pi R^8(z)}{4608 D_m \mu_m^2} \left(\frac{dp}{dz}\right)^2 \frac{\partial C_{mm}}{\partial z} \left[\frac{72}{\alpha} + 57\alpha^2 - 18\alpha^3 + 63\alpha^4 + 40\alpha^5 - 24\alpha^5 \log\alpha - 27\alpha^6 - 18\alpha^9\right] \tag{48}$$

$$M_m = \frac{-\pi}{72 D_m} \frac{\partial C_{mm}}{\partial z} \left[\frac{\bar{u}_m^2 R^6(z)}{\left((1-\alpha^2)R(z) + 4\beta\mu_m\right)^2}\right] \left[\frac{72}{\alpha} + 57\alpha^2 - 18\alpha^3 + 63\alpha^4 + 40\alpha^5 - 24\alpha^5 \log\alpha - 27\alpha^6 - 18\alpha^9\right] \tag{49}$$

Equation (49) shows that C_{mm} is dispersed relative to a tube which moves with the mean velocity of mucus region $\bar{u}_m$ exactly as though it has been diffused by a process which obeys the same law as molecular diffusion but with a relative diffusion coefficient $(D_m)_0$ called Taylor diffusion coefficient

$$M_m = -(D_m)_0 (F_m)_0 \frac{\partial C_{mm}}{\partial z} \tag{50}$$

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$$(D_m)_0 = \frac{\bar{u}_m^2}{D_m}(F_m)_0 \tag{51}$$

$$(F_m)_0 = \frac{\pi R^6(z)}{72\left((1-\alpha^2)R(z)+4\beta\mu_m\right)^2}\left[\frac{72}{\alpha}+57\alpha^2-18\alpha^3+63\alpha^4+40\alpha^5 - 24\alpha^5\log\alpha-27\alpha^6-18\alpha^9\right] \tag{52}$$

Also from equation (48)

$$M_m = -(D_m)_1\,(F_m)_1\,\frac{\partial C_{mm}}{\partial z} \tag{53}$$

$$(D_m)_1 = \frac{1}{D_m}\left(\frac{dp}{dz}\right)^2(F_m)_1 \tag{54}$$

$$(F_m)_1 = \frac{\pi R^8(z)}{4068\mu_m^2}\left[\frac{72}{\alpha}+57\alpha^2-18\alpha^3+63\alpha^4+40\alpha^5-24\alpha^5\log\alpha-27\alpha^6-18\alpha^9\right] \tag{55}$$

The fact that the no material is lost in the process is expressed by the continuity of C_{mm} namely

$$\frac{\partial M_m}{\partial z} = -\frac{\partial C_{mm}}{\partial t} \tag{56}$$

where $\frac{\partial}{\partial t}$ represents differentiation with respect to time at point where z is constant. Equations (50) and (53) using (56) become

$$\frac{\partial C_{mm}}{\partial t} = (D_m)_0\frac{\partial^2 C_{mm}}{\partial \xi^2} \tag{57}$$

and

$$\frac{\partial C_{mm}}{\partial t} = (D_m)_1\frac{\partial^2 C_{mm}}{\partial \xi^2} \tag{58}$$

Equations (57) and (58) are the governing equations for the longitudinal dispersion where $(D_m)_0$ and $(D_m)_1$ are Taylor diffusivity coefficients of mucus region given by equation (51) and (54).

The expressions for axial velocity distribution, ratio of flow rate, resistance to flow, wall shear stress distribution, and concentration and Taylor diffusivity coefficients of air and mucus region are numerically computed for different values of the parameters and the results are depicted graphically and conclusions are drawn in next section.

IV. CONCLUSIONS

The objective of this paper is to study the basic equation mechanisms between the air region and the mucus region and to predict the Taylor diffusion coefficients and its effects on constriction of the trachea.

A simplified case of steady state model for laminar flow of air region and the mucus region in the constricted trachea by considering constant pressure gradient simulating mucus transport in the upper airways during inhalation and has been obtained.

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The axial velocity distribution in the constricted region, given by equation (31), is computed for different constriction length L_0, thickness δ_h by varying slip parameter β and the results are depicted in Figure 2(a) and 2(b).From the Figure 2(a) we conclude that an increase in thickness δ_h increases the axial velocity u with radial position r for fixed slip parameter β = 0.05 gm^{-1}.cm^2 sec and fixed length L_0 = 0.5cm. From the Figure 2(b) we conclude that the increase in slip parameter decreases the axial velocity u for fixed thickness δ_h = 0.2cm with radial position r for fixed length L_0=0.5cm. Also the velocity moves constantly in the mucus region of the trachea.The resistance to flow λ has been computed using equation (30) and the results are depicted in Figure 3. This figure is concerned with resistance for different constricted lengths L_0 and thickness δ_h and different slip parameter. From this figure, we conclude that the increase in thickness δ_h and length L_0, increases the resistance λ for fixed slip parameter β, also increase in slip parameter β decreases the resistance λ for fixed thickness δ_h with axial position of length L_0/L.

The depth of mucus could be many times higher than normal and the contribution of mucus to resistance could conceivably become important. Our results are similar to the results of Scherer and Burtz (1978) and John Blake (1973). They found that straight tube was lined with mucus stimulant airflow resistance was higher than that expected for simple constriction in trachea. Hence in the diseased state the ratio of flow to resistance increases as increase with the length of constricted region for fixed thickness and slip coefficient.

The wall shear stress τ has been computed using equation (32) for different constricted length L_0 and height δ_h and slip parameter β and the results are depicted in Figure 4. From this Figure we conclude that the increase in thickness δ_h increases the wall shear stress τ for fixed slip parameter β and also an increase in slip parameter β decreases the wall shear stress for fixed thickness δ_h with axial position of length z. Our results similar to the results of John Blake (1973). Hence the resistance and shear stress increases in the diseased state. The ratio of the flow rate Q_a/Q_m has been computed using equation (28) for different values of thickness δ_h and different values of the ratio of the viscosity $\mu' = \mu_m/\mu_a$ where μ_a is the viscosity of air region (μ_a = 1500 poise) and δ_h is the viscosity of mucus region (μ_m = 1.00 – 10.00 poise) for fixed length L_0 = 0.5 and slip parameter β = 0.05, and the results are depicted in Figure 5(a) and Figure 5(b).From the Figure 5(a) we conclude that the increases in the ratio of the viscosity $\mu' = \mu_m/\mu_a$ decreases the ratio of the flow rate Q_m/Q_a with thickness of the trachea δ_h for fixed length L_0=0.5 and slip parameter β = 0.05.

From the Figure 5(b), we conclude that as increase of ratio of viscosity decreases the ratio of the flow rate Q_a/Q_m with axial position interface α. The concentration C_a of the air region has been computed using equation (33) and the concentration C_m of the mucus region has been computed using equation (34) for different values of the thickness δ_h and length z of the constructed trachea and for different values of slip parameters β and the results are depicted in Figure 6 and Figure 7.

From the Figure 6, we conclude that an increase in thickness δ_h increases the concentrations C_a and C_m with radial position γ for fixed length z = 0.5 and slip parameter β = 0.5. Also an increase of β decreases the concentration C_a and C_m with radial position η for fixed thickness δ_h = 0.2 length z =0.5. Also the concentration C_a of air region is more parabolic than the concentration C_m of the mucus region.From the Figure 7, we conclude that an increase of slip parameter β decreases the concentrations C_a and C_m at the interface of the radial position r = 0.95 (α=R′(z) /R(z) = 0.95) with axial constricted length z, for fixed thickness δ_h = 0.2 . Also from this figure we conclude that the concentration

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C_a of the air region is more constricted than the concentration of mucus region C_m with axial length of the constricted length z.The variations of diffusivity coefficients for both air region and mucus region have been presented in the Table I and Table II.

From Table I diffusivity coefficients $(F_a)_0$ *and* $(F_a)_1$ in the air region increase with an increase of ratio of viscosity of mucus region to air region $\mu' = \mu_m/\mu_a$ for fixed thickness δ_h of the constricted trachea. Also its decreases with an increase of thickness δ_h for fixed ratio of the viscosity of $\mu' = \mu_m/\mu_a$.From the Table II the diffusivity coefficients $(F_m)_0$ *and* $(F_m)_1$ in the mucus region decreases with an increase of constricted thickness δ_h and increases with viscosity of the mucus region μ_m for fixed constricted length z = 0.5.These results show that the effect of the mucus layer in a constricted trachea cannot be neglected as people with chronic bronchitis have increased numbers of secretary cells in the tracheobronchial tree (see Blames *et al.* 1987). They produce an excess of mucus and have recurrent cough. The excess secretion of mucus may lead to impairment of normal clearance mechanisms; consequently it takes longer for particles to be removed from the scrap of patient with chronic bronchitis than it does in healthy people. This reduced clearance makes people with chronic bronchitis more susceptible to respiratory infections, because bacteria entering the respiratory tract are not removed efficiently (see Barker and Osmond 1986).

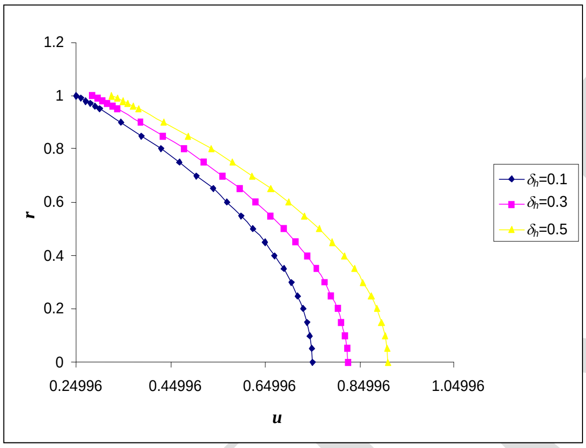

Fig. 2(a) Variation of axial velocity ***u*** with radial position ***r*** for different thickness of the constricted trachea

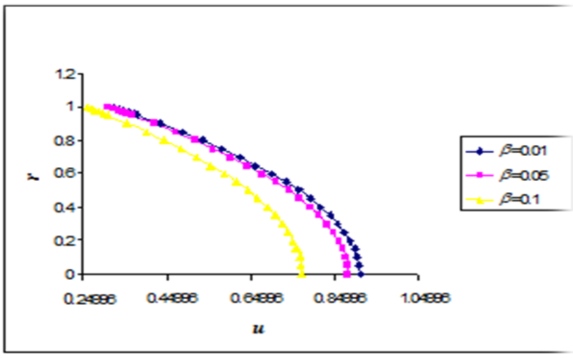

Fig. 2(b) Variation of axial velocity ***u*** with radial position ***r*** for different slip parameter β of the constricted trachea

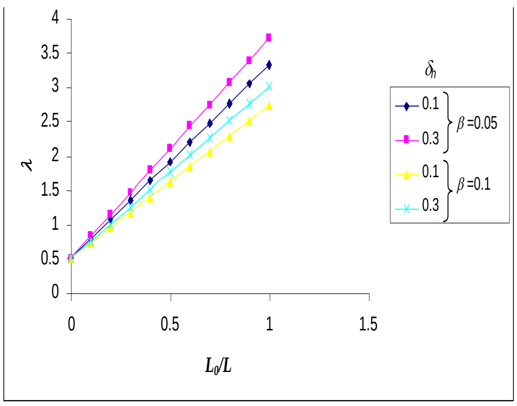

Fig. 3 Variation of resistance λ with axial position L_0/L for different slip parameter β thickness δ_h of the constricted trachea

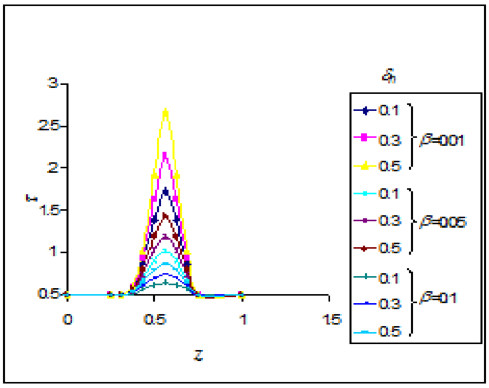

Fig. 4 Variation of wall shear stress τ with axial position z for different slip parameter β and thickness δ_h of the constricted trachea

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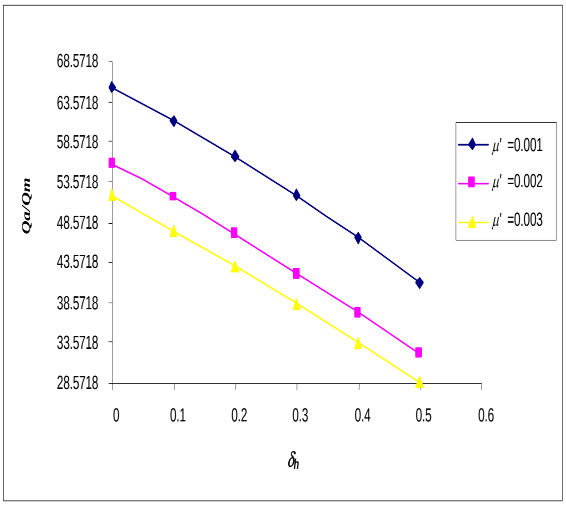

Fig. 5(a) Variation of ratio of flow rate Q_a/Q_m with axial position of the thickness δ_h for different ratio of the viscosity $\mu' = \mu_m/\mu_a$

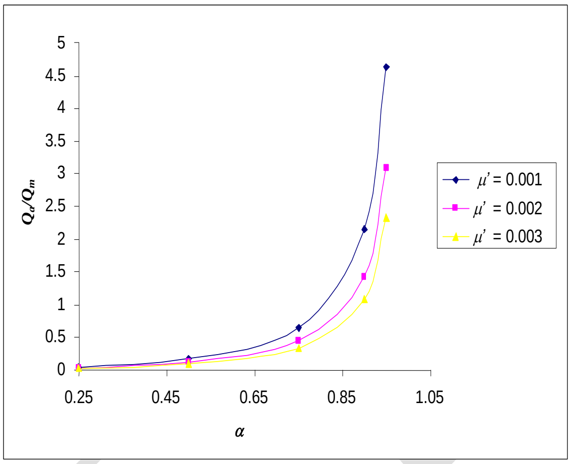

Fig. 5(b) Variation of ratio of flow rate Q_a/Q_m with axial position of the interface α for different ratio of the viscosity $\mu' = \mu_m/\mu_a$

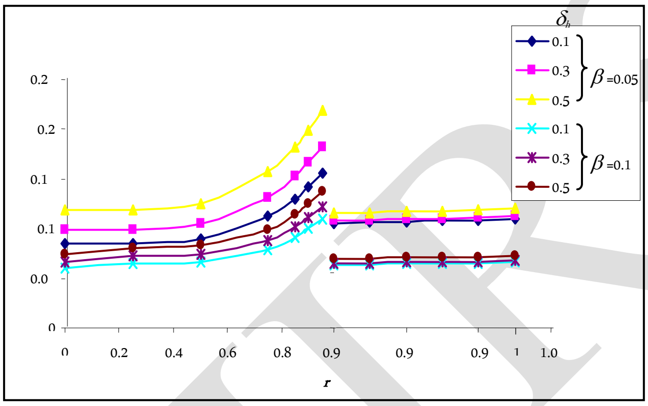

Figure 6 Variation of the Concentration ***C*** with radial position ***r*** for different values of slip parameter β and thickness δ_h of the constricted trachea

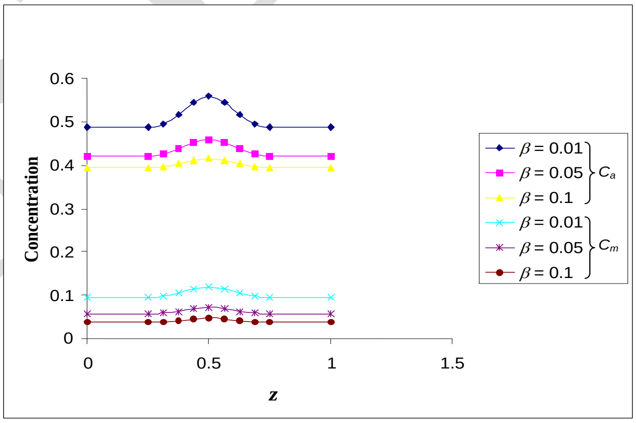

Fig. 7 Variation of the Concentration C_a of the air region and concentration C_m of the mucus region at the interface α=0.95 with axial position *z* for different slip Viscosity parameter β of the constricted trachea

TABLE I

Diffusivity Coefficient of Air Region $(f_a)_0$ and $(f_a)_1$ for Different Ratio of $\mu' = \mu_m/\mu_a$ and Thickness δ_h of the Constricted Trachea

Diffusivity Coefficient-Air Region-$(F_a)_0$: β = 0.05, L_0=0.5				
μ'/δ_h	**0**	**0.1**	**0.3**	**0.5**
0.001	0.000104	0.000045	0.000006	0
0.002	0.000233	0.0001	0.000013	0.000001
0.003	0.000415	0.000178	0.000024	0.000002

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Diffusivity Coefficient Air Region-$(F_a)_1$: β = 0.05, L_0=0.5			
μ'/δ_h	**0.1**	**0.3**	**0.5**
0.001	0.007376	0.000813	0.00004
0.002	0.009972	0.001147	0.000061
0.003	0.011816	0.001396	0.000077

	Diffusivity Coefficient-Mucus Region-$(F_m)_1$: β=0.05, L_0=0.5				
μ_m/δ_h	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**
1	41.6287	21.9998	10.60418	4.528504	1.637805
2	14.48868	7.441364	3.480175	1.439452	0.503245
3	7.287227	3.698699	1.708751	0.697854	0.240787
4	4.373631	2.205562	1.012202	0.410578	0.140679
5	2.913166	1.463089	0.668668	0.270081	0.092139

	Diffusivity Coefficient-Mucus Region-$(F_m)_0$: β=0.05, L_0=0.5					
μ_m/δ_h	**0**	**0.1**	**0.2**	**0.3**	**0.4**	**0.5**
1	0.1013	0.0436	0.01700	0.00584	0.00170	0.000396
2	0.0253	0.0109	0.00425	0.00146	0.00042	0.000099
3	0.0112	0.0048	0.00188	0.00064	0.00018	0.000044
4	0.0063	0.0027	0.00106	0.00036	0.00010	0.000025
5	0.0040	0.0017	0.00068	0.00023	0.00006	0.000016

TABLE II

Diffusivity Coefficient of Mucus Region $(F_m)_0$ and $(F_m)_1$ for Different Ratio of Viscosity $\mu' = \mu_m/\mu_a$ and Thickness δ_h of the Constricted Trachea

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