

Characterization of Planar Graph with Edge Series

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ABSTRACT: Kuratowski's Theorem, perhaps the most famous result in graph theory, states that K_5 and $K_{3,3}$ are the only non-planar graphs for which both $G \setminus e$, the deletion of the edge e , and G/e , the contraction of the edge e , are planar for all edges e of G . We characterize the planar graphs edge series, those non-planar graphs for which $G \setminus e$ or G/e is planar for all edges e of G . This paper gives five characterizations of the planar graphs: an explicit description of the structure of planar graphs; and an excluded minor criterion. We also give a best possible bound on the number of edges of an planar graph.

I. INTRODUCTION

Planar graph from a map by representing countries by vertices, and placing edges between countries that touch each other[15]. Assuming each country is continuous, this gives a planar graph. While planar graphs were introduced for practical reasons, they possess many remarkable mathematical properties. This is one reason we will study them. Our main result is the characterization of planar graphs as the graphs whose order-dimension does not exceed three. Strong versions of several known properties of planar graphs are implied by this characterization. These properties include[14]: each planar graph has arboricity at most three and each planar graph has a plane embedding whose edges are straight line segments. A nice feature of this embedding is that the coordinates of the vertices have a purely combinatorial meaning[13,12,11].

Creation of edges in series:1

If the degree of the vertex v is 2, then the two edges incident on a vertex v are said to be edges in series[10].

Let $e = uv$ be an edge. Take a new vertex w . Remove the edge e and introduce two new edges. $e_1 = uw$ and $e_2 = vw$ by this subdivision of a edge e we are creating a new vertex w with two edges e_1, e_2 in series at w . So the subdivision of an edge is same as creation of edges in a series.

The Removal of edges in series or creation of edges in series does not affect the planarity or otherwise of the graph[8].

Note:

Creation of edges in series (subdivision of an edge) increases the number of vertices and edges each by one[9].

Merger of edges:2

If the two edges in a series at a vertex are replaced by a single edge, then this process is called merger of edges we may get parallel edges[7].

Removal of all parallel edges except one between two vertices does not affect the planarity or otherwise of the graph.

Also the removal of loops does not affect the planarity[6].

Note:

The merger of edges decreases the number of vertices and edges each by one.

Homeomorphism of graph:3

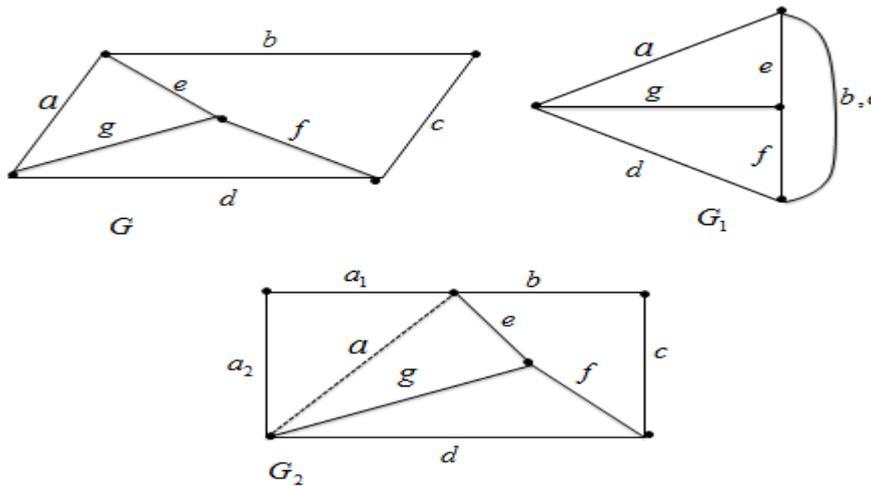
Two graphs are said to be homeomorphic[5]. If one can be obtained from the other by creation of edges in series or by merger of edges in series.

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Example:



G_1 is obtained from G by merging the edges b and c . G_2 is obtained from G by creating the edges a_1, a_2 in series. All the three graphs G, G_1, G_2 are homeomorphic to one another[5].

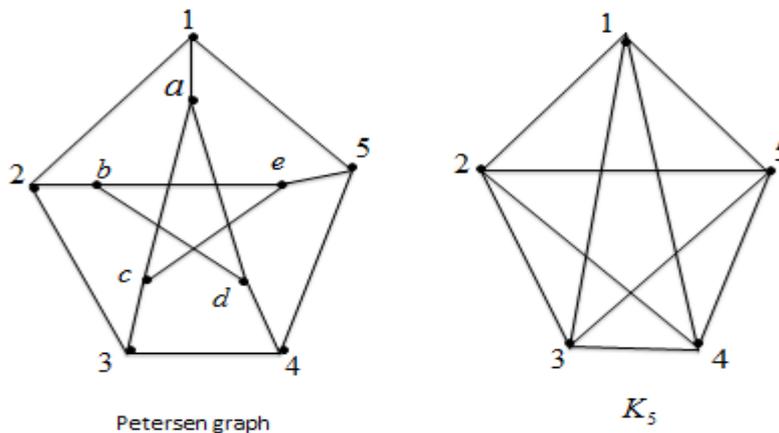
Contractible graph:4

Let u and v be two adjacent vertices in a graph G . The graph obtained from G by the removal of u and v the addition of vertex w adjacent to those vertices to which u or v are adjacent is called an elementary contraction of G .

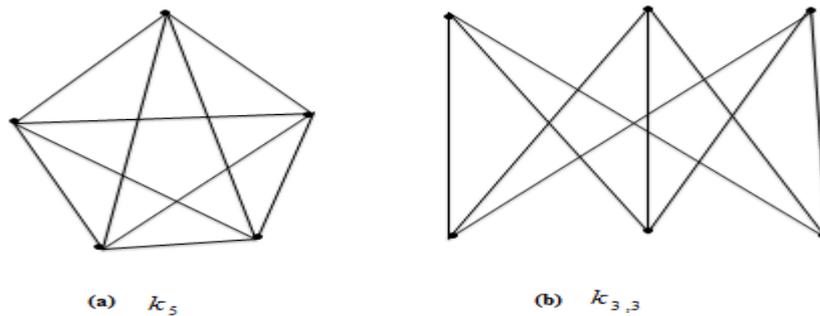
i.e. A graph G is contractible to a graph H if H can be obtained from G by a sequence of elementary contractions[5].

Example :

The Petersen graph is contraction to K_5 by contracting the vertices $1a, 2b, 3c, 4d$ and $5e$.



Kuratowski's two graphs: 5



Theorem:2.1.8

Kuratowski's graphs k_5 and $k_{3,3}$ are non-planar[4].

Proof:

- i) For k_5
 the number of vertices $p = 5$.
 the number of edges $q = 10$.
 If the graph is to be planar, $q \leq 3p - 6$
 But for k_5 , the above inequality is not satisfied for $p = 5$ and $q = 10$.
 $\therefore k_5$ is non-planar.
- ii) For $k_{3,3}$
 the number of vertices $p = 6$.
 the number of edges $q = 9$.
 Now $k_{3,3}$ is free from triangles.
 So if it is to be planar $q \leq 2p - 4$.
 Clearly this inequality is not satisfied[3]
 When $p = 6$ and $q = 9$.
 $\therefore k_{3,3}$ is non-planar.

II. CONCLUSION

We have given a characterization of planar graphs edge series in terms of minimal forbidden subgraphs. Characterization of families of graphs by creation of edge series, Merger of edges. We have shown that for the [2] family of homeomorphism of planar (directed) graphs, and the contractible edge series, Kuratowski's graphs with theorems [1].

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