ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

*(An ISO 3297: 2007 Certified Organization)*

Vol. 3, Issue 6, June 2014

# Bianchi Type-I (Kasner Form) Cosmological Model in F($R$) Theory of Gravity

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**ABSTRACT:** The vacuum solution of Bianchi types-I (Kasner form) cosmological model in $f(R)$ theory of gravity has been obtained. The general solution of the field equations in respect of Bianchi type-I space-time in Kasner form have been obtained with the assumption of special form of deceleration parameter. The physical behavior of the model has been discussed by obtaining some physical quantities.

**KEYWORDS:** $f(R)$ theory of gravity, Bianchi type-I space-time in Kasner form, Special form of deceleration parameter.

## I. INTRODUCTION

The $f(R)$ theory of gravity plays an important role in describing the evolution of the universe. The $f(R)$ theory of gravity is the modification of the general theory of relativity proposed by Einstein. The $f(R)$ actions were first studied by Weyl [1] and Eddington [2]. The $f(R)$ gravity provides a very natural unification of the early-time inflation and late-time acceleration. Carroll *et al*.[3] explained the presence of a late time cosmic acceleration of the universe in $f(R)$ gravity. As per Nojiri and Odintsov ( [4],[5] ) a unification of the early time inflation and late time acceleration is allowed in $f(R)$ gravity.

## II. RELATED WORK

Multamaki & Vilja ( [6],[7] ) investigated static spherically symmetric vacuum solutions of the field equations and non-vacuum solutions by taking perfect fluid respectively. Sharif & Shamir [8] studied exact vacuum solutions of Bianchi type-I & type-V space times in $f(R)$ theory of gravity. Non-vacuum solutions in Bianchi type-I & type-V using perfect fluid in $f(R)$ gravity have been obtained by Sharif & Shamir [9]. Shamir [10] discussed the plane symmetric vacuum Bianchi type-III cosmology in $f(R)$ gravity. Bianchi type-III space time with anisotropic fluid in $f(R)$ gravity has been dealt with by Sharif & Kausar [11]. Aktas *et al.* [12] have studied anisotropic models in $f(R)$ theory of gravity. Adhav [13] discussed the Kantowski–Sachs string cosmological model in $f(R)$ theory of gravity. Singh *et al.* [14] studied functional form of $f(R)$ with power-law expansion in anisotropic model. Recently, Reddy *et al.* [15] studied vacuum solution of Bianchi type-I and V models in $f(R)$ gravity with a special form of deceleration parameter.

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 3, Issue 6, June 2014

Motivated by the above research, in this paper, the general solutions of the field equations of $f(R)$ gravity for Bianchi type-I space-time in Kasner form have been obtained with the assumption of special form of deceleration parameter. The physical behavior of the model is also discussed.

## III. $f(R)$ THEORY OF GRAVITY AND DECELERATION PARAMETER

The $f(R)$ theory of gravity is nothing but modification of general theory of relativity. The action for $f(R)$ theory of gravity is given by

$$S = \int \sqrt{-g}\left(\frac{1}{16\pi G} f(R) + L_m\right) d^4x \ . \tag{1}$$

Here $f(R)$ is a general function of the Ricci scalar and $L_m$ is the matter Lagrangian.
Note that this action is obtained just by replacing $R$ by $f(R)$ in the standard Einstein-Hilbert action.
Now by varying the action with respect to the metric $g_{ij}$, we get the corresponding field equations as

$$F(R)R_{ij} - \frac{1}{2} f(R) g_{ij} - \nabla_i \nabla_j F(R) + g_{ij} \Box F(R) = \kappa T_{ij} , \tag{2}$$

where $F(R) = {df(R)}/{dR}$ , $\Box \equiv \nabla^i \nabla_i$ , (3)

$\nabla_i$ is the covariant derivative, $T_{ij}$ is the standard matter energy-momentum tensor and $\kappa$ is the coupling constant in gravitational units. These are the fourth order partial differential equations in the metric tensor $g_{ij}$. The fourth order is due to the last two terms on the left hand side of the equation. If we consider $f(R) \equiv R$, these equations of $f(R)$ theory of gravity reduce to the field equations of Einstein's general theory of relativity.
After contraction of the field equations (2), we get

$$F(R)R - 2f(R) + 3 \ \Box F(R) = \kappa T , \tag{4}$$

In vacuum this field equation (4) reduced to

$$F(R)R - 2f(R) + 3 \ \Box F(R) = 0 , \tag{5}$$

This gives a relationship between $f(R)$ and $F(R)$.

From equation (5), we get

$$f(R) = \frac{F(R)R}{2} + \frac{3}{2} \ \Box F(R). \tag{6}$$

Putting this value of $f(R)$ in the vacuum field equations (2), we have

$$\frac{1}{4}\left[F(R)R - \ \Box F(R)\right] = \frac{F(R)R_{ij} - \nabla_i \nabla_j F(R)}{g_{ij}} . \tag{7}$$

Since the left hand side of equation (7) does not depend on the index $i$ , so the field equation can be expressed as

$$K_i \equiv \frac{F(R)R_{ii} - \nabla_i \nabla_i F(R)}{g_{ii}} \tag{8}$$

is independent of the index $i$ and hence $K_i - K_j = 0$ for all $i$ and $j$.

Here $K_i$ is just a notation for the traced quantity.

Many authors assume various physical or mathematical conditions to obtain exact solutions of the modified Einstein field equations in *f*(*R*) theory of gravity. Berman in [16] proposed a special law of variation for Hubble parameter which yields constant deceleration parameter models of the universe. Akarsu and Kilinc [17], Pradhan *et al.* [18] and Adhav ( [19],[20] ) have extended this law for Bianchi models. Cunha & Lima [21] favours recent acceleration and past deceleration with high degree of statistical confidence level by analyzing
three SNe type Ia samples. Recently, Singha and Debnath **[**22**]** has defined a special form of deceleration parameter for FRW metric as

$$q = -\frac{\ddot{a}a}{\dot{a}^2} = -1 + \frac{\alpha}{1+a^{\alpha}}, \tag{9}$$

where $\alpha > 0$ is a constant and $a$ is mean scale factor of the universe.

After solving equation (9) one can obtain the mean Hubble parameter $H$ as

$$H = \frac{\dot{a}}{a} = k\left(1 + \frac{1}{a^{\alpha}}\right), \tag{10}$$

where $k$ is a constant of integration.

On integration equation (10) gives the mean scale factor as

$$a = (e^{k\alpha t} - 1)^{1/\alpha}. \tag{11}$$

The physical parameters that are of cosmological importance are

The mean anisotropy parameter: $$\Delta = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2. \tag{12}$$

The shear scalar: $$\sigma^2 = \frac{1}{2}\sum_{i=1}^{3}\left(H_i^2 - 3H^2\right). \tag{13}$$

The expansion scalar : $$\theta = 3H. \tag{14}$$

## IV. BIANCHI TYPE-I MODEL

We consider the anisotropic Bianchi type-I space time in Kasner form

$$ds^2 = dt^2 - t^{2q_1}dx^2 - t^{2q_2}dy^2 - t^{2q_3}dz^2, \tag{15}$$

where $q_1$, $q_2$ and $q_3$ are three parameters that we shall required to be constant.
The corresponding Ricci scalar is

$$R = -\left[S^2 - 2S + \phi\right] t^{-2}, \tag{16}$$

where $S = q_1 + q_2 + q_3$ and $\phi = q_1^2 + q_2^2 + q_3^2$ .

Since the equation (7) does not depend on index *i*, so $K_i - K_j = 0$ for all *i* and *j*. With the help of equation (8) we can obtain the corresponding field equations for the metric (15) as

$$\left[(S-\phi) + q_1(S-1)\right] t^{-2} + \frac{q_1}{t}\frac{\dot{F}}{F} - \frac{\ddot{F}}{F} = 0 \quad , \tag{17}$$

$$\left[(S-\phi) + q_2(S-1)\right] t^{-2} + \frac{q_2}{t}\frac{\dot{F}}{F} - \frac{\ddot{F}}{F} = 0 \quad , \tag{18}$$

$$\left[(S-\phi) + q_3(S-1)\right] t^{-2} + \frac{q_3}{t}\frac{\dot{F}}{F} - \frac{\ddot{F}}{F} = 0 \quad , \tag{19}$$

where dot ($\cdot$) denotes derivative with respect to time *t*.

The field equations (17)-(19) are three non-linear differential equations with four unknowns $q_1$, $q_2$, $q_3$ and *F*. In order to solve the system completely we impose a special form of deceleration parameter as given in equation (9) and corresponding mean Hubble parameter *H* and mean scale factor *a* is given in equation (10) and (11) respectively.

We define the spatial volume $V$ for Bianchi type-I universe as

$$V = a^3 = t^s . \tag{20}$$

The mean Hubble parameter *H* for Bianchi type-I universe is defined as

$$H = \frac{\dot{a}}{a} = \frac{1}{3}\left(H_x + H_y + H_z\right) , \tag{21}$$

where $H_x = \frac{q_1}{t}$, $H_y = \frac{q_2}{t}$, $H_z = \frac{q_3}{t}$ are the directional Hubble parameters in the directions of $x, y, z$ respectively.

Subtracting Equations (18) from (17), (19) from (18) and (19) from (17) respectively, we get

$$\left[(q_1 - q_2)(S-1)\right] t^{-2} + \frac{\dot{F}}{F}\frac{(q_1 - q_2)}{t} = 0 \quad , \tag{22}$$

$$\left[(q_2 - q_3)(S-1)\right] t^{-2} + \frac{\dot{F}}{F}\frac{(q_2 - q_3)}{t} = 0 \quad , \tag{23}$$

$$\left[(q_1 - q_3)(S-1)\right] t^{-2} + \frac{\dot{F}}{F}\frac{(q_1 - q_3)}{t} = 0 \quad , \tag{24}$$

After solving equations (22)-(24), we get

$$t^{q_1} = t^{q_2} n_1 \exp\left(m_1 \int \frac{dt}{t^S F}\right) \quad , \tag{25}$$

$$t^{q_2} = t^{q_3} n_2 \exp\left(m_2 \int \frac{dt}{t^S F}\right) \quad , \tag{26}$$

ISSN: 2319-8753

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 3, Issue 6, June 2014

$$t^{q_1} = t^{q_3} n_3 \exp\left( m_3 \int \frac{dt}{t^S F} \right) \quad , \tag{27}$$

where $m_1$ , $m_2$ , $m_3$ and $n_1$ , $n_2$ , $n_3$ are constants of integration which satisfy the relation

$$m_1 + m_2 + m_3 = 0 \quad , \quad n_1 n_2 n_3 = 1 . \tag{28}$$

Using Equations (25)- (27) we can write the metric functions explicitly as

$$t^{q_1} = t^{S/3} r_1 \exp\left( p_1 \int \frac{dt}{t^S F} \right) , \tag{29}$$

$$t^{q_2} = t^{S/3} r_2 \exp\left( p_2 \int \frac{dt}{t^S F} \right) , \tag{30}$$

$$t^{q_3} = t^{S/3} r_3 \exp\left( p_3 \int \frac{dt}{t^S F} \right) , \tag{31}$$

where $r_1 = (n_1 n_3)^{\frac{1}{3}}$ , $r_2 = \left(n_1^{-2} n_3\right)^{\frac{1}{3}}$ , $r_3 = \left(n_1 n_3^{-2}\right)^{\frac{1}{3}}$ (32)

and

$$p_1 = \frac{m_1 + m_3}{3} , \quad p_2 = \frac{m_3 - 2m_1}{3} \quad , p_3 = \frac{m_1 - 2m_3}{3} \tag{33}$$

Note that $r_1$ , $r_2$ , $r_3$ $p_1$ , $p_2$ , $p_3$ also satisfy the relation

$$r_1 r_2 r_3 = 1 \text{ and } p_1 + p_2 + p_3 = 0 . \tag{34}$$

Now, we use the power law to solve the integral part in the above equations. In a recent paper, Kotub Uddin *et al.*[23] has established a result in the context of $f(R)$ gravity which shows that

$$F \propto a^m \quad , \quad \text{i.e. } F = h a^m \quad , \tag{35}$$

where $h$ is the constant of proportionality and $m$ is any integer [here $m = -2$ ].

Using equations (11) and (35) for $k = 1, \alpha = 2$ and $m = -2$ and equation (20) in the equations (29)-(31) , we obtain the scale factors as

$$t^{q_1} = r_1 (e^{2t} - 1)^{\frac{1}{2}} \exp\left[ \frac{p_1}{h} \tan^{-1} \left(e^{2t} - 1\right)^{\frac{1}{2}} \right] , \tag{36}$$

$$t^{q_2} = r_2 (e^{2t} - 1)^{\frac{1}{2}} \exp\left[ \frac{p_2}{h} \tan^{-1} \left(e^{2t} - 1\right)^{\frac{1}{2}} \right] , \tag{37}$$

$$t^{q_3} = r_3 (e^{2t} - 1)^{\frac{1}{2}} \exp\left[ \frac{p_3}{h} \tan^{-1} \left(e^{2t} - 1\right)^{\frac{1}{2}} \right] , \tag{38}$$

where $r_1$ , $r_2$ , $r_3$ and $p_1, p_2$ , $p_3$ are the constants of integration.

Using equations (36)-(38) , the directional Hubble parameters in the directions of $x$ , $y$ and $z$ -axes are

ISSN: 2319-8753

**International Journal of Innovative Research in Science, Engineering and Technology**

*(An ISO 3297: 2007 Certified Organization)*

**Vol. 3, Issue 6, June 2014**

$$H_x = \frac{p_1}{h\left(e^{2t}-1\right)^{\frac{1}{2}}} + \frac{e^{2t}}{(e^{2t}-1)}, H_x = \frac{p_2}{h\left(e^{2t}-1\right)^{\frac{1}{2}}} + \frac{e^{2t}}{(e^{2t}-1)} \text{ and } H_x = \frac{p_3}{h\left(e^{2t}-1\right)^{\frac{1}{2}}} + \frac{e^{2t}}{(e^{2t}-1)}. \tag{39}$$

The Mean Hubble parameter is given by

$$H = \frac{e^{2t}}{(e^{2t}-1)} . \tag{40}$$

The volume of the universe is given by

$$V = \left(e^{2t}-1\right)^{\frac{3}{2}} . \tag{41}$$

The expansion scalar $\theta = 3H$ is given by

$$\theta = \frac{3e^{2t}}{(e^{2t}-1)} . \tag{42}$$

The mean anisotropy parameter $\Delta$ is given by

$$\Delta = \frac{\left(p_1^2 + p_2^2 + p_3^2\right)}{3h^2} \frac{(e^{2t}-1)}{e^{4t}} . \tag{43}$$

The shear scalar $\sigma^2$ is given by

$$\sigma^2 = \frac{\left(p_1^2 + p_2^2 + p_3^2\right)}{2h^2} \frac{1}{(e^{2t}-1)} . \tag{44}$$

The deceleration parameter $q$ is given by

$$q = \frac{2}{e^{2t}} - 1 . \tag{45}$$

Using equation (6), we obtain the function $f(R)$ as

$$f(R) = \frac{1}{2\left(e^{2t}-1\right)^3}\left[6h\left(2-e^{2t}\right) + h\left(e^{2t}-1\right)^2 R\right]. \tag{46}$$

The Ricci scalar is given by

$$R = -\left[\frac{6e^{2t}}{\left(e^{2t}-1\right)} - \frac{\left(p_1 p_2 + p_2 p_3 + p_3 p_1\right)}{h^2\left(e^{2t}-1\right)}\right]. \tag{47}$$

## V. CONCLUSION

i) Exact vacuum solution of Bianchi Type-I (Kasner form) model in $f(R)$ theory of gravity is obtained by using a special form of deceleration parameter.

ii) From equation (41), it is observed that the spatial volume $V$ vanishes at $t = 0$. It expands exponentially as $t$ increases and becomes infinitely large as $t \mapsto \infty$.

iii) From equation (42), it is observed that the expansion scalar $\theta$ starts with infinite value at $t = 0$ and then becomes constant after some finite time.

ISSN: 2319-8753

**International Journal of Innovative Research in Science, Engineering and Technology**

*(An ISO 3297: 2007 Certified Organization)*

**Vol. 3, Issue 6, June 2014**

iv) It is observed that in Bianchi type-I model, the anisotropy increases as time increases and then it decreases to zero after some time and remains zero. Hence, the model reaches to isotropy after some finite time.

v) It is seen that, the results obtained here are similar to the results obtained earlier by Reddy *et al.*[15]..

## REFERENCES

[1] Weyl H.: Ann. Phys., 59, 101 (1919).

[2] Eddington A.S.: The mathematical theory of relativity (Cambridge university press, Cambridge, (1923)

[3] Carroll, S.M., et al. "Is cosmic speed-up due to new gravitational physics?" Phys. Rev. D, 70, pp.043528, (2004).

[4] Nojiri S and Odintsov S D "Modified gravity and its reconstruction from the universe expansion history" J. Phys. Conf.Ser. 66, 012005,(2007).

[5] Nojiri S and Odintsov S D . " Future evolution and finite-time singularities in F(R) gravity unifying inflation and cosmic acceleration" Phys. Rev. D, 78, 046006, (2008).

[6] Multamaki,T., Vilja, I. "Spherically symmetric solutions of modified field equations in f(R) theories of gravity" Phys. Rev. D 74, 064022 (2006).

[7] Multamaki,T., Vilja, I. "Static spherically symmetric perfect fluid solutions in f(R) theories of gravity" Phys. Rev. D 76, 064021(2007).

[8] Sharif,M.,Shamir,M.F. : Class. Quantum Grav. 26 , 235020 (2009).

[9] Sharif,M.,Shamir,M.F. "Energy distribution in $f(R)$ gravity" Gen. Relativ. Grav. 42, pp.1557-1569 (2010).

[10] Shamir,M.F. "Plane Symmetric Vacuum Bianchi Type III Cosmology in f(R) Gravity" Int. J. Theor. Phys. 50, pp.637-643 (2011).

[11] Sharif, M., Kausar, H.R." Anisotropic fluid and Bianchi type III model in $f(R)$ gravity" Phys. Lett. B, 697, pp.1-6, (2011).

[12] Aktas, C., et al." Behaviors of dark energy and mesonic scalar field for anisotropic universe in $f(R)$ gravity" Phys. Lett. B,707, pp.237-242 (2012).

[13] Adhav, K.S." Kantowski–Sachs string cosmological model in f(R) theory of gravity" Can.J.Phys.,90,pp.119-123, (2012).

[14] Singh,V . and Singh, C.P. "Functional form of $f(R)$ with power-law expansion in anisotropic model" Astrophys Space Sci, 346, pp.285-289, (2013).

[15] Reddy, DRK et al." Vacuum Solutions of Bianchi Type-I and V Models in f(R) Gravity with a Special Form of Deceleration Parameter" Int.Jou.Sci.Adv.Tech., 4, 3, pp.23-33 (2014).

[16] Berman, M.S." A special law of variation for Hubble's parameter" Nuovo Cimento B, 74, 2, pp.182-186, (1983)

[17] Akarsu O.and Kilinc C.B. "Bianchi type III models with anisotropic dark energy" Gen. Relat. Gravit. 42, pp.763-775 (2010).

[18] Pradhan et al." Bianchi Type-I Anisotropic Dark Energy Model with Constant Deceleration Parameter" Int J Theor Phys, 50(9), pp.2923-2938 (2011).

[19] Adhav. K.S. et al. "Bianchi type-$VI_0$ cosmological models with anisotropic dark energy" Astrophysics and Space Science, 332(2), pp.497-502(2011).

[20] Adhav. K.S. et al. "Kantowski-Sachs cosmological models with anisotropic dark energy" Cent. Eur. J. Phys., 9(4) , pp.919-925 (2011). [21] Cunha J.V. and Lima J.A.S." Transition redshift: new kinematic constraints from supernovae" Mon. Not. R. Astron. Soc.: 390(1),pp.210-217 (2008).

[22] Singha A.K. and Debnath U."Accelerating Universe with a Special Form of Decelerating Parameter" Int. J. Theor Phys., 48(2), pp.351-356 (2009).

[23] Kotub Uddin et al. "Cosmological perturbations in Palatini-modified gravity" Class. Quantum Grav. 24 , pp.3951-3962 (2007).