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Apodised optical systems in resolving two point objects with shrink apertures

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Abstract: The influence of apodisation on the resolution of two point objects in the case of shrink apertures of rotationally symmetric optical systems has been studied by applying the modified Sparrow criterion introduced by Asakura to suit the case of unequally bright object points. The results are presented in terms of Rayleigh and Sparrow limits obtained for coherent, partially coherent and incoherent illuminations. It is found that shaping of the aperture along with the chosen Hanning amplitude filter is effective in increasing the resolving power of the optical imaging systems.

Keywords: Rotationally Symmetric Systems, Shrink aperture, Hanning amplitude filters.

I. INTRODUCTION

Two-point resolution studies forms an interesting area when the two point objects are situated within a close neighborhood of each other. The resolution necessary to satisfy astrophysical observations requires large-diameter optical systems. However, the size required for high-resolution optical systems is not practical in implementation. A shrink aperture system is composed to assess the performance of optical imaging systems in judging depending upon its ability in resolving closely situated object points. There are a number of quality criteria in the field of image science; two-point resolution criterion is one of the most important and most basic measures proposed for judging the quality of optical imaging systems [1]. Rayleigh [2] criterion states that the two point sources are just resolved if the central maximum of one falls on the first minimum of central maxima produced by the other point source. Sparrow [3] proposed an alternative criterion according to which the two points are just resolved if the second derivative of the resultant image intensity distribution vanishes at the point midway between the respective Gaussian image points. Asakura [4] has modified the Sparrow criterion to be applicable for the more realistic case of unequally bright object points. In the present study this modified Sparrow criterion has been applied to optical imaging systems apodised by the first order Hanning amplitude filters.

International Journal of Innovative Research in Science, Engineering and Technology

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Vol. 3, Issue 3, March 2014

II. THEORY

The theoretical expression as given by Hopkins and Barham [5], for the total image intensity distribution in the image plane of an optical system, as a function of the reduced co-ordinate Z , is given by,

$$I(z) = |A(Z+B)|^2 + \alpha |A(Z-B)|^2 + 2\sqrt{\alpha} \gamma(Z_0) |A(Z+B)| |A(Z-B)| \quad \text{where } 2B=Z_0 \text{ is the separation} \tag{1}$$

between the two object points, α is the ratio of their intensities, $\gamma(Z_0)$ is the degree of spatial coherence of the illumination. Z is the dimensionless diffraction variable. $\alpha = 1$ gives the case of equal intensities while $\alpha \neq 1$ corresponds to unequally bright object points. The coherent and the incoherent extremes of illuminations are given by $\gamma = 1$ and $\gamma = 0$, respectively; whereas $0 < \gamma < 1$ is for the partially coherent illumination. The two object points are situated with a separation Z_0 at the same distance $Z_0/2$ on either side of the optical axis. $I(Z)$ is image intensity distribution as a function of Z , which is measured from the axis of the optical system. $A(Z+B)$ and $A(Z-B)$ are the normalized amplitude point spread functions for circular apertures and are given by the following expressions:

$$A(Z+B) = 2 \int_0^\epsilon J_0[(Z+B)r] e^{-i\phi_d} r dr \tag{2}$$

$$A(Z-B) = 2 \int_0^\epsilon J_0[(Z-B)r] e^{-i\phi_d} r dr \tag{3}$$

J_0 being the Bessel function of first kind and zero order, r is the normalized distance of a general point on the exit pupil varying from 0 to 1. In the present case, as the first order Hanning amplitude filter is used for apodizing the optical system, the above Eqs. (2) and (3) become,

$$A(Z+B) = 2 \int_0^\epsilon f(r) J_0[(Z+B)r] e^{-i\phi_d} r dr \tag{4}$$

$$A(Z-B) = 2 \int_0^\epsilon f(r) J_0[(Z-B)r] e^{-i\phi_d} r dr \tag{5}$$

where $f(r)$ is the pupil function of the first order Hanning amplitude filter [6] and is given by

$$f(r) = \text{Cos}(\pi \beta r) \tag{6}$$

The variable β is the apodization parameter controlling the degree of non-uniformity of the transmission over the exit pupil in the range of $0 \leq \beta \leq 1$. For $\beta = 0$, $f(r) = 1$ which implies the uniform transmittance

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 3, Issue 3, March 2014

over the exit pupil. Thereby, for $\beta = 0$ the filter function becomes an Airy pupil. Φ_d if the defocus parameter. For chosen apodizer the amplitude transmittance decreases monotonically from the center towards the edges of the pupil. Higher spatial frequency components of the object are diffracted by a larger angle and hence these go predominantly through the edge of the aperture. As the pupil transmittance is decreased at the edges as compared to that of the center, which results in the reduction in the higher spatial frequency components in the image. This manifests as partial or complete suppression of the undesired optical side lobes, which consequently enhances imaging characteristics. However, for shrink apertures the expressions for the amplitude PSF can be expressed as

$$A(Z \pm B) = 2 \int_{\varepsilon}^{\varepsilon} f(r) J_0 [(Z \pm B)r] e^{-i\phi_d} r dr \quad (7)$$

where ε is the symmetric obscuration parameter whose value specifies the amount of shrink in the aperture. Therefore, the expression for the total image intensity distribution is given by

$$I(Z) = |A(Z + B)|^2 + \alpha |A(Z - B)|^2 + 2\sqrt{\alpha} \gamma |A(Z + B)| |A(Z - B)| \quad (8)$$

The modified Sparrow criterion introduced by Asakura [4] states that, “the resolution is retained when the second derivative of the image intensity distribution vanishes at a certain point ($Z = Z_0^1$) between two Gaussian image points, with the condition that this point Z_0^1 should be a solution for the first derivative of the image intensity distribution becoming zero”. This can be expressed mathematically as

$$\left. \frac{\partial^2 I(Z)}{\partial Z^2} \right|_{Z=Z_0^1} = 0 \quad (9)$$

III. RESULTS AND INTERPRETATIONS

Expressions (8) and (9) have been used to compute the image intensity distribution of the two point objects and the Sparrow and Rayleigh limits of the optical systems with circular and shrink apertures apodised with the chosen amplitude filter. The results have been compared with that of the Airy case. Table-1 gives the computed values of Sparrow and Rayleigh limits (SL & RL) for clear aperture ($\beta = 0$), for various degrees of defect-of-focus when the optical imaging system is illuminated with coherent light and the two object points are of equal intensities ($c = 1$). With shrink aperture ($\varepsilon \neq 1$) the sparrows limit as

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 3, Issue 3, March 2014

well as the Rayleigh limits decreases with decrease in the value of ϵ , thereby enhancing the resolving power of the optical system. This feature is observed for various defocused planes.

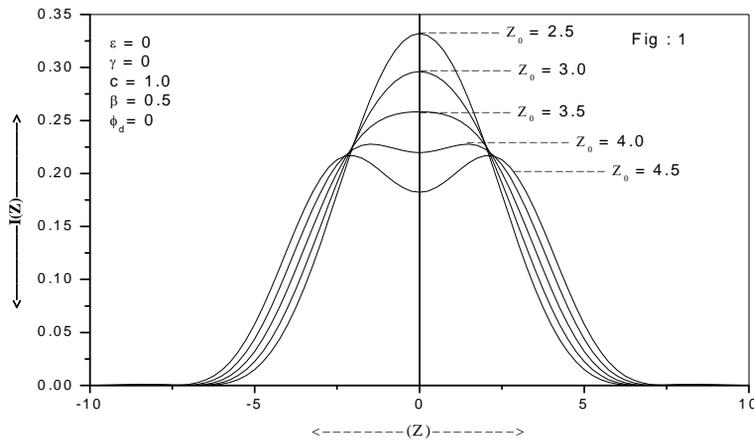


Fig.1 depicts the intensity distribution curves for various values of separation between two object points (z_0), when the apodisation parameter $\beta = 0.5$. The two points are said to be resolved in Sparrow sense for $z_0 = 3.7650$ dimensionless diffraction units. There is a well defined resolution for $z_0 = 4.5$.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 3, Issue 3, March 2014

TABLE - 1

$\beta = 0, \gamma = c = 1.0$

ϵ	SL	RL	SL	RL	SL	RL	SL	RL	SL	RL	
$\phi_d = 0$	0.20	4.2170	4.8810	4.0440	4.6810	3.8480	4.5020	3.6350	4.3330	3.2980	4.1640
	0.40	4.5810	5.1940	4.3810	4.9800	4.1770	4.8000	3.9610	4.6330	3.6210	4.4670
	0.60	4.9170	5.4380	4.7010	5.2330	4.4970	5.0610	4.2820	4.9010	3.9440	4.7400
	0.80	5.1720	5.5610	4.9860	5.4200	4.8000	5.2760	4.5990	5.1320	4.2790	4.9830
	1.00	5.1670	6.0570	5.1570	5.5210	5.0480	5.4310	4.9000	5.3180	4.6380	5.1890
$\phi_d = \pi/2$	0.20	4.2510	4.9620	4.0550	4.7250	3.8520	4.5310	3.6350	4.3550	3.2940	4.1830
	0.40	4.6420	5.3080	4.4110	5.0440	4.1940	4.8460	3.9700	4.6700	3.6240	4.4980
	0.60	5.0180	5.5740	4.7530	5.3160	4.5320	5.1250	4.3060	4.9540	3.9590	4.7860
	0.80	5.2880	5.6810	5.0620	5.5110	4.8580	5.3530	4.6450	5.2000	4.3110	5.0440
	1.00	5.2280	6.0180	5.2250	5.6030	5.1170	5.5100	4.9670	5.3940	4.7020	5.2610
$\phi_d = \pi$	0.20	4.3160	5.1110	4.0790	4.7990	3.8630	4.5820	3.6380	4.3950	3.2910	4.2160
	0.40	4.7560	5.5200	4.4620	5.1570	4.2260	4.9270	3.9900	4.7340	3.6330	4.5530
	0.60	5.2050	5.7980	4.8470	5.4610	4.5940	5.2370	4.3490	5.0470	3.9860	4.8660
	0.80	5.4420	5.8490	5.1940	5.6600	4.9600	5.4820	4.7260	5.3160	4.3690	5.1490
	1.00	5.2970	6.2070	5.3200	5.7230	5.2230	5.6360	5.0800	5.5190	4.8220	5.3820
$\phi_d = 3\pi/2$	0.20	4.4190	5.3800	4.1200	4.9260	3.8820	4.6680	3.6460	4.4610	3.2890	4.2720
	0.40	4.9550	5.9080	4.5520	5.3520	4.2800	5.0650	4.0230	4.8430	3.6480	4.6450
	0.60	5.5700	6.1070	5.0150	5.7000	4.7010	5.4250	4.4230	5.2030	4.0310	5.0010
	0.80	5.5710	6.0160	5.3940	5.8710	5.1390	5.6870	4.8680	5.5060	4.4700	5.3220
	1.00	5.3590	6.4490	5.4320	5.8680	5.3610	5.8070	5.2450	5.7080	5.0630	5.5740
$\phi_d = 2\pi$	0.20	4.5980	6.0180	4.1880	5.1580	3.9130	4.8180	3.6590	4.5740	3.2880	4.3640
	0.40	5.4230	6.4330	4.7110	5.7190	4.3680	5.3140	4.0780	5.0340	3.6750	4.8020
	0.60	5.8310	6.3420	5.3570	6.0570	4.9020	5.7460	4.5550	5.4780	4.1080	5.2370
	0.80	5.6510	6.1360	5.6010	6.0990	5.4210	5.9720	5.1500	5.8080	—	—
	1.00	5.4170	6.6070	5.5430	6.0110	5.5140	5.9970	5.4380	5.9400	—	—

Fig.2 shows the intensity distribution curves for shrink aperture with $\epsilon = 0.2$ for incoherent illumination when the intensity ratio $c = 0.8$ for defocused plane with $\phi_d = \pi$. It is clear observed that for $z_0 = 4.5$ the two points are well resolved.

Table-2 provides the computed values of Sparrow and Rayleigh limits (SL & RL) for highly apodised aperture ($\beta = 1$), for various degrees of defect-of-focus when the optical imaging system is illuminated with coherent light and the two object points are of equal intensities ($c = 1$). With shrink aperture ($\epsilon \neq 1$)

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 3, Issue 3, March 2014

the sparrow limit as well as the Rayleigh limits decreases with decrease in the value of ϵ . On comparing the tabulated values from both the tables 1 & 2, it can be concluded that with apodisation the resolution limit decreases thereby enhancing the resolving power of the optical system.

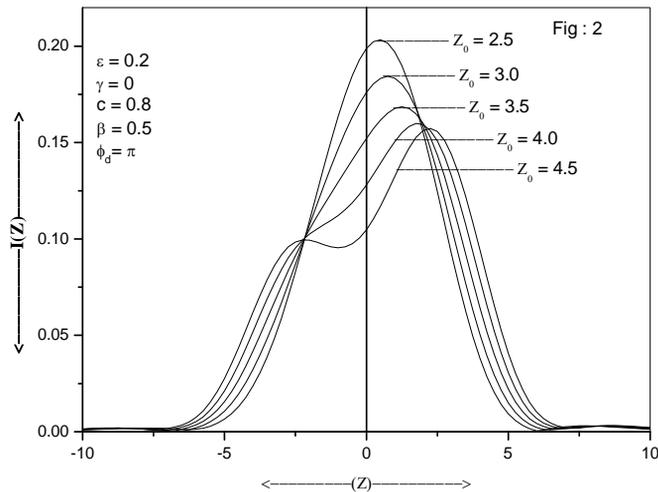


Fig.2 shows the intensity distribution curves for shrink aperture with $\epsilon = 0.2$ for incoherent illumination when the intensity ratio $c = 0.8$ for defocused plane with $\phi_d = \pi$. It is clear observed that for $z_0 = 4.5$ the two points are well resolved

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 3, Issue 3, March 2014

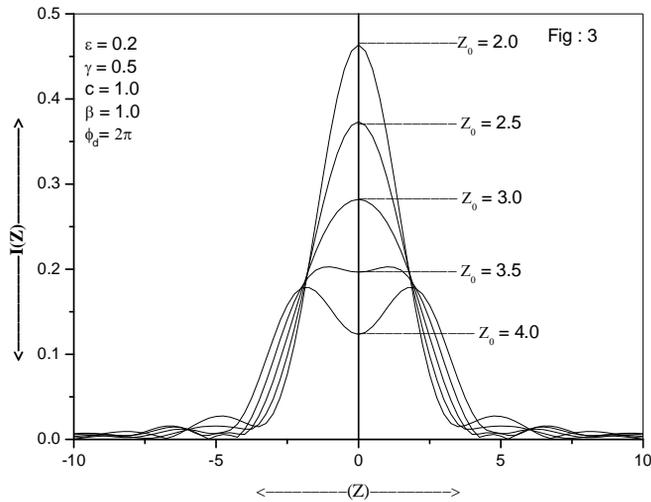


Fig.3 shows the intensity distribution curves for shrink aperture with $\epsilon = 0.2$ for partially coherent illumination when the intensity ratio $c = 1$ for defocused plane with $\phi_d = 2\pi$. It is clear observed that for $z_0 = 3.5$ the two points are well resolved in the Sparrow sense. There is a well defined resolution for $z_0 = 4.0$.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 3, Issue 3, March 2014

TABLE - 2

$\beta = \gamma = c = 1.0$

\mathcal{E}	SL	RL	SL	RL	SL	RL	SL	RL	SL	RL	
$\phi_d = 0$	0.20	3.3610	3.9360	3.4310	3.9310	3.3390	3.8390	3.1960	3.7260	2.9350	3.6010
	0.40	3.6990	4.2220	3.7390	4.2050	3.6400	4.1120	3.4940	3.9990	3.2310	3.8740
	0.60	4.0060	4.4660	4.0350	4.4450	3.9350	4.3560	3.7910	4.2500	3.5330	4.1280
	0.80	4.2410	6.0010	4.2990	4.6210	4.2200	4.5590	4.0920	4.4690	3.8510	4.3600
	1.00	4.0650	6.2010	4.3600	5.6500	4.4170	4.6740	4.3690	4.6320	4.2080	4.5510
$\phi_d = \pi/2$	0.20	3.5170	4.0750	3.5190	4.0310	3.4040	3.9220	3.2470	3.7990	2.9720	3.6660
	0.40	3.8380	4.3440	3.8220	4.2940	3.7030	4.1880	3.5450	4.0680	3.2690	3.9380
	0.60	4.1210	4.5490	4.1060	4.5140	3.9910	4.4190	3.8370	4.3090	3.5690	4.1850
	0.80	4.2830	6.0010	4.3420	4.6570	4.2600	4.6000	4.1290	4.5120	3.8830	4.4030
	1.00	4.1000	6.0000	4.3700	5.6600	4.4310	4.6980	4.3850	4.6570	4.2280	4.5780
$\phi_d = \pi$	0.20	3.6630	4.2170	3.6060	4.1340	3.4660	4.0070	3.2950	3.8740	3.0070	3.7330
	0.40	3.9730	4.4680	3.9050	4.3880	3.7660	4.2680	3.5940	4.1400	3.3050	4.0040
	0.60	4.2350	4.6290	4.1810	4.5890	4.0500	4.4880	3.8860	4.3740	3.6070	4.2460
	0.80	4.3040	6.0000	4.3920	4.7030	4.3080	4.6520	4.1730	4.5640	3.9200	4.4550
	1.00	4.1000	6.0000	4.3900	5.6800	4.4540	4.7340	4.4130	4.6950	4.2610	4.6170
$\phi_d = 3\pi/2$	0.20	3.8160	4.3760	3.6950	4.2460	3.5290	4.1000	3.3420	3.9540	3.0390	3.8040
	0.40	4.1190	4.6080	3.9960	4.4950	3.8320	4.3600	3.6450	4.2210	3.3420	4.0780
	0.60	4.3550	5.8450	4.2670	4.6780	4.1180	4.5710	3.9410	4.4510	3.6480	4.3190
	0.80	4.3110	6.0010	4.4530	4.7660	4.3690	4.7190	4.2290	4.6320	3.9660	4.5210
	1.00	4.1610	6.0510	4.4220	5.6000	4.4920	4.7880	4.4550	4.7500	4.3130	4.6730
$\phi_d = 2\pi$	0.20	3.9850	4.5770	3.7890	4.3820	3.5940	4.2090	3.3880	4.0480	3.0680	3.8870
	0.40	4.2950	4.7760	4.1020	4.6290	3.9080	4.4740	3.7020	4.3220	3.3800	4.1690
	0.60	4.4950	5.8050	4.3770	4.7910	4.2040	4.6790	4.0090	4.5520	3.6960	4.4130
	0.80	4.3540	6.0440	4.5300	4.8530	4.4530	4.8110	4.3060	4.7240	4.0300	4.6130
	1.00	4.2450	6.0010	4.4690	5.6010	4.5450	4.8660	4.5170	4.8310	4.4010	4.7560

III. CONCLUSIONS

The process of apodising the optical system with Hanning amplitude filter, suppresses fully or partially the optical side-lobes of the individual point spread functions. The apodisation parameter is effective in shaping the point spread function so that there is an improvement of resolution of two point objects rendering the optical system to be more effective in their resolution.. Hence, the Hanning amplitude filters are effective in enhancing the resolving power of the given optical imaging system.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 3, Issue 3, March 2014

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