

# **Recovery Service Policy in Railway System during Disruption**

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**Abstract:** This paper is based on the decisions on recovery services for short-term disruptions during emergencies in railways system. The basis is to examine whether the railway management should collaborate with a local bus provider to provide the recovery service and how to price to compensate the service. The average arrival time of bus which is the leading decision variable, is formulated and analyzed. Both theoretical and numerical sensitivity analyses are measured to discuss the critical factors affecting the decisions.

**Keywords:** Transport systems, Disruption, Recovery service, Railway systems, Optimization

## **I. INTRODUCTION**

Developing better solution for train rescheduling problems is drawing the attention of researchers for decades. Railway authorities aim to provide transportation services to their customers in a safe as well as effective and efficient manner but the railway system always faces disruptions caused by unexpected events such as track misalignment, power failure and natural disasters etc., which result in inconvenience to passengers and reduction of the desired service levels. Therefore, it is important to have an effective disruption handling procedure to provide high-quality passenger service. The challenge is to respond to short-term, unplanned disruptions that require real-time and fast recovery decisions especially for passenger trains in India. When such a short-term disruption occurs, how should the railway system react? – should it wait till the problem is solved and let the passengers wait or find their own transport alternatives, or offer a temporary recovery service to the impacted passengers?

This turns out that railway system in many countries has attempted to apply similar strategies for un-planned short-term disruptions in the past. As, they use their own buses to provide temporary service to the passengers. Their other alternative is to hire buses from a local bus provider to ensure faster rail replacement service to minimize customer inconvenience. As, buses can easily arrive on the spot and are a perfect solution for replacement in the beginning. Through temporary bus service for recovery the railway system expect a reduced recovery time. In addition, it also relieves the pressure of railway system's control centres, as the arrangement of the buses is conducted by the local bus provider. This gives ample time to concentrate on recovering the system.

The recovery method: Buses will be ordered only for the longer duration of the disruption. The railway management order a required number of buses. The buses will immediately arrive at the affected station and then take the passengers to the next station or any nearby bus or subway station, without deviating the route. Service charges to the bus provider will be paid by the railway. Passengers will be advised not to call the transport on their own; otherwise they would have to pay for the service.

## **II. RELATED LITERATURE**

Firstly Clausen, J., Larsen, J., Larsen, A., Hansen, J., (2001) presented a theory on disruption management between planning and execution. In the same year Bertsimas, D., Perakis, G., (2001) proposed a dynamic pricing approach. Kachani, S., Perakis, G., (2002) discussed a fluid model of dynamic pricing and inventory management for make-to-stock manufacturing systems. Bichler, M., Kalagnanam, J., Katircioglu, K., King, A.J., Lawrence, R.D., Lee, H.S., Lin, G.Y., Lu, Y., (2002) share the applications of flexible pricing in business-to-business electronic commerce. Schade, J., Schlag, B., (2003) coded the acceptability of urban transport pricing strategies. Perakis, G., Sood, A., (2004) focused

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on competitive Multi-period Pricing with Fixed Inventories. Klahn, V., (2004) discuss the effective management of reduced track capacity caused by maintenance possessions can minimize disruption to passengers and offer practical transport alternatives. Freling, R., Lentink, R.M., Waglmans, A.P.M., (2004) present a decision support system for crew planning in passenger transportation using a flexible branch-and-price algorithm. Clausen, J., (2007) proposed a disruption management in passenger transportation—from air to tracks. D’Ariano, A., Pranzo, M., (2009) presented an advanced real-time train dispatching system for minimizing the propagation of delays in a dispatching area under severe disturbances. Corman, F., D’Ariano, A., Pacciarelli, D., Pranzo, M., (2010) showed a tabu search algorithm for rerouting trains during rail operations. Jia, J., Hu, Q., (2011) discuss a dynamic ordering and pricing for a perishable goods supply chain.

### III. NOTATION AND ASSUMPTIONS

In the decision-making process, time is a critical factor. There are three time related parameters in this paper. The first two input parameters whose values can be calculated at the time of disruption are as follows:

1. The average duration of a disruption  $t_d$  and
2.  $t_s$  is the average time of round-trip for each bus.
3. The last parameter  $t_c$ , is the average time it takes for the buses to arrive at the disruption site, and is treated as a decision variable for the local bus provider.

Clearly,  $t_c$  affects the railways recovery service payment, which in turn impacts the railways additional profit. To proceed with our modeling process, we make the following assumption:

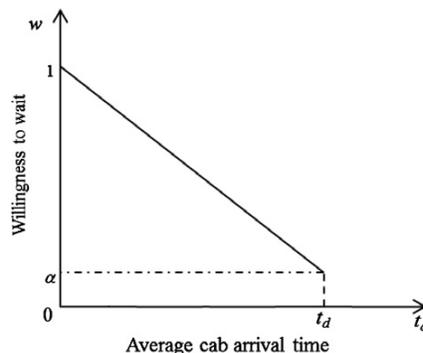
1. Aggregate passenger’s behaviour is assumed instead of each one. The aggregate passenger’s willingness to wait,  $w(t_c)$ , is supposed to decrease linearly as the bus’s arrival time ( $t_c$ ) increases and can be calculated from equation (1). Fig. 2 shows the relationship between the willingness to wait and the average bus’s arrival time. This assumption includes the fact that if buses arrive instantaneously, then all the passengers will stay to use the service; and if bus’s arrival time is about the same as the duration of the disruption, then only a small portion of passengers will wait for the service.

$$w(t_c) = 1 - (1 - \alpha) \frac{t_c}{t_d}, 0 < t_c \leq t_d, \quad \text{where} \begin{cases} w(t_c) \rightarrow 1, & \text{if } t_c \rightarrow 0 \\ w(t_c) \rightarrow \alpha & \text{if } t_c \rightarrow t_d \end{cases} \quad (1)$$

2. Let  $C(t_c)$  be the cost of arranging required number of buses available for the purpose. This cost includes the drivers’ efforts and all related a service cost which is a nonlinear decreasing function with respect to  $t_c$ .

$$C(t_c) = kt_c^{-1} \quad (2)$$

Where  $k$  is a scale factor.



**Fig.2.** Aggregate passenger’s willingness to wait vs. average buses arrival time.

We consider two pricing methods to pay the local bus provider:

- (i) Fixed payment with  $p_0$  being the constant unit time payment; and

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(ii) Linear pricing with the payment rate decreasing linearly with the bus's arrival time ( $t_c$ ), shown in the Fig. 3. This pricing scheme is defined by a pair of parameters,  $(p_m, \beta)$ .

$$p(t_c) = p_m \left( 1 - (1 - \beta) \frac{t_c}{t_d} \right), 0 < t_c \leq t_d; \text{ where } \begin{cases} p(t_c) \rightarrow p_m, & \text{if } t_c \rightarrow 0 \\ p(t_c) \rightarrow \beta p_m, & \text{if } t_c \rightarrow t_d \end{cases} \quad (3)$$

Where

$c$ : The capacity of each bus;

$P$ : Average passenger rate during a disruption;

$k$ : A scale factor for the total service costs during a disruption;

$p_m$ : Maximum payment rate for service during a disruption;

$p_0$ : Fixed payment rate for service during a disruption;

$\alpha$ : The average passenger's lowest level of willingness to wait for the service during a disruption;

$\beta$ : Fraction of the maximum payment for the service during a disruption;

$c_i$ : The loss to bus provider from each passenger that is not served or cannot wait to use the recovery service;

$t_c$ : The average arrival time of bus to the disruption site which is a decision variable;

$t_d$ : The average duration of a typical disruption;

$t_s$ : The service time of bus from the disruption site to the next station, and then return to the disruption site for more pickups.

### IV. MODELS AND ANALYSES

#### Scenario1. Fixed Payment Scheme

The additional profit  $\Pi_F(t_c)$  of local bus provider for a fixed payment rate  $p_0$ , during a disruption, is calculated as

$$\Pi_F(t_c | p_0) = p_0 t_s \cdot \frac{P}{c} \left( 1 - (1 - \alpha) \frac{t_c}{t_d} \right) - k t_c^{-1} \quad (4)$$

Here the first terms calculate the total revenue received from recovery service and the second term is the aggregate cost of arranging buses, drivers' adjustments and efforts, and all other cost elements needed for the recovery service. Setting the first-order derivative of (4) results in the following result:

$$t_{(F)c}^* = \left( \frac{k c t_d}{p_0 t_s P (1 - \alpha)} \right)^{1/2} \quad (5)$$

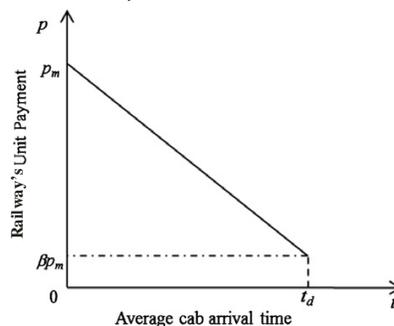


Fig.3. payment rate vs. Average bus arrival time.

Let  $O_F(t_c)$  is the difference between the loss of doing nothing during a disruption and the total cost of working to provide recovery service and the partial loss of passengers unwilling to wait. So  $O(t_c)$  can be computed by

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$$O_F(t_c) = P_{C_l} - \left[ p_0 t_s \frac{P}{C} \left( 1 - (1 - \alpha) \frac{t_c}{t_d} \right) + c_l P (1 - \alpha) \frac{t_c}{t_d} \right] \quad (6)$$

After some simplification, (6) becomes

$$O_F(t_c) = P \left( 1 - (1 - \alpha) \frac{t_c}{t_d} \right) \left( C_{(F)l} - p_0 t_s / C \right) \quad (7)$$

It is shown clearly that as long as the following condition is met:

$$C_{(F)l} > p_0 t_s / C \quad (8)$$

**Scenario2. Linear Payment Scheme**

Under this scenario, the additional profit  $\Pi_L(t_c)$  of local bus provider during a disruption with a linearly decreasing payment rate, is calculated as

$$\Pi_L(t_c) = p_m \left( 1 - (1 - \beta) \frac{t_c}{t_d} \right) t_s \cdot \frac{P}{c} \left( 1 - (1 - \alpha) \frac{t_c}{t_d} \right) - kt_c^{-1} \quad (9)$$

$$\frac{d\Pi_L}{dt_c} = -\frac{p_m t_s \cdot P}{ct_d} \left[ (2 - \alpha - \beta) - 2(1 - \alpha)(1 - \beta) \frac{t_c}{t_d} \right] + kt_c^{-2} \quad (10)$$

$$\frac{d^2\Pi_L}{dt_c^2} = \frac{2p_m t_s \cdot P}{ct_d^2} (1 - \alpha)(1 - \beta) - 2kt_c^{-3} \quad (11)$$

It will be concave if and only if  $t_c < \left( \frac{kct_d^2}{p_m t_s P (1 - \alpha)(1 - \beta)} \right)^{1/3}$

Under this condition, equate (10) to zero and letting  $\frac{p_m t_s P}{ct_d} = M : 1 - \alpha = \bar{\alpha}; 1 - \beta = \bar{\beta}$ ,

lead to the following equation:

$$t_c^3 - \frac{(\bar{\alpha} + \bar{\beta})t_d}{2\bar{\alpha}\bar{\beta}} t_c^2 - \frac{t_d k}{2M\bar{\alpha}\bar{\beta}} = 0 \quad (12)$$

Using the cubic root formula or a numerical search technique can identify the real value of the best  $t_c$ , which is denoted as  $t_{(L),c}^*$

Similar to Scenario 1, we examine the reduced loss given the best arrival time as follows

$$\begin{aligned} O_L(t_c) &= P_{C_l} - \left[ p_m \left( 1 - (1 - \beta) \frac{t_c}{t_d} \right) t_s \frac{P}{c} \left( 1 - (1 - \alpha) \frac{t_c}{t_d} \right) + c_l P (1 - \alpha) \frac{t_c}{t_d} \right] \\ &= P \left( 1 - (1 - \alpha) \frac{t_c}{t_d} \right) \left[ c_l - \frac{p_m t_s}{C} \left( 1 - (1 - \beta) \frac{t_c}{t_d} \right) \right] \end{aligned} \quad (13)$$

if the unit passenger loss,  $c_l$ , under linear payment scheme satisfies the following relationship,

$$c_{(L)l} > \left( p_m t_s / c \right) \left( 1 - (1 - \beta) \frac{t_c}{t_d} \right) \quad (14)$$

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then the railway should consider working with the bus provider to provide a recovery service. The condition in (14) indicates a upper bound of the unit passenger loss as

$$c_{(L)} > p_m t_s / c \tag{15}$$

Using the results obtained above, we state the railways decision as to whether to use the fixed or linear pricing scheme.

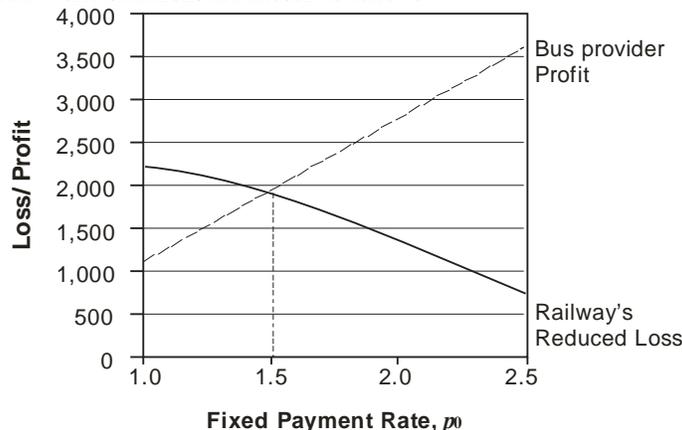
### V. NUMERICAL ILLUSTRATIONS

We give a few numerical examples to show the behaviour of the railways reduced loss and the profit of bus provider when the pricing parameters change. The first example considers the fixed-pricing situation, under which the fixed payment rate  $p_0$ , is varied from 1.0 to 2.5. The optimal average bus arrival time ( $t_c$ ) under each payment rate (Eq. (5) is first calculated, and the associated average passengers' willingness to wait (Eq. (1), the profit of bus provider (Eq. (4), and the railways reduced loss (Eq. (7) are then computed. The results are plotted in Fig. 4, which shows that the reduced loss decreases monotonically with the increase of the fixed payment rate, and such decrease is caused by higher recovery service cost needed to compensate the bus provider. Notice that the two curves in Fig. 4 intersect at one point, where the values of the two factors decision functions reach equilibrium. This chart provides a global view for the railways managers to make a decision on payment rate, and the break-even point indicates such a decision; in particular, the payment rate to be adopted in this example is  $p_0^* = 1.5$  (\$/min).

We have also calculated the same items when the maximum linear payment rate,  $p_m$ , changes from 2.0 to 3.5 (while the decrease rate,  $\beta$ , holds at 0.3), and report the results along with the fixed-pricing case in Table1. What is most interesting from the comparison is that the optimal time ( $t_c^*$ ) changes noticeably when the fixed payment rate ( $p_0$ ) varies slightly, but can stay fixed when the linear payment rate ( $p_m$ ) increases slowly. For example, as  $p_m$  grows from 2.6 to 3.0, the optimal arrival time stays at 11, indicating that a slightly larger maximum payment rate is not necessarily sufficient to improve the arrival time. On the other hand, the optimal average arrival time is more sensitive to the fixed payment rate. Additionally, the values of reduced loss,  $O(t_c^*)$ , under fixed payment rate are all larger than those under linear payment scheme. Consequently, the fixed pricing appears more favourable. Finally, it is evident from Table1 that the two parties benefits are identical at  $p_0 = 1.5$  under fixed pricing (as also shown in Fig. 4) and at  $p_m = 2.6$  under linear pricing.

### VI. SENSITIVITY ANALYSIS

To understand the impacts of the input parameters on the two parties' decisions, namely the optimal average arrival time  $t_c$ , and the railways pricing scheme, we conduct a series of numerical experiments. The base values of the input parameters based on the real-world situations are chosen as follows:



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**Fig.4.** Behaviors of the decision functions vs. the fixed payment rate ( $c = 4, P = 400, c_l = 15, t_d = 40, t_s = 20, \alpha = 0.2, k = 20,000$ ).

**Table 1**  
Impacts of the payment methods.

Fixed pricing					Linear pricing ( $\beta = 0.3$ )				
$P_0$	$t_c^*$	$w(t_c^*)$	$O(t_c^*)$	$\Pi(t_c^*)$	$P_m$	$t_c^*$	$w(t_c^*)$	$O(t_c^*)$	$\Pi(t_c^*)$
1.0	22	0.55	2211	1106	2.0	13	0.74	2153	748
1.1	21	0.57	2180	1262	2.1	13	0.74	2039	862
1.2	20	0.59	2130	1420	2.2	13	0.74	1925	977
1.3	20	0.61	2066	1580	2.3	12	0.76	1798	1095
1.4	19	0.62	1991	1742	2.4	12	0.76	1678	1215
1.5	18	0.63	1905	1905	2.5	12	0.76	1558	1335
1.6	18	0.65	1810	2069	2.6	11	0.78	1405	1457
1.7	17	0.66	1708	2234	2.7	11	0.78	1279	1583
1.8	17	0.67	1600	2400	2.8	11	0.78	1153	1709
1.9	16	0.68	1486	2567	2.9	11	0.78	1027	1835
2.0	16	0.68	1368	2735	3.0	11	0.78	901	1961
2.1	15	0.69	1245	2904	3.1	10	0.80	708	2092
2.2	15	0.70	1118	3073	3.2	10	0.80	576	2224
2.3	15	0.71	987	3244	3.3	10	0.80	444	2356
2.4	14	0.71	854	3414	3.4	10	0.80	312	2488
2.5	14	0.72	717	3586	3.5	10	0.80	180	2620

$$c = 4, P = 400, c_l = 15, t_d = 40, t_s = 20, p_m = 3. \tag{18}$$

## VII. CONCLUSION

Thus, the major contributions of this research contain two main steps: first step was to formulate the decision-making situation and to decide whether to collaborate with a second party for the recovery service whereas, second step was to decide how to compensate the recovery service so that the majority of the passengers affected by the disruption can be benefitted from the recovery arrangement. As a result, two sets of mathematical models are developed and analyzed, and a series of hypothetical numerical and sensitivity studies is conducted. Finally and essentially to balance the two dimensions: the passengers' satisfaction and the recovery cost, the railways should prefer the fixed pricing method for the passengers' satisfaction in spite of the few limitations in the mathematical models so that the short-term disruptions may be avoided during emergencies especially for the passenger trains.

So, particularly the well-known concept in supply chain management namely, the supply contract can be introduced to disruption recovery planning in railway systems as opportunities for future research are plentiful in the same field.

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