

Dominance Property Involved In Graphs

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ABSTRACT: Domination in graphs has been studied extensively in recent years. The study of domination in graphs originated around 1850 with the problems of placing minimum number of queens on an $n \times n$ chess board so as to cover or dominate every square. With very few exceptions these problems still remain unsolved today. The theory of domination in graphs introduced by Ore and Berge is an emerging area of research in graph theory today. Berge presents the problem of five queens, namely, place five queens on the chess board so that every square is covered by at least one queen. The solution to these problems are nothing but dominating sets in the graph, whose vertices are the queens of the chess board and vertices u, v are adjacent if a queen move from u to v in one move. This leads to domination in graphs. In this paper, we define the domination in graphs.

KEYWORDS: Graph theory, domination in graph.

I. INTRODUCTION

The theory of domination has been the nucleus of research activity in graph theory in recent times. This is largely due to a variety of new parameters that can be developed from the basic definition of domination[11]. The rigorous study of dominating sets in graph theory began around 1960, even though the subject has historical roots dating back to 1862 when de Jaenisch studied the problems of determining the minimum number of queens which are necessary to cover or dominate a $n \times n$ chessboard. In 1958, Berge defined the concept of the domination number of a graph, calling this as “coefficient of External Stability”. In 1962, Ore used the name “dominating set” and “domination number” for the same concept. In 1977 Cockayne and Hedetniemi made an interesting and extensive survey of the results known at that time about dominating sets in graphs[10,9]. In the sequel, the following definitions are needed[5].

Definition 1: Let G be a finite connected graph with vertex set $V = V(G)$ and edge set $E = E(G)$ [8,7]. The **neighborhood** of a vertex $v \in V(G)$, denoted by $N(v)$, is the set of vertices adjacent to v . The **open neighbourhood** of a vertex v in a graph G is defined as the set $N_G(v) = \{u \in V(G) : uv \in E(G)\}$. The **closed neighborhood** of v denoted by $N[v] = N(v) \cup \{v\}$ and the degree of v in G by $d(v)$. If S is a subset of V , we set $N(S) = \bigcup_{v \in S} N(v)$ and $N[S] = \bigcup_{v \in S} N[v] = S \cup N(S)$. We use $|X|$ for the cardinality of a set X . Let $\Delta(G)$ and $\delta(G)$ denote the maximum degree and the minimum degree of G [6].

We now introduce the concept of dominating sets in graphs[5].

Definition 2: A set $D \subseteq V$ of vertices in a graph $G = (V, E)$ is a **dominating set** if every vertex $v \in V$ is an element of D or adjacent to an element of D [4]. Alternatively, we can say that $D \subseteq V$ is a dominating set of G if $N[D] = V(G)$.

Definition 3: A dominating set D is a **minimal dominating set** if no proper subset $D' \subset D$ is a dominating set[3]. The domination number $\gamma(G)$ of a graph G is the **minimum cardinality** of a dominating set of G . We call such a set a γ -set of G [2].

Definition 4: For a graph $G = (V, E)$ and $S \subseteq V$ a vertex $v \in D$ is an **enclave** of D if $N[v] \subseteq D$. For $D \subseteq V$ a vertex $v \in D$ is an **isolate** of D if $N(v) \subseteq V - D$. We say that a set is **enclaveless** if it does not contain any enclaves. Note that S is a dominating set of a graph $G = (V, E)$ if and only if $V - D$ is enclaveless[1].

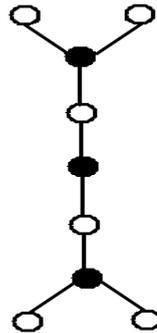
International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 3, Issue 11, November 2014

Example

D = Shaded vertices



Theorem 1 . A dominating set S of a graph G is a minimal dominating set if and only if for any $u \in S$,

1. u is an isolate of S , or
2. There is $v \in V - S$ for which $N[v] \cap S = \{u\}$.

$S = \{u\}$.

Proof . Let S be a γ -set of G . Then for every vertex $u \in S$, $S - \{u\}$ is not a dominating set of G . Thus, there is a vertex $v \in (V - S) \cup \{u\}$ that is not dominated by any vertex in $S - \{u\}$. Now, either $v = u$, which implies u is an isolate of S ; or $v \in V - S$, in which case v is not dominated by $S - \{u\}$, and is dominated by S . This shows that $N[v] \cap S = \{u\}$.

In order to prove the converse, we assume S is a dominating set and for all $u \in S$, either u is an isolate of S or there is $v \in V - S$ for which $N[v] \cap S = \{u\}$. We assume to the contrary that S is not a γ -set of G . Thus, there is a vertex $u \in S$ such that $S - \{u\}$ is a dominating set of G . Hence, u is adjacent to at least one vertex in $S - \{u\}$, so condition (1) does not hold. Also, if $S - \{u\}$ is a dominating set, then every vertex in $V - S$ is adjacent to at least one vertex in $S - \{u\}$, so condition (2) does not hold for u . Therefore, neither (1) nor (2) holds, contradicting our assumption. ■

REFERENCES

1. M.Borowiecki and D.Michalak, Generalized Independence and domination in graphs, *Discrete Mathematics* **191** (1998), 51–56.
2. Jaikumar S., Ramaswamy S., Asokan B.R., Mohan T., Gnanavel M., "Anti ulcer activity of methanolic extract of *Jatropha curcas* (Linn.) on Aspirin-induced gastric lesions in wistar strain rats", *Research Journal of Pharmaceutical, Biological and Chemical Sciences*, ISSN : 0975-8585, 1(4) (2010) PP.886-897.
3. E.J.Cockayne, R.M.Dawes, and S. T.Hedetniemi, Total domination in graphs, *Networks* **10** (1980), 211–219.
4. Kumar S.S., Rao M.R.K., Balasubramanian M.P., "Anticarcinogenic effects of *indigofera aspalathoides* on 20-methylcholanthrene induced fibrosarcoma in rats", *Research Journal of Medicinal Plant*, ISSN : 5(6) (2011) PP. 747-755.
5. W.Goddard and M. A. Henning, Generalised domination and independence in graphs, *Congressus Numerantium* **123** (1997), 161–171.
6. Sharmila S., Jeyanthi Rebecca L., Saduzzaman M., "Biodegradation of domestic effluent using different solvent extracts of *Murraya koenigii*", *Journal of Chemical and Pharmaceutical Research*, ISSN : 0975 – 7384, 5(2) (2013) PP.279-282.
7. J.Harant and M. A. Henning, On double domination in graphs, *Discussiones Mathematicae Graph Theory* **25** (2005), 29–34.
8. Beula Devamalar P.M., Thulasi Bai V., Srivatsa S.K., "Design and architecture of real time web-centric tele health diabetes diagnosis expert system", *International Journal of Medical Engineering and Informatics*, ISSN : 1755-0661, 1(3) (2009) PP.307-317.
9. T. W. Haynes, S. T. Hedetniemi, and P. J. Slater, *Fundamentals of Domination in Graphs*, Marcel Dekker, Inc., New York, 1998.
10. Niranjan U., Subramanyam R.B.V., Khanaa V., "Developing a Web Recommendation System Based on Closed Sequential Patterns", *Communications in Computer and Information Science*, ISSN : 1865-0929, 101() (2010) PP.171-179.
11. T. W. Haynes, S. T. Hedetniemi, and P. J. Slater, eds. *Domination in Graphs: Advanced Topics*, Marcel Dekker, Inc., New York, 1998.