

Effect of Rotation and Initial Stress on Generalized Thermoelastic Medium with Two-Temperature under Three-Phase-Lag Model

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ABSTRACT: Three-phase-lag theory of thermoelasticity is employed to study the deformation of thermoelastic solid half-space under hydrostatic initial stress and rotation with two-temperature. The normal mode analysis is used to obtain the analytical expressions of the displacement components, force stress, thermodynamic temperature and conductive temperature. The numerical results are given and presented graphically when mechanical and thermal force is applied. Comparisons are made with the results predicted by three phase lag, Green-Naghdi III and Lord-Shulman in the presence and absence of initial stress as well as rotation.

KEYWORDS: Hydrostatic initial stress, Three-phase-lag, Thermoelastic, Thermodynamic temperature, Conductive temperature, Normal-mode.

I. INTRODUCTION

The generalized theory of thermoelasticity is one of the modified versions of classical uncoupled and coupled theory of thermoelasticity and has been developed in order to remove the paradox of physical impossible phenomena of infinite velocity of thermal signals in the classical coupled thermoelasticity. Five generalizations of the coupled theory of thermoelasticity have investigated [1]. The first generalization formulates the generalized thermoelasticity theory involving one thermal relaxation time [2]. The temperature rate-dependent thermo-elasticity is developed where includes two thermal relaxation times and does not violate the classical Fourier's law of heat conduction, when the body under consideration has a center of symmetry [3]. One can review and presentation of generalized theories of thermoelasticity [4]. The wave propagation in anisotropic solids in generalized theories of thermoelasticity has studied [5-7]. The third generalization of the coupled theory of thermoelasticity is developed by Hetnarski and Ignaczak and is known as low-temperature thermoelasticity. The fourth generalization to the coupled theory of thermoelasticity introduced by Green and Naghdi and this theory is concerned with the thermoelasticity theory without energy dissipation, referred to as G-N theory of type II in which the classical Fourier law is replaced by a heat flux rate-temperature gradient relation and Green and Naghdi with energy dissipation referred to as G-N theory of type III. The fifth generalization of the coupled theory of thermoelasticity is referred to the dual-phase-lag thermoelasticity [8, 9]. Recently the three-phase-lag, heat conduction equation in which the Fourier law of heat conduction is replaced by an approximation to a modification of the Fourier law with the introduction of three different phases-lags for the heat flux vector, the temperature gradient and the thermal displacement gradient [10]. The stability of the three-phase-lag, the heat conduction equation is discussed [11]. Subsequently, this theory has employed of thermoelasticity with three-phase-lag to discuss a problem of thermoelastic interactions on functional graded orthotropic hollow sphere under thermal shock [12].

Some researchers in the past have investigated different problems of rotating media. The propagation of plane harmonic waves in a rotating elastic medium without a thermal field has been studied [13]. It was shown there that the rotation causes the elastic medium to be depressive and anisotropic. An investigation of the distribution of deformation, stresses and magnetic field in a uniformly rotating, homogeneous, isotropic, thermally and electrically

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conducting elastic half-space was presented [14]. The effect of rotation on elastic waves has been studied [15, 16]. The effect of rotation in a magneto-thermoelastic medium was discussed [17].

The two-temperature theory of thermoelasticity was introduced [18-20], in which the classical Clausius-Duhem inequality was replaced by another one depending on two-temperature; the conductive temperature and the thermodynamic temperature, the first is due to the thermal processes, and the second is due to the mechanical processes inherent between the particles and the layers of elastic material, this theory was also investigated [21]. The two-temperature model was underrated and unnoticed for many years thereafter. Only in the last decade the theory has been noticed, developed in many works, and find its applications, mainly in the problems in which the discontinuities of stresses have no physical interpretations. Among the authors who contribute to developing this theory, it has studied existence, structural stability, convergence and spatial behavior of this theory [22], it was introduced the generalized Fourier law to the field equations of the two-temperature theory of thermoelasticity and proved the uniqueness of the solution for homogeneous isotropic material [23], the propagation of harmonic plane waves studied [24]. Recently, authors have studied the uniqueness and growth solutions [25], for the model proposed [23]. The effect of three-phase-lag on a two-temperature generalized thermoelasticity for the infinite medium with a spherical cavity had studied [26].

The initial stress present in the medium also has a considerable effect on the propagation of waves (Biot, 1965). Initial stress in solids has the significant influence on the mechanical response of the material from an initially stressed configuration and has applications in geophysics, engineering structures, and in the behavior of soft biological tissues. Initial stress arises from processes, such as manufacturing or growth and is present in the absence of applied loads. It has studied the effect of hydrostatic initial stress and rotation with Green-Naghdi (type III) thermoelastic half space with two-temperature [27]. It has studied the effect of initial stress on a generalized thermoelastic medium with the three-phase-lag model under temperature-dependent properties [28]. Effect of rotation and initial stress on an infinite generalized magneto-thermoelastic diffusing body with a spherical cavity had investigated [29]. Hall effects on unsteady flow due to no coaxial rotating disk and a fluid at infinity were studied [30]. Flow induced by non-coaxial rotation of a porous disk executing non-torsion oscillating and second grade fluid rotating at infinity was discussed [31].

The present paper is concerned with the investigations related to the effect of rotation and initial stress with two-temperature on a generalized thermoelastic medium based on the three-phase-lag model by applying the normal mode analysis. Also, the effects of rotation and initial stress on the physical quantities are discussed numerically and illustrated graphically.

Nomenclature

λ, μ	Lame's parameter counterparts	u_i	displacement vector
T	thermodynamic temperature	p	initial stress
b	constant material "two temperature parameter "	ρ	mass density
c_e	specific heat at constant strain	$K (\geq 0)$	thermal conductivity
K^*	material characteristic of the theory	δ_{ij}	Kronecker delta
τ_v	phase lag of thermal displacement gradient	ϕ	conductive temperature
τ_t	phase lag of temperature gradient	τ_q	phase lag of heat flux
ω_{ij}	skew symmetric tensor called the rotation tensor		

II. FORMULATION OF THE PROBLEM

We consider a homogeneous thermoelastic half-space with two-temperature rotating uniformly with angular velocity $\Omega = \Omega \mathbf{n}$, where \mathbf{n} is a unit vector representing the direction of the axis of rotation. All quantities are considered are functions of the time variable t and of the coordinates x and y . The displacement equation in the rotating frame has two

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additional terms [Schoenberg and Censor (1973)]: Centripetal acceleration $\boldsymbol{\Omega} \times (\boldsymbol{\Omega} \times \mathbf{u})$ due to time varying motion only and Coriolis acceleration $2\boldsymbol{\Omega} \times \dot{\mathbf{u}}$ where $\mathbf{u} = (u_1, u_2, 0)$ is the dynamic displacement vector and angular velocity is $\boldsymbol{\Omega} = (0, 0, \Omega)$. These terms, do not appear in non-rotating media.

III. BASIC EQUATIONS

The field equations and constitutive relations for a homogeneous isotropic thermoelastic solid with hydrostatic initial stress and rotation in the absence of incremental body forces and heat source are given by three phase lag model,

$$\sigma_{ij} = 2\mu e_{ij} + [\lambda e - \gamma (T - T_0)] \delta_{ij} - p (\omega_{ij} + \delta_{ij}). \quad (1)$$

The equation of motion

$$\sigma_{ij,j} = \rho [\ddot{u}_i + \{\boldsymbol{\Omega} \wedge (\boldsymbol{\Omega} \wedge \mathbf{u})\}_i + (2\boldsymbol{\Omega} \wedge \dot{\mathbf{u}})_i]. \quad (2)$$

The equation of heat conduction under three phase lag model

$$K^* \nabla^2 \phi + \tau_v^* \nabla^2 \dot{\phi} + K \tau_t \nabla^2 \ddot{\phi} = (1 + \tau_q \frac{\partial}{\partial t} + \frac{\tau_q^2}{2!} \frac{\partial^2}{\partial t^2}) (\rho c_e \dot{T} + \gamma T_0 \ddot{e}). \quad (3)$$

Where, $\tau_v^* = (K + K^* \tau_v)$,

$$T = (1 - b \nabla^2) \phi, \quad (4)$$

$$e_{ij} = \frac{1}{2} (u_{i,j} + u_{j,i}), \quad (5)$$

$$\omega_{ij} = \frac{1}{2} (u_{j,i} - u_{i,j}). \quad (6)$$

For a two-dimensional problem in xy -plane, Eqs. (3.2) and (3.3) can be written as:

$$(\mu - \frac{p}{2}) \nabla^2 u_1 + (\lambda + \mu + \frac{p}{2}) \frac{\partial e}{\partial x} - \gamma (1 - b \nabla^2) \frac{\partial \phi}{\partial x} = \rho [\frac{\partial^2 u_1}{\partial t^2} - u_1 \Omega^2 - 2\Omega \frac{\partial u_2}{\partial t}], \quad (7)$$

$$(\mu - \frac{p}{2}) \nabla^2 u_2 + (\lambda + \mu + \frac{p}{2}) \frac{\partial e}{\partial y} - \gamma (1 - b \nabla^2) \frac{\partial \phi}{\partial y} = \rho [\frac{\partial^2 u_2}{\partial t^2} - u_2 \Omega^2 + 2\Omega \frac{\partial u_1}{\partial t}], \quad (8)$$

$$K^* \nabla^2 \phi + \tau_v^* \nabla^2 \frac{\partial \phi}{\partial t} + K \tau_t \nabla^2 \frac{\partial^2 \phi}{\partial t^2} = (1 + \tau_q \frac{\partial}{\partial t} + \frac{\tau_q^2}{2!} \frac{\partial^2}{\partial t^2}) [(\rho c_e (1 - b \nabla^2) \ddot{\phi} + \gamma T_0 \frac{\partial^2 e}{\partial t^2})]. \quad (9)$$

For the purpose of numerical evaluation, we introduce dimensionless variables

$$(x', y', u'_i) = \frac{1}{c_0 \omega} (x, y, u_i), \quad (t', \tau'_v, \tau'_q, \tau'_t) = \frac{1}{\omega} (t, \tau_v, \tau_q, \tau_t), \quad T' = \frac{\gamma}{\lambda + 2\mu} T, \quad \sigma'_{ij} = \frac{1}{\lambda + 2\mu} \sigma_{ij}, \quad P' = \frac{P}{\lambda + 2\mu}, \quad \Omega' = \omega \Omega,$$

$$\phi' = \frac{\gamma}{\lambda + 2\mu} \phi, \quad c_0^2 = \frac{\lambda + 2\mu}{\rho}, \quad \omega = \frac{K}{\rho c_e c_0^2}.$$

Using the above dimensionless quantities, Eqs. (7)-(9) become

$$(\frac{\mu}{\lambda + 2\mu} - \frac{p}{2}) \nabla^2 u_1 + (\frac{\lambda + \mu}{\lambda + 2\mu} + \frac{p}{2}) \frac{\partial e}{\partial x} - (1 - b^* \nabla^2) \frac{\partial \phi}{\partial x} = [\frac{\partial^2 u_1}{\partial t^2} - u_1 \Omega^2 - 2\Omega \frac{\partial u_2}{\partial t}], \quad (10)$$

$$(\frac{\mu}{\lambda + 2\mu} - \frac{p}{2}) \nabla^2 u_2 + (\frac{\lambda + \mu}{\lambda + 2\mu} + \frac{p}{2}) \frac{\partial e}{\partial x} - (1 - b^* \nabla^2) \frac{\partial \phi}{\partial x} = [\frac{\partial^2 u_2}{\partial t^2} - u_2 \Omega^2 - 2\Omega \frac{\partial u_1}{\partial t}], \quad (11)$$

$$\varepsilon_1 \nabla^2 \phi + \varepsilon_2 \nabla^2 \dot{\phi} + \tau_t \nabla^2 \ddot{\phi} = (1 + \tau_q \frac{\partial}{\partial t} + \frac{\tau_q^2}{2!} \frac{\partial^2}{\partial t^2}) [(1 - b^* \nabla^2) \ddot{\phi} + \varepsilon_3 \ddot{e}]. \quad (12)$$

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Where, $\varepsilon_1 = \frac{K^*}{\rho c_e c_0^2}$, $\varepsilon_2 = 1 + \varepsilon_1 \tau_v$, $\varepsilon_3 = \frac{\gamma^2 T_0}{\rho c_e (\lambda + 2\mu)}$, $b^* = \frac{b}{c_0^2 \omega^2}$.

We define displacement potentials q and ψ which relate to displacement components u_1 and u_2 as,

$$u_1 = q_{,x} - \psi_{,y}, \quad u_2 = q_{,y} + \psi_{,x}. \tag{13}$$

Using Eq. (13) in Eqs. (10)-(12), we obtain:

$$[\nabla^2 - (\frac{\partial^2}{\partial t^2} - \Omega^2)]q + 2\Omega \frac{\partial \psi}{\partial t} - (1 - b^* \nabla^2)\phi = 0, \tag{14}$$

$$- 2\Omega \frac{\partial q}{\partial t} + [a_1 \nabla^2 - (\frac{\partial^2}{\partial t^2} - \Omega^2)]\psi = 0, \tag{15}$$

$$\varepsilon_1 \nabla^2 \phi + \varepsilon_2 \nabla^2 \dot{\phi} + \tau_v \nabla^2 \ddot{\phi} = (1 + \tau_q \frac{\partial}{\partial t} + \frac{\tau_q^2}{2!} \frac{\partial^2}{\partial t^2})[(1 - b^* \nabla^2)\ddot{\phi} + \varepsilon_3 \nabla^2 \dot{q}]. \tag{16}$$

IV. NORMAL MODE ANALYSIS

The solution of the considered physical variable can be decomposed in terms of normal mode as the following form

$$[u_1, u_2, e, T, \phi, q, \psi, \sigma_{ij}](x, y, t) = [u_1^*, u_2^*, e^*, T^*, \phi^*, q^*, \psi^*, \sigma_{ij}^*](y) \exp(\omega t + iax). \tag{17}$$

Where, ω is the complex time constant and a is the wave number in x -direction.

Using (17) in Eqs. (14)-(16), we obtain

$$(D^2 - A_1)q^* + (b^* D^2 - A_2)\phi^* + 2\Omega \omega \psi^* = 0, \tag{18}$$

$$(a_1 D^2 - A_3)\psi^* - 2\Omega \omega q^* = 0, \tag{19}$$

$$(A_4 D^2 - A_5)\phi^* - A_6(D^2 - a^2)q^* = 0. \tag{20}$$

Where,

$$A_1 = (a^2 + \omega^2 - \Omega^2), \quad A_2 = 1 - b^* a^2, \quad A_3 = (a_1 a^2 + \omega^2 - \Omega^2), \quad A_4 = (\varepsilon_1 + \varepsilon_2 \omega + \tau_v \omega^2) + b^* (1 + \tau_q \omega + \frac{\tau_q^2 \omega^2}{2}),$$

$$A_5 = a^2 (\varepsilon_1 + \varepsilon_2 \omega + \tau_v \omega^2) + (1 + a^2 b^*) [1 + \tau_q \omega + \frac{\tau_q^2}{2!} \omega^2], \quad A_6 = \varepsilon_3 \omega^2 [1 + \tau_q \omega + \frac{\tau_q^2}{2!} \omega^2].$$

Eliminating ϕ^* and ψ^* between Eqs. (4.18)-(4.20), we get

$$[D^6 - A D^4 + B D^2 - E]\{q^*(y), \phi^*(y), \psi^*(y)\} = 0. \tag{21}$$

Where,

$$A = \frac{a_1 \{A_5 + A_1 A_4 + A_2 A_6 + b^* a^2 A_6\} + A_3 (A_4 + b^* A_6)}{a_1 (A_4 + A_6 b^*)},$$

$$B = \frac{A_3 A_5 + a_1 A_1 A_5 + A_1 A_3 A_4 + 4A_4 \Omega^2 \omega^2 + A_2 A_3 A_6 + b^* a^2 A_3 A_6 + a_1 a^2 A_2 A_6}{a_1 (A_4 + A_6 b^*)},$$

$$E = \frac{A_1 A_3 A_5 + 4A_5 \Omega^2 \omega^2 + a^2 A_2 A_3 A_6}{a_1 (A_4 + A_6 b^*)}, \quad D = \frac{d}{dy}.$$

The solution of Eq. (21) which are bound as $y \rightarrow \infty$, has the form

$$q^* = \sum_{n=1}^3 M_n e^{-k_n y}, \tag{22}$$

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$$\psi^* = \sum_{n=1}^3 H_{1n} M_n e^{-k_n y}, \quad (23)$$

$$\phi^* = \sum_{n=1}^3 H_{2n} M_n e^{-k_n y}, \quad (24)$$

$$T^* = \sum_{n=1}^3 H_{3n} M_n e^{-k_n y}. \quad (25)$$

Where M_n ($n=1,2,3$) are some constants, k_n^2 are the roots of the characteristic equation of Eq. (21). Dimensionless variables of the stress components as the following,

$$\sigma_{xx} = u_{1,x} + \left(1 - \frac{2}{\beta^2}\right) u_{2,y} - T - P, \quad (26)$$

$$\sigma_{yy} = u_{2,y} + \left(1 - \frac{2}{\beta^2}\right) u_{1,x} - T - P, \quad (27)$$

$$\sigma_{xy} = a_2 u_{1,y} + a_1 u_{2,x}, \quad (28)$$

$$\sigma_{yx} = a_1 u_{1,y} + a_2 u_{2,x}. \quad (29)$$

Using Eq. (13) and Eqs. (22)-(25) in (26)-(29)

$$u_1 = \sum_{n=1}^3 H_{4n} M_n e^{(\omega t - k_n y + i a x)}, \quad (30)$$

$$u_2 = \sum_{n=1}^3 H_{5n} M_n e^{(\omega t - k_n y + i a x)}, \quad (31)$$

$$\sigma_{xx} = \sum_{n=1}^3 H_{6n} M_n e^{(\omega t - k_n y + i a x)} - P, \quad (32)$$

$$\sigma_{yy} = \sum_{n=1}^3 H_{7n} M_n e^{(\omega t - k_n y + i a x)} - P, \quad (33)$$

$$\sigma_{xy} = \sum_{n=1}^3 H_{8n} M_n e^{(\omega t - k_n y + i a x)}, \quad (34)$$

$$\sigma_{yx} = \sum_{n=1}^3 H_{9n} M_n e^{(\omega t - k_n y + i a x)}, \quad (35)$$

$$T = \sum_{n=1}^3 H_{3n} M_n e^{(\omega t - k_n y + i a x)}. \quad (36)$$

Where,

$$H_{1n} = \frac{2\Omega\omega}{(a_1 k_n^2 - A_3)}, \quad H_{2n} = \frac{(k_n^2 - a^2)A_6}{(A_4 k_n^2 - A_5)}, \quad H_{3n} = H_{2n} [1 - b^* (k_n^2 - a^2)], \quad H_{4n} = (ia + k_n H_{1n}), \quad H_{5n} = (-k_n + ia H_{1n}),$$

$$H_{6n} = [ia H_{4n} - (1 - \frac{2}{\beta^2}) k_n H_{5n} - H_{3n}], \quad H_{7n} = [-k_n H_{5n} + ia H_{4n} (1 - \frac{2}{\beta^2}) - H_{3n}], \quad H_{8n} = (-a_2 k_n H_{4n} + ia a_2 H_{5n}),$$

$$H_{9n} = (-a_1 k_n H_{4n} + i a a_2 H_{5n}), \quad a_1 = \frac{1}{\beta^2} - \frac{p}{2}, \quad a_2 = \frac{1}{\beta^2} + \frac{p}{2}, \quad \beta^2 = \frac{\lambda + 2\mu}{\mu}.$$

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V. THE BOUNDARY CONDITIONS

The boundary conditions on the plane surface $y = 0$ are:

$$\sigma_{xx} = -P + P_1 e^{(\omega t + i a x)}, \quad T = P_2 e^{(\omega t + i a x)}, \quad \sigma_{xy} = 0. \quad (37)$$

Using Eqs. (32), (34) and (36) in boundary conditions (37), we get three equations in three constants M_n ($n = 1, 2, 3$) as:

$$\sum_{n=1}^3 H_{6n} M_n = P_1, \quad (38)$$

$$\sum_{n=1}^3 H_{8n} M_n = 0, \quad (39)$$

$$\sum_{n=1}^3 H_{3n} M_n = P_2. \quad (40)$$

Solving Eqs. (38)-(40) the constants M_n ($n = 1, 2, 3$) are defined as follows:

$$M_1 = \frac{\Delta_1}{\Delta}, \quad M_2 = \frac{\Delta_2}{\Delta}, \quad M_3 = \frac{\Delta_3}{\Delta}.$$

Where, $\Delta = H_{61}(H_{82}H_{33} - H_{83}H_{32}) - H_{62}(H_{81}H_{33} - H_{83}H_{31}) + H_{63}(H_{81}H_{32} - H_{82}H_{31})$,

$\Delta_1 = P_1(H_{82}H_{33} - H_{83}H_{32}) + P_2(H_{83}H_{62} - H_{63}H_{82})$, $\Delta_2 = -P_1(H_{81}H_{33} - H_{83}H_{31}) + P_2(H_{63}H_{81} - H_{61}H_{83})$,

$\Delta_3 = P_1(H_{81}H_{32} - H_{82}H_{31}) + P_2(H_{61}H_{82} - H_{62}H_{81})$.

VI. NUMERICAL RESULTS

To study the effect of rotation and initial stress, we now present some numerical results. For this purpose, copper is taken as the thermoelastic material for which we take the following values of the different physical constants.

$\lambda = 7.7 \times 10^{10} \text{ N.M}^{-2}$, $\mu = 3.86 \times 10^{10} \text{ Kg.m}^{-1}.\text{s}^{-2}$, $K = 300 \text{ w.m}^{-1}.\text{K}^{-1}$, $\alpha_t = 1.78 \times 10^{-5} \text{ K}^{-1}$, $\rho = 8954 \text{ Kg.m}^{-3}$,
 $C_E = 383.1 \text{ J.Kg}^{-1}.\text{K}^{-1}$, $T_0 = 293 \text{ K}$, $a = 0.5$, $\omega = \omega_0 + i\xi$, $\omega_0 = -0.1$, $K^* = 2.97 \times 10^{13}$, $\xi = 0.6$, $x = 0.5$,
 $t = 0.3$, $p_1 = 2$, $p_2 = 3$, $b = 0.1$, $\tau_v = 0.05$, $\tau_t = 0.2$, $\tau_q = 0.8$.

The numerical technique, outlined above, was used for the distribution of the real part of the temperature T , the displacement components u, v and the stress components $\sigma_{xx}, \sigma_{yy}, \sigma_{xy}$ for the problem. Here, all the variables are taken in non-dimensional form the result. Fig. 1-6 show the distribution of the physical quantities based on L-S, G-N III and 3PHL in the case of $p = 0, 1$. Fig. 1 depicts that the displacement component u increases with the decrease of initial stress in the three theories. In the absence of initial stress (i.e. $p = 0$), u begins to increase then smooth decreases again to try to return to zero at infinity and in the presence of initial stress (i.e. $p = 1$), u decreases and increases until it develops to zero.

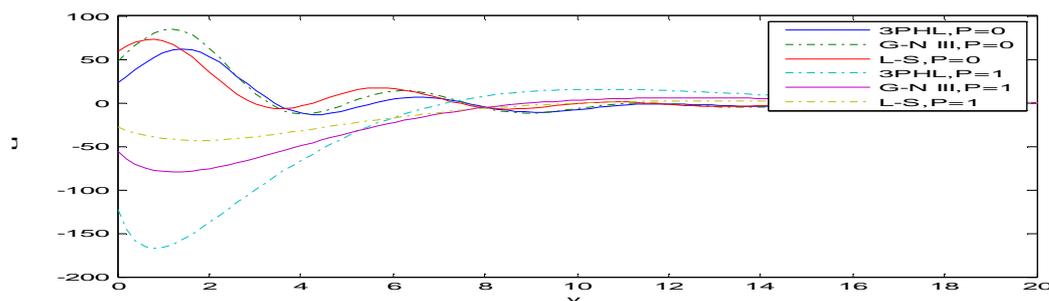


Fig. 1 Distribution of displacement component u in the absence and presence of p

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Fig. 2 exhibits that the displacement component v decreases with the decrease of initial stress and it is increasing with the increase of initial stress, based on L-S, G-N III and 3PHL, that's means that the displacement v is directly proportional to initial stress, and take the form of a wave until it develops to zero. Fig. 3 demonstrates that the behavior of temperature T decays meaning that the temperature decrease for $p = 0,1$ and take the form of a wave until it develops to zero.

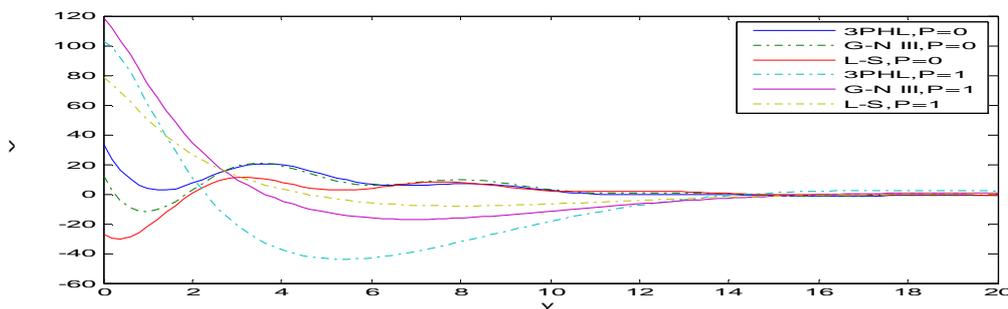


Fig. 2 Distribution of displacement component v in the absence and presence of p

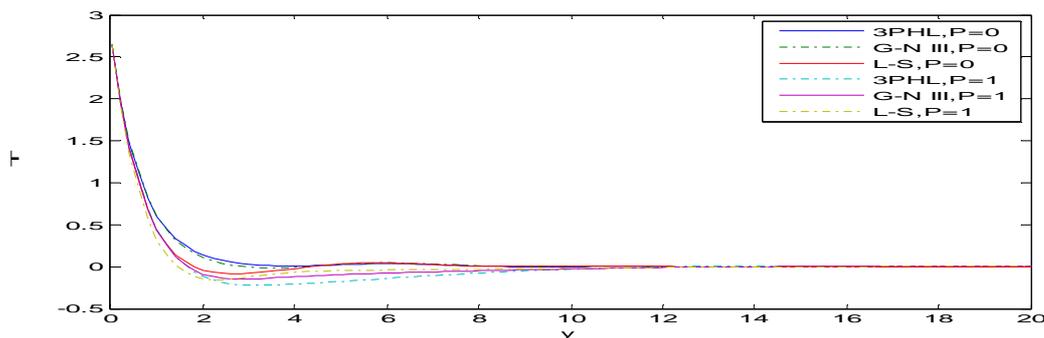


Fig. 3 Distribution of the temperature T in the absence and presence of p

Fig. 4 represents that the stress component σ_{xy} increase with the increase of initial stress p in three theories and take the form of the wave until it develops to zero, and decreasing with the decrease of the initial stress in three theories and take the form of the wave until it develops to zero.

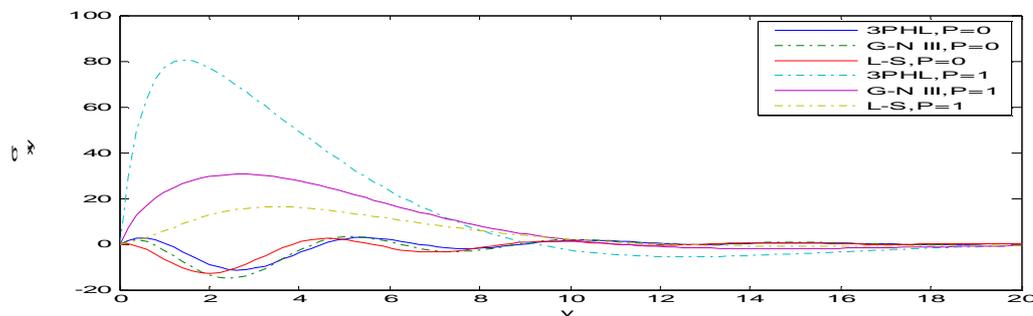


Fig. 4 Distribution of the stress component σ_{xy} in the absence and presence of p

Fig. 5 depicts that the stress component σ_{xx} begins from the value 2 and satisfies the boundary condition at $y = 0$ in three theories. The stress component σ_{xx} increase with the decrease of initial stress and decays to zero. Fig. 6 explains that the stress component σ_{yy} increase with the decrease of initial stress and take the form of the wave until it

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develops to zero. Figs. 7-12 exhibit the distribution of the physical quantities based on L-S, G-N III and 3PHL in the case of $\Omega = 0, 0.2$.

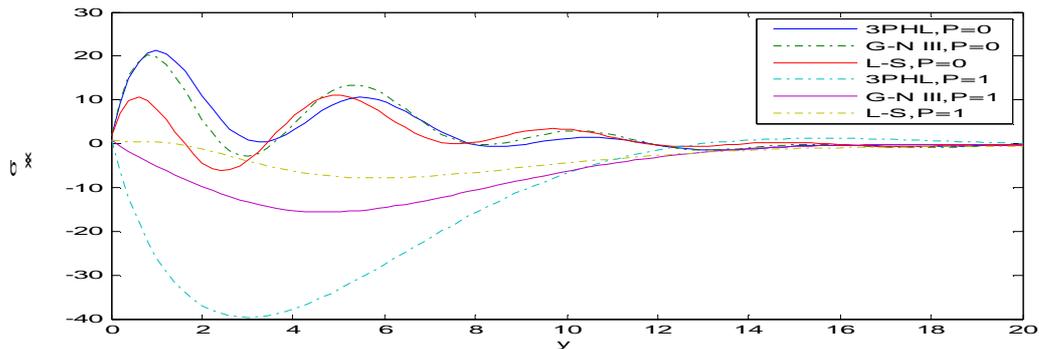


Fig. 5 Distribution of the stress component σ_{xx} in the absence and presence of p

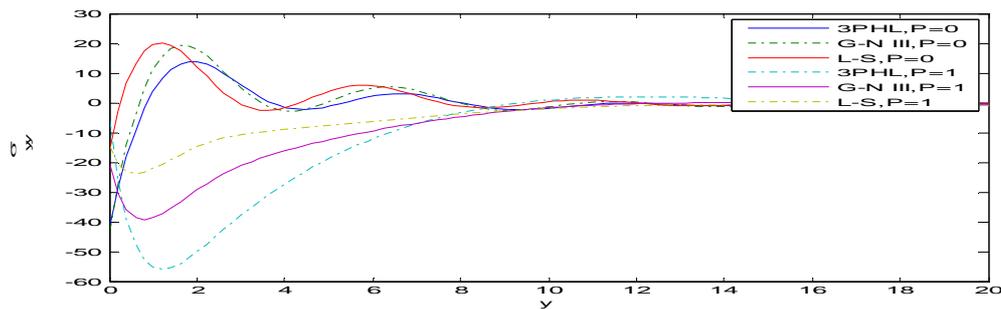


Fig. 6 Distribution of the stress component σ_{yy} in the absence and presence of p

Fig. 7 shows that at $\Omega = 0$, the stress component σ_{yy} is decreasing to a minimum value in the range $0 \leq y \leq 2$, while, increases in the range $2 \leq y \leq 4$ and decays to zero in L-S, G-N III and 3PHL theories, but at $\Omega = 0.2$, it decreases to a minimum value in the range $0 \leq y \leq 1$, and increases in the range $1 \leq y \leq 3$, until it develops to zero.

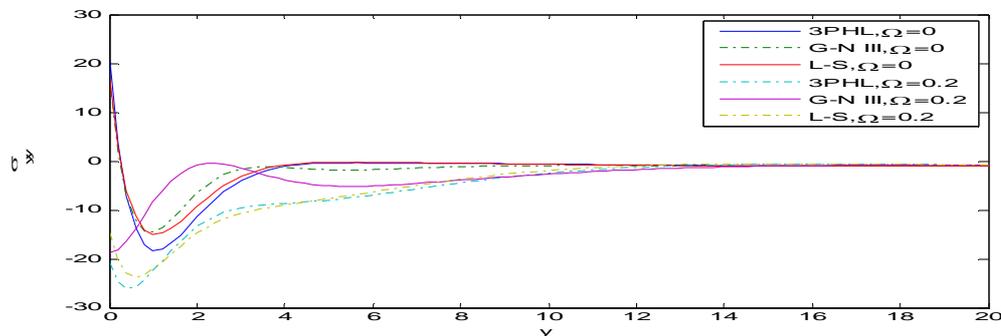


Fig. 7 Distribution of the stress component σ_{yy} in the absence and presence of rotation

Fig. 8 shows that at $\Omega = 0$, the stress component σ_{xx} satisfies the boundary condition and decreasing to a minimum value in the range $0 \leq y \leq 1$, while, increases in the range $1 \leq y \leq 6$ and decays to zero in the context of three theories. However, at $\Omega = 0.2$, it increases in the range $0 \leq y \leq 2$, then, decreases in the range $2 \leq y \leq 8$ and decays to zero in the three theories.

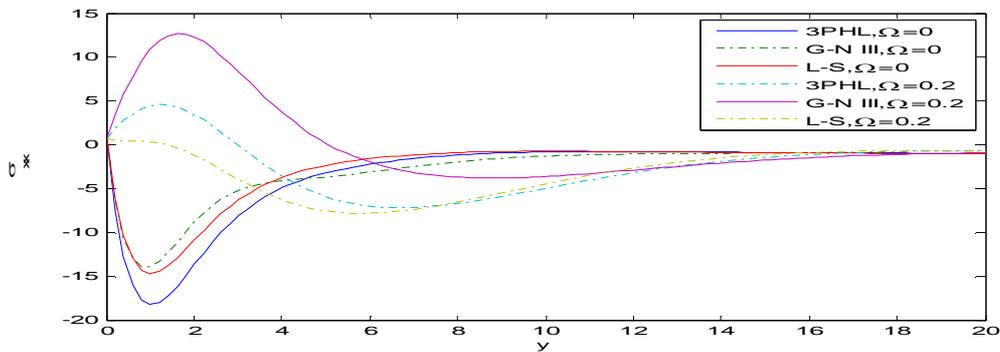


Fig. 8 Distribution of the stress component σ_{xx} in the absence and presence of rotation

Fig. 9 explains that in the absence of rotation the stress component σ_{xy} decreases in the range $0 \leq y \leq 1$, in three theories, and increases in the range $1 \leq y \leq 3$. While in the presence of rotation, σ_{xy} decreases in the range $0 \leq y \leq 1$, then, increases in the range $2 \leq y \leq 5$ and take the form of the wave until it develops to zero in L-S and G-N III and 3PHL theories.

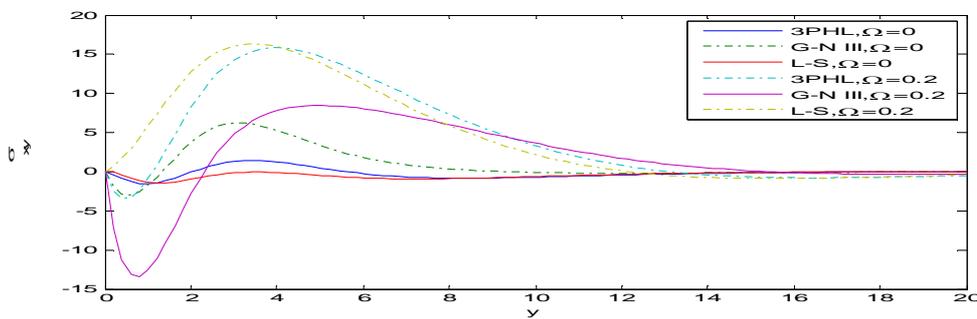


Fig. 9 Distribution of the stress component σ_{xy} in the absence and presence of rotation

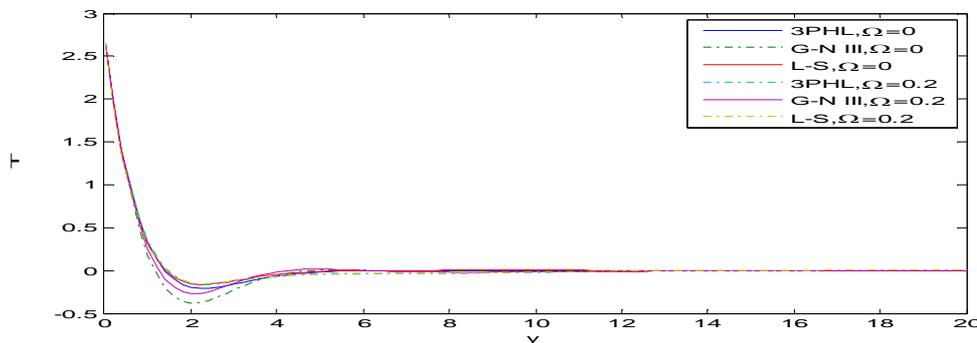


Fig. 10 Distribution of the temperature T in the absence and presence of rotation

Fig. 10 demonstrates that the temperature satisfies the boundary condition at $y = 0$ and decreases, in the three theories at $\Omega = 0, 0.2$, to a minimum value in the range $0 \leq y \leq 2$ and increasing in the range $2 \leq y \leq 4$, until it decays to zero. Fig. 11 shows that the displacement component v increases at $\Omega = 0.2$, in three theories, and decreases at $\Omega = 0$, that's mean that the displacement component v increases with the increase of rotation and take the form of the wave

until it develops to zero. Fig. 12 depicts that the displacement component u increases with the decrease of rotation and try to return to zero at infinity.

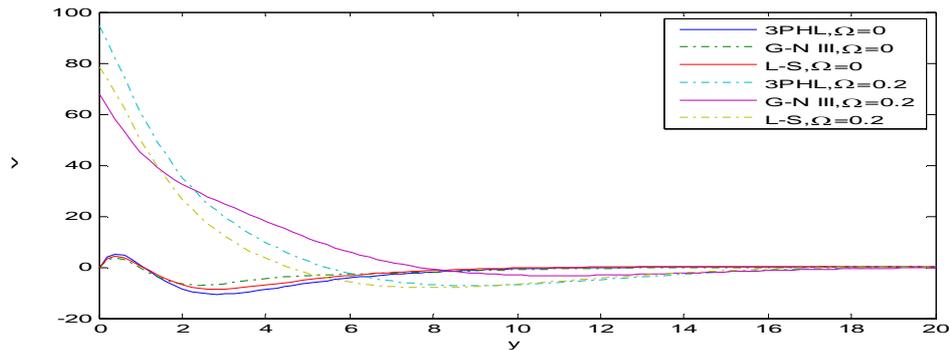


Fig. 11 Distribution of displacement component v in the absence and presence of rotation

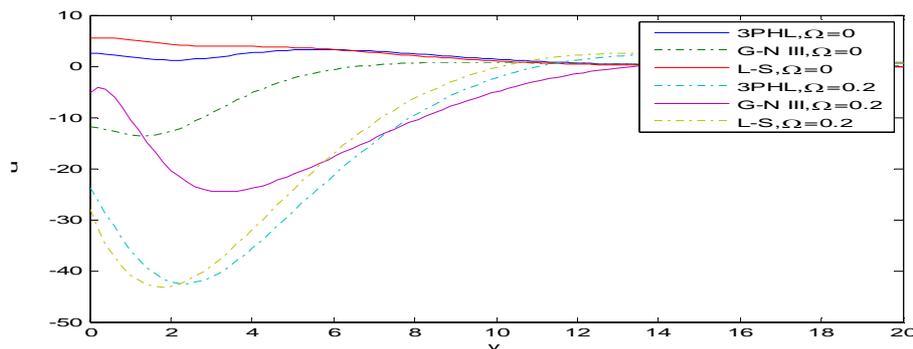


Fig. 12 Distribution of displacement component u in the absence and presence of rotation

VII. CONCLUSION

The figures obtained by comparing the three theories, important phenomena are observed:

1. Analytical solutions based upon normal mode analysis of the thermoelastic problem in solids have been developed.
2. The method that is used in the present article is applicable to a wide range of problems in hydrodynamics and thermoelasticity.
3. There are significant differences in the field quantities under GN-III, 3PHL and L-S theories.
4. The presence of the initial stress and rotation plays a significant role in all physical quantities.
5. All the physical quantities satisfy the boundary conditions.
6. The comparison of different theories of thermoelasticity, i.e. L-S theory, the model of 3PL and the GN-III is carried out.
7. The value of all the physical quantities converges to zero, and all the functions are continuous.

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