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Markovian Model on the District Size Distribution

R. Selvam

Assistant Professor, Department of Statistics, Government Arts college (Autonomous), Salem, Tamil Nadu, India.

ABSTRACT: In this paper, the mobility of district size from one size class to another size class using Markovian model is discussed. This study uses 2001 and 2011 census data from the primary census abstract of Tamil Nadu state. The urban population of all districts in Tamil Nadu state were considered for the empirical analysis. As per this study, the proportional change of district with respect to urban population in the class interval of district size 1541350-4700000 are obtained as 3.3% in 2001, 20% in 2011, 29.9% in 2021, 34.9% in 2031, 39.7% in 2041 and 44.9% in 2051. The more number of districts are moved towards the highest proportion of district size interval than the other classes when time is increased.

KEYWORDS: Chapman-Kolmogorov equation, District size distribution, Markovian model, Transition Probability, Urban population.

I. INTRODUCTION

Mobility is the process of movement of population from one region to another region. It is one of the important aspects of human nature, which is often guided by socio economic, political as well as environmental factors. The nature, pattern and direction of population mobility may vary across the space. Mobility refers to the movement of people from region to region, or country to country, or job to job, or social class to social class. Population mobility refers to the geographic movement of people where there has been a change in the place of usual residence. People move for many reasons, and factors that drive mobility often stem from unequal distribution of resources, climatic instability, conflict and political unrest. Movements have often been conceptualised as being driven by push or pull factors or a combination of the two. Many are pushed out of a country or a region by political instability, violence and natural disasters, and pulled towards other regions and countries with stronger economies and better opportunities. The reasons to mobility are in many cases not just economic, social, political and cultural aspects also play its part. A migrant from an impoverished backward region may be attracted to more prosperous region because of better public services, higher wages, more business opportunities, and more employment opportunities, etc.,.

The concept of population mobility now connotes much wider dimension that varies from daily commuting to nearby places on one hand, to permanent shift of residence to distant places on the other. The nature and pattern of mobility varies from one social group of migrants to another because of the fact that the rural migrants are not a homogenous group. Rural to urban mobility has attracted the attention of academicians as well as the policymakers throughout the world in recent years, only after the seminal work of Lewis (1954) because of its wide range of socio-economic, political, demographic, ecological and environmental implications. It plays an important role in the process of economic development and social transformation because the shifting of the work force from primary to secondary and tertiary sectors that result in structural change. The improvement in infrastructure and communication accompanying economic development could result in increased population mobility. But from the earlier studies, it is evident that mobility as a percentage of total population has been declining up to 1991 census, whereas the rural to urban mobility is showing an increasing trend over the period 1971 to 2001. The recent report of United Nations Development Programme (UNDP, 2009) entitled 'Mobility and Human Development' stated that population shift from villages to cities is natural and should be encouraged and that it results in concentration of production, mobility of people and economic integration to lift rural people out of poverty in India. Hence, any long-term plan aiming at an increase in labour productivity and rural employment opportunities should consider the possibility of migration of a relatively large proportion of population away from areas with relatively low agronomic potential. The availability of

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good agricultural land continues to be one of the most powerful economic factors determining the magnitude and direction of population mobility. The agricultural unemployment and underemployment, lower agricultural wage and poverty are the major factors pushing labourers towards job opportunities in urban areas, while the availability of

employment opportunities, relatively higher wages, better educational opportunities, better availability of transportation and infrastructure etc. are the major factors in the rural-urban mobility. In this study, mobility is the movement of district of urban population from one size class to other size classes in Tamil Nadu state. The movement of urban district size is random in nature and it motivates the researcher to develop the stochastic model to describe the movement of district size. Markovian model is used to analyse the mobility of district size from one size class to another size class to another size classes.

II. SCOPE OF RESEARCH

To analyse the mobility of district size from one size class to another size class of urban population. This study uses data from 2001 and 2011 census. The urban population of the districts are the unit of the study. All districts in Tamil Nadu state were considered in this study.

III. METHODOLOGY

The Markovian Model used to analyse the mobility of district size from one size class to another size class of urban population.

3.1 Markovian Model

Let X_n represents the urban population size of district at time n, the stochastic process $\{X_n, n = 0,1,2, \dots\}$ is called Markov chain, if for $j,k,j_1,\dots,j_{n-1} \in N$, if, for j, k.

$$P_r\{X_n = k | X_{n-1} = j, X_{n-2} = j_1, \dots, X_0 = j_{n-1}\} = P_r\{X_n = k | X_{n-1} = j\} = P_{jk}$$

The values of variable X_n are called the states of the Markov chain, if X_n has the outcome $X_n = j$, the process is said to be at state j. The transition probability states that the movement of the populations from one state at a trial to the next state at the next following trial. The transition probability P_{jk} states that the movement of the populations from the states j at the n^{th} trial to state k at $(n + 1)^{th}$ trial. The transition probability is the basis to the study of the structure of the Markov chain. When the transition is one step, P_{jk} is one-step transition probability. In general, the pair of states (j, k) at non-successive trials, say, state j at n^{th} trial and state k at the $(n + m)^{th}$ trial, the corresponding transition probability is called as the m-step transition probability and is denoted by $P_{jk}^{(m)}$,

(i.e.) $P_{jk}^{(m)} = P_r\{X_{(n+m)} = k | X_n = j\}$

The transition probability P_{jk} satisfy the conditions such that $P_{jk} \geq 0, \sum P_{jk} = 1$ for all j.

These probabilities may be written in the matrix form as follows

$$P = \begin{pmatrix} P_{11} & P_{12} & P_{13} & \dots \\ P_{21} & P_{22} & P_{23} & \dots \\ P_{31} & P_{32} & P_{33} & \dots \\ \dots & \dots & \dots & \dots \end{pmatrix}$$

This is called the transition probability matrix or matrix of transition probability of the Markov chain. The matrix p is called stochastic or Markov matrix.

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3.2 Chapman-Kolmogorov Equation

Let P_{jk} gives the probability of one step transition from the state j at a trial to the state k at the next following trial, the m-step transition probability is given by

$$P_r\{X_{m+n} = k | X_n = j\} = P_{jk}^{(m)}$$

$P_{jk}^{(m)}$ gives the probability that from the state j at n^{th} trial, the state k is reached at $(n + m)^{\text{th}}$ trial in m-steps. (i.e.) the probability of transition from the state j to the state k in exactly m- steps is denoted by $P_{jk}^{(m)}$. The one-step transition probabilities $P_{jk}^{(1)}$ are denoted by P_{jk} for simplicity.

Consider,

$$P_{jk}^{(2)} = P_r\{X_{n+2} = k | X_n = j\}$$

The state k can be reached from the state j in two steps through some intermediate state r.

Consider a fixed value of r, we have

$$P_r\{X_{n+2} = k, X_{n+1} = r | X_n = j\} = P_{rk}^{(1)} P_{jr}^{(1)} = P_{jr}^{(1)} P_{rk}^{(1)}$$

When the intermediate state r have the mutually exclusive values, it is observed that

$$P_{jk}^{(2)} = P_r\{X_{n+2} = k | X_n = j\} = \sum P_r\{X_{n+2} = k, X_{n+1} = r | X_n = j\} = \sum P_{jr} P_{rk}$$

By induction, higher step transition probability are obtained,

$$P_{jk}^{(m+1)} = P_r\{X_{n+m+1} = k | X_n = j\} = \sum P_r\{X_{n+m+1} = k | X_{n+m} = r\} P_r\{X_{n+m} = r | X_n = j\} = \sum P_{rk}^{(1)} P_{jr}^{(m)}$$

In general, it is observed as

$$P_{jk}^{(m+n)} = \sum P_{rk}^{(n)} P_{jr}^{(m)} = \sum P_{jr}^{(n)} P_{rk}^{(m)}$$

In matrix form, it is represented as

$$p^{(m+n)} = p^{(m)} p^{(n)}$$

This equation is a special case of Chapman-Kolmogorov equation and it is satisfied by the transition probabilities of Markov chain.

Using higher step transition probability matrix and initial probability vector are forecasted.

$$\begin{aligned} P_1 &= P_0 P \\ P_2 &= P_1 P = P_0 P^2 \\ P_3 &= P_2 P = P_0 P^3 \\ P_4 &= P_3 P = P_0 P^4 \\ &\vdots \\ P_m &= P_0 P^m \end{aligned}$$

The computation of m-step transition probability is the computation of m^{th} power of the matrix.

IV. EMPIRICAL RESULT

Urban population of all districts in Tamil Nadu state as per 2001 and 2011 census data were used to examine the mobility of district size distribution of over time. The urban population in the districts of Tamil Nadu state will be divided into five categories: I: 70000-641350, II: 641350-941350, III: 941350-1241350, IV: 1241350-1541350, and V: 1541350-4700000. There is no criterion for setting up the population of 70000. The mobility of district size distribution will be analysed by Markovian model.

A bivariate distribution of urban size of districts in Tamil Nadu state were obtained using the information obtained from 2001 and 2011 census data and represented in Table (1).

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Table 1. Bivariate Distribution Representing the Urban Size of Districts

District size Class (in ten thousand) for 2001 census	District size Class(in ten thousand) for 2011 census				
	7-64.135	64.135- 94.135	94.135 -124.135	124.135- 154.135	154.135- 470
7- 64.135	12	3	-	-	-
64.135- 94.135	-	4	1	-	-
94.135- 124.135	-	-	1	3	-
124.135-154.135	-	-	-	-	5
154.135- 470	-	-	-	-	1

A transition probability matrix using the information given in Table (1) has been obtained and it is presented in Table (2).

Table 2. Transition Probability Matrix for Urban Size of Districts

District size class	S1	S2	S3	S4	S5
S1	0.8	0.2	-	-	-
S2	-	0.8	0.2	-	-
S3	-	-	0.25	0.75	-
S4	-	-	-	-	1
S5	-	-	-	-	1

The initial probability vector P_0 can be obtained as follows

$$P_0 = [0.5 \quad 0.1667 \quad 0.1333 \quad 0.1667 \quad 0.0333]$$

The transition probability matrix P is described as follows

$$P = \begin{pmatrix} 0.8 & 0.2 & - & - & - \\ - & 0.8 & 0.2 & - & - \\ - & - & 0.25 & 0.75 & - \\ - & - & - & - & 1 \\ - & - & - & - & 1 \end{pmatrix}$$

$$P_1 = P_0 \cdot P$$

$$P_1 = [0.5 \quad 0.1667 \quad 0.1333 \quad 0.1667 \quad 0.0333] \times \begin{pmatrix} 0.8 & 0.2 & - & - & - \\ - & 0.8 & 0.2 & - & - \\ - & - & 0.25 & 0.75 & - \\ - & - & - & - & 1 \\ - & - & - & - & 1 \end{pmatrix}$$

$$P_1 = [0.4 \quad 0.23336 \quad 0.066665 \quad 0.099975 \quad 0.2]$$

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$$P_2 = P_1 \cdot P$$

$$= [0.4 \quad 0.23336 \quad 0.066665 \quad 0.099975 \quad 0.2] \times \begin{pmatrix} 0.8 & 0.2 & - & - & - \\ - & 0.8 & 0.2 & - & - \\ - & - & 0.25 & 0.75 & - \\ - & - & - & - & 1 \\ - & - & - & - & 1 \end{pmatrix}$$

$$P_2 = [0.32 \quad 0.266688 \quad 0.063338 \quad 0.049999 \quad 0.299975]$$

$$P_3 = P_2 \cdot P$$

$$= [0.32 \quad 0.266688 \quad 0.063338 \quad 0.049999 \quad 0.299975] \times \begin{pmatrix} 0.8 & 0.2 & - & - & - \\ - & 0.8 & 0.2 & - & - \\ - & - & 0.25 & 0.75 & - \\ - & - & - & - & 1 \\ - & - & - & - & 1 \end{pmatrix}$$

$$P_3 = [0.256 \quad 0.27735 \quad 0.069172 \quad 0.047504 \quad 0.349974]$$

$$P_4 = P_3 \cdot P$$

$$= [0.256 \quad 0.27735 \quad 0.069172 \quad 0.047504 \quad 0.349974] \times \begin{pmatrix} 0.8 & 0.2 & - & - & - \\ - & 0.8 & 0.2 & - & - \\ - & - & 0.25 & 0.75 & - \\ - & - & - & - & 1 \\ - & - & - & - & 1 \end{pmatrix}$$

$$P_4 = [0.2048 \quad 0.2730 \quad 0.0727 \quad 0.0518 \quad 0.3974]$$

$$P_5 = P_4 \cdot P = [0.2048 \quad 0.2730 \quad 0.0727 \quad 0.0518 \quad 0.3974] \times \begin{pmatrix} 0.8 & 0.2 & - & - & - \\ - & 0.8 & 0.2 & - & - \\ - & - & 0.25 & 0.75 & - \\ - & - & - & - & 1 \\ - & - & - & - & 1 \end{pmatrix}$$

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$P_5 = [0.16384 \ 0.259424 \ 0.072807 \ 0.05457 \ 0.44935]$

Table 3. Estimate of Proportion of Districts with Size of Urban area (as per census 2001 and 2011)

District size class in(Ten Thousand)	Census Year					
	2001	2011	2021	2031	2041	2051
7-64.135	0.5	0.4	0.32	0.256	0.204	0.163
64.135-94.135	0.166	0.233	0.266	0.277	0.273	0.259
94.135-124.135	0.133	0.066	0.063	0.069	0.072	0.072
124.135-154.135	0.166	0.099	0.049	0.047	0.051	0.054
154.135-470	0.033	0.2	0.299	0.349	0.397	0.449

V. DISCUSSION

The proportional change of district with respect to urban population in the class interval of district size 1541350-4700000 are obtained as 3.3% in 2001, 20% in 2011, 29.9% in 2021, 34.9% in 2031, 39.7% in 2041 and 44.9% in 2051. This indicates that the proportion of districts has the increasing tendency when time is increasing and more number of districts are moved towards with the highest proportion in the district size interval (1541350 - 4700000) than the other classes.

VI. CONCLUSION

The proportion of districts has the increasing tendency when time is increased and more number of districts are moved towards with the highest proportion in the district size interval (1541350 - 4700000) than the other classes as per census 2001 and 2011.

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BIOGRAPHY



Mrs. R. Selvam, M.Sc., M.Phil., is currently an Assistant Professor, Department of Statistics, Government Arts College (Autonomous), Salem-636007. Prior to her recent appointment at Govt. Arts College (Autonomous), Salem-7, she was a Lecturer in Statistics at various colleges affiliated to Periyar University, Salem, Tamil Nadu, India. She received her M.Sc and M.Phil from Annamalai University, Chidambaram, Tamil Nadu, India and her Ph.D programme is going under process. In 1993, she has been awarded 'The Sir Norman Strathic Prize' by Annamalai University for her excellence in academic. She has teaching experience in Statistics of more than 15 years. She has presented research papers in National and International conference, National level Seminar and National Workshop. She also published research papers in International Journal. Her current research focus is in the area of Mathematical Demography.