

An Approximate Analytical Solution of Nonlinear Differential Equation Arising In Fluid Flow through Homogeneous Porous Media

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ABSTRACT: The present paper discusses an approximate analytical solution of one dimensional nonlinear partial differential equation arising in oil-water displacement process (instability) in a homogeneous porous medium. The homotopy analysis method has been applied to this governing equation by using appropriate boundary conditions. The initial approximate value of saturation of injected water has been satisfying boundary conditions. The numerical value is obtained by using Mathematica software and it is compared with the exact solution. It is concluded that the saturation of injected water increases when the length of fingers and time increases.

KEYWORDS: HOMOTOPY ANALYSIS METHOD, INSTABILITY PHENOMENA, FLUID FLOW, SERIES SOLUTION

I. INTRODUCTION

During the displacement process of two immiscible fluids through homogeneous porous media, the phenomenon of fingering (instabilities) occurs which is frequently encountered in many engineering fields such as soil mechanics, agriculture engineering, ground water and hydrology and petroleum engineering [2, 16, 19]. This type of instability can also be observed in the oil recovery procedure that takes place in oil reservoirs. In the oil recovery technology, it is common practice to inject water into oil field at certain spots in an attempt to drive oil towards a production well. This stage of oil recovery is called secondary recovery process.

This paper presents the important phenomenon of fingering in a double phase flow through the homogeneous porous medium. When a fluid flowing through a porous medium is displaced by another fluid of lesser viscosity instead of regular displacement of the whole front, perturbation takes place which shoot through the porous medium at a relatively great speed. This phenomenon is called instability.

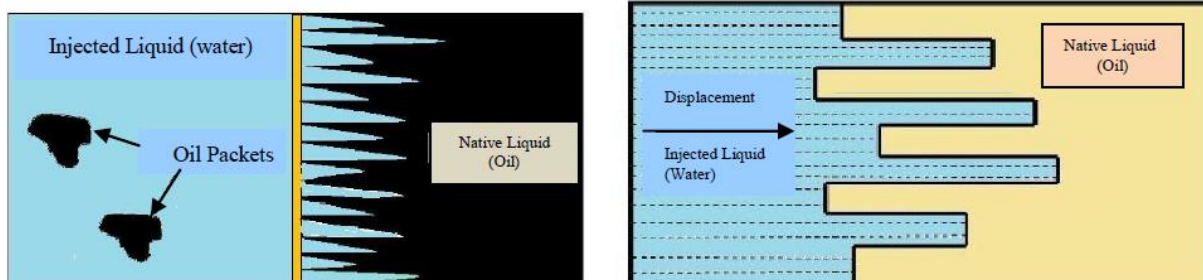


Figure 1

In figure1 fingering process has been shown between oil – water flow into a porous medium. In the statistical treatment of fingering only average cross – sectional area occupied by the fingers was observed while the size and shape of the

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individual fingers are neglected. Scheidegger and Johnson [18] discussed the statistical behaviour of instabilities in homogeneous porous media using method of characteristics. Verma [20] has examined the statistical behaviour of the fingering phenomenon in a displacement process through heterogeneous porous media with capillary pressure using perturbation solution. Patel [14] has solved this problem by using the confluent hypergeometric function. Patel [14] has discussed this problem by using the advection diffusion concept. Mehta and Joshi [12] have discussed the solution by the group invariant method of the instability phenomenon arising in fluid flow through porous media.

In this paper, we have obtained an approximate analytical solution of one dimensional nonlinear partial differential equation arising in oil-water displacement process (instability) in a homogeneous porous medium using the homotopy analysis method. The Mathematica software is used to obtain the numerical values at different time and the values are compared with the exact solution.

II. STATEMENT OF THE PROBLEM

During secondary oil recovery process, when water is injected into oil formation to drive oil towards a production well, the phenomenon of fingering occurs. It is considered that there is a uniform water injection into an oil in a saturated homogeneous porous medium such that the injected water shoots through the native oil and gives rise to perturbances. This furnishes a well-developed fingers flow. Since the entire oil at the initial boundary $x = 0$ (x being measured in the direction of displacement) is displaced through a small distance due to the water injection, therefore, we assume further that complete water saturation exists at the initial boundary.

For mathematical formulation, it has been assumed that Darcy's law is valid for the investigated flow system and assumed further that the macroscopic behaviour of fingers is governed by a statistical treatment. Hence, only the average behaviour of two fluids involved is taken into consideration. The saturation of the injected water is then defined as the average cross-sectional area occupied by the injected water at the distance x . In this paper, the phenomenon is discussed by the homotopy analysis solution.

III. MATHEMATICAL FORMULATION

During water injection in secondary oil recovery process, let injected water and native oil be two immiscible fluids governed by Darcy's law expressed [1] as

$$V_w = -\frac{k_w}{\delta_w} K \left[\frac{\partial P_w}{\partial x} + \rho_w g \sin \alpha \right] \quad (1)$$

$$V_o = -\frac{k_o}{\delta_o} K \left[\frac{\partial P_o}{\partial x} + \rho_o g \sin \alpha \right] \quad (2)$$

where V_w and V_o are the seepage velocities of water and oil respectively, K is the permeability of the homogeneous medium, k_w and k_o are the relative permeabilities of water and oil respectively which are functions of saturation of water S_w and saturation of oil S_o , δ_w and δ_o are the constant kinematic viscosities, P_w and P_o are the pressures of water and oil respectively, ρ_w and ρ_o are constant densities of water and oil respectively, α is the inclination of the bed and g is acceleration due to gravity.

The equations of continuity of the two phases are

$$P \frac{\partial S_w}{\partial t} + \frac{\partial V_w}{\partial x} = 0 \quad (3)$$

$$P \frac{\partial S_o}{\partial t} + \frac{\partial V_o}{\partial x} = 0 \quad (4)$$

where P is the porosity of the medium. From the definition of phase saturation, it is evident that

$$S_w + S_o = 1. \quad (5)$$

The capillary pressure P_c , defined as the pressure discontinuity of the flowing phases across their common interface is a function of the phase saturation. For definiteness we assume a continuous linear function of the form

$$P_c = -\beta S_w \quad \text{Mehta [11]} \quad (6)$$

where β is a constant quantity.

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$$P_c = P_o - P_w \quad (7)$$

For definiteness of the mathematical analysis, we assume the standard relationship due to Scheidegger and Johnson [18], between phase saturation and relative permeability as,

$$k_w = S_w \quad (8)$$

$$k_o = S_o = 1 - S_w \quad (9)$$

The equation of motion for saturation can be obtained by equations (1) - (4), we get

$$P \frac{\partial S_w}{\partial t} = \frac{\partial}{\partial x} \left[K \frac{k_w}{\delta_w} \frac{\partial P_w}{\partial x} + K \frac{k_w}{\delta_w} \rho_w g \sin \alpha \right] \quad (10)$$

$$P \frac{\partial S_o}{\partial t} = \frac{\partial}{\partial x} \left[K \frac{k_o}{\delta_o} \frac{\partial P_o}{\partial x} + K \frac{k_o}{\delta_o} \rho_o g \sin \alpha \right] \quad (11)$$

These are the general flow equations of the phases in homogeneous porous medium when effect due to pressure discontinuity and gravity term in inclined porous medium is considered.

Neglecting the effect of inclination and gravity, the equations of motion for saturation obtained from equations (10) and (11) are

$$P \frac{\partial S_w}{\partial t} = \frac{\partial}{\partial x} \left(\frac{k_w}{\delta_w} K \frac{\partial P_w}{\partial x} \right) \quad (12)$$

$$P \frac{\partial S_o}{\partial t} = \frac{\partial}{\partial x} \left(\frac{k_o}{\delta_o} K \frac{\partial P_o}{\partial x} \right) \quad (13)$$

Using equation (7), equation (12) reduces to

$$P \frac{\partial S_w}{\partial t} = \frac{\partial}{\partial x} \left[\frac{k_w}{\delta_w} K \left(\frac{\partial P_o}{\partial x} - \frac{\partial P_c}{\partial x} \right) \right] \quad (14)$$

Using equations (13) and (5), eliminating $\frac{\partial S_w}{\partial t}$ from (14), we get

$$\frac{\partial}{\partial x} \left[K \left(\frac{k_w}{\delta_w} + \frac{k_o}{\delta_o} \right) \frac{\partial P_o}{\partial x} - K \frac{k_w}{\delta_w} \frac{\partial P_c}{\partial x} \right] = 0 \quad (15)$$

Integrating both sides with respect to x , we have

$$K \left(\frac{k_w}{\delta_w} + \frac{k_o}{\delta_o} \right) \frac{\partial P_o}{\partial x} - K \frac{k_w}{\delta_w} \frac{\partial P_c}{\partial x} = -C \quad (16)$$

where C is integrating constant.

Solving (16) for $\frac{\partial P_o}{\partial x}$,

$$\frac{\partial P_o}{\partial x} = \frac{-C}{K \left(\frac{k_w}{\delta_w} + \frac{k_o}{\delta_o} \right)} + \frac{\frac{\partial P_c}{\partial x}}{1 + \frac{k_o \delta_w}{k_w \delta_o}} \quad (17)$$

Using (17) in (14) we have

$$P \frac{\partial S_w}{\partial t} + \frac{\partial}{\partial x} \left[\frac{K \frac{k_o}{\delta_o} \frac{\partial P_c}{\partial x}}{1 + \frac{k_o \delta_w}{k_w \delta_o}} + \frac{C}{1 + \frac{k_o \delta_w}{k_w \delta_o}} \right] = 0 \quad (18)$$

The value of the pressure of oil P_o can be written in the form

$$P_o = \frac{P_o + P_w}{2} + \frac{P_o - P_w}{2} = \bar{P} + \frac{1}{2} P_c \quad (19)$$

where \bar{P} is the mean pressure which is constant, therefore (16) implies

$$C = \frac{K}{2} \left(\frac{k_w}{\delta_w} - \frac{k_o}{\delta_o} \right) \frac{\partial P_c}{\partial x} \quad (20)$$

Equation (18) becomes

$$P \frac{\partial S_w}{\partial t} + \frac{1}{2} \frac{\partial}{\partial x} \left[K \frac{k_w}{\delta_w} \frac{\partial P_c}{\partial S_w} \frac{\partial S_w}{\partial x} \right] = 0 \quad (21)$$

Since $k_w = S_w$ & $P_c = -\beta S_w$, we have

$$P \frac{\partial S_w}{\partial t} - \frac{\beta K}{2} \frac{\partial}{\partial x} \left(S_w \frac{\partial S_w}{\partial x} \right) = 0 \quad (22)$$

Using dimensionless variables $X = \frac{x}{L}$, $T = \frac{K \beta t}{2 \delta_w L^2 P}$,

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equation (22) reduces to

$$\frac{\partial S_w}{\partial T} = \frac{\partial}{\partial X} \left(S_w \frac{\partial S_w}{\partial X} \right) = S_w \frac{\partial^2 S_w}{\partial X^2} + \left(\frac{\partial S_w}{\partial X} \right)^2 \quad (23)$$

where $S_w(x, t) = S_w(X, T)$.

The set of boundary conditions for instability phenomenon are

$$S_w(0, T) = T \quad (24)$$

$$S_w(1, T) = 1 - T \quad (25)$$

Here, $S_w(0, T) = T$ is saturation of injected water at $X = 0$ and $S_w(1, T) = 1 - T$ is saturation of injected water at $X = 1$.

IV. BASIC IDEA OF THE HOMOTOPY ANALYSIS METHOD (HAM)

The HAM [4, 7] was first proposed by Liao in 1992 to solve nonlinear problems. The HAM has been successfully applied to many nonlinear problems. This technique is very powerful and has been already successfully applied to various complicated problems.

Consider the following nonlinear differential equation

$$\mathcal{N}[u(r, t)] = 0 \quad (26)$$

where \mathcal{N} is a nonlinear operator, r and t are independent variables, $u(r, t)$ is an unknown function.

❖ The zero order deformation equation

For simplicity, we ignore all boundary and/or initial conditions. By means of the homotopy analysis method, Liao [7] constructs the so called zero order deformation equation

$$(1 - q)\mathcal{L}[\varphi(r, t; q) - u_0(r, t)] = qc_0H(r, t)\mathcal{N}(\varphi(r, t; q)) \quad (27)$$

where $q \in [0, 1]$ is the embedding parameter, c_0 is a nonzero convergence control parameter, $H(r, t)$ is nonzero auxiliary function, \mathcal{L} is an auxiliary linear operator, $u_0(r, t)$ is an initial guess of $u(r, t)$, $\varphi(r, t; q)$ is an unknown function.

It is important that one has great freedom to choose nonzero convergence control parameter, an auxiliary function, an auxiliary linear operator and an initial guess in the HAM.

When $q = 0$ and $q = 1$, it holds

$$\varphi(r, t; 0) = u_0(r, t), \quad (28)$$

$$\text{and } \varphi(r, t; 1) = u(r, t) \quad (29)$$

respectively. Thus, as the embedding parameter q increases from 0 to 1, the solution $\varphi(r, t; q)$ varies (or deforms) from the initial guesses $u_0(r, t)$ to the solution $u(r, t)$ of the original equation (26). Due to Taylor's Theorem and (28), we expand $\varphi(r, t; q)$ in Taylor series with respect to q , we have

$$\varphi(r, t; q) = u_0(r, t) + \sum_{m=1}^{\infty} u_m(r, t)q^m \quad (30)$$

$$\text{where } u_m(r, t) = \frac{1}{m!} \left. \frac{\partial^m \varphi(r, t; q)}{\partial q^m} \right|_{q=0} \quad (31)$$

The above series is called the homotopy series. If the auxiliary linear operator, the initial guess, the convergence control parameter c_0 and the auxiliary function are properly chosen, the homotopy series converges at $q = 1$. Then we have due to equation (29)

$$u(r, t) = u_0(r, t) + \sum_{m=1}^{\infty} u_m(r, t) \quad (32)$$

❖ The high order deformation equation

Differentiating the zero order deformation equation (27) m times with respect to the embedding parameter q and then dividing them by $m!$ and finally setting $q = 0$, we have the so called high order deformation equation

$$\mathcal{L}[u_m(r, t) - \chi_m u_{m-1}(r, t)] = c_0 H(r, t) \mathcal{R}_m(r, t) \quad (33)$$

$$\text{where } \mathcal{R}_m(r, t) = \frac{1}{(m-1)!} \left. \frac{\partial^{m-1} \mathcal{N}(\varphi(r, t; q))}{\partial q^{m-1}} \right|_{q=0} \quad (34)$$

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$$\text{and } \chi_m = \begin{cases} 0, & \text{when } m \leq 1 \\ 1, & \text{when } m > 1 \end{cases} \quad (35)$$

Applying \mathcal{L}^{-1} on both sides of equation (33), we get

$$u_m(r, t) = \chi_m u_{m-1}(r, t) + c_0 \mathcal{L}^{-1}[H(r, t)\mathcal{R}_m(r, t)] \quad (36)$$

In this way, it is easy to obtain u_m for $m \geq 1$.

V. APPLYING HOMOTOPY ANALYSIS METHOD

To solve the equation (23) by means of the HAM, according to the boundary conditions, it is straight forward to choose initial approximation as

$$S_{w_0}(X, T) = (1 - T)X + T \quad (37)$$

We choose the linear operator as $\mathcal{L}[\varphi(X, T; q)] = \frac{\partial^2 \varphi(X, T; q)}{\partial X^2}$ (38)

with the property $\mathcal{L}[C_1 + C_2 X] = 0$, where C_1 & C_2 are constants.

Now, we define a nonlinear operator as

$$\mathcal{N}(\varphi(X, T; q)) = \varphi(X, T; q) \frac{\partial^2 \varphi(X, T; q)}{\partial X^2} + \left(\frac{\partial \varphi(X, T; q)}{\partial X} \right)^2 - \frac{\partial \varphi(X, T; q)}{\partial T} \quad (39)$$

Using assumption $H(X, T) = 1$, we construct the zero order deformation equation

$$(1 - q)\mathcal{L}[\varphi(X, T; q) - S_{w_0}(X, T)] = qc_0 \mathcal{N}(\varphi(X, T; q)) \quad (40)$$

Obviously, when $q = 0$ and $q = 1$,

$$\varphi(X, T; 0) = S_{w_0}(X, T) \quad (41)$$

$$\text{and } \varphi(X, T; 1) = S_w(X, T) \quad (42)$$

Thus, we obtain the high order deformation equation

$$\mathcal{L}[S_{w_m}(X, T) - \chi_m S_{w_{m-1}}(X, T)] = C_0 \mathcal{R}_m(X, T) \quad (43)$$

$$\text{where } S_w(X, T) = S_{w_0}(X, T) + \sum_{m=1}^{\infty} S_{w_m}(X, T) \quad (44)$$

$$\& \mathcal{R}_m(X, T) = \sum_{j=0}^{m-1} S_{w_j}(X, T) \frac{\partial^2 S_{w_{m-1-j}}(X, T)}{\partial X^2} + \sum_{j=0}^{m-1} \frac{\partial S_{w_j}(X, T)}{\partial X} \frac{\partial S_{w_{m-1-j}}(X, T)}{\partial X} - \frac{\partial S_{w_{m-1}}(X, T)}{\partial T} \quad (45)$$

Now, the solution of high order deformation equation (43) is

$$S_{w_m}(X, T) = \chi_m S_{w_{m-1}}(X, T) + c_0 \mathcal{L}^{-1}[\mathcal{R}_m(X, T)] \quad (46)$$

subject to boundary conditions $S_{w_m}(0, T) = 0$, $S_{w_m}(1, T) = 0$, $m \geq 1$

In this way, we get $S_{w_m}(X, T)$ for $m = 1, 2, 3, \dots$ successively.

$$\begin{aligned} \text{Hence, } S_w(X, T) = & (T + X - TX) + c_0 \left(-\frac{X}{6} + TX - \frac{T^2 X}{2} - TX^2 + \frac{T^2 X^2}{2} + \frac{X^3}{6} \right) \\ & + c_0 \left[-\frac{X}{6} + TX - \frac{T^2 X}{2} - TX^2 + \frac{T^2 X^2}{2} + \frac{X^3}{6} + c_0 \frac{X}{12} + c_0 \frac{TX}{12} + c_0 T^2 X - c_0 \frac{T^3 X}{2} - c_0 \frac{X^2}{6} + c_0 \frac{5TX^2}{6} \right. \\ & \left. - c_0 \frac{5T^2 X^2}{2} + c_0 T^3 X^2 - c_0 \frac{X^3}{6} - c_0 \frac{2TX^3}{3} + c_0 \frac{3T^2 X^3}{2} - c_0 \frac{T^3 X^3}{2} + c_0 \frac{X^4}{4} - c_0 \frac{TX^4}{4} \right] + \dots \quad (47) \end{aligned}$$

is an infinite series solution which represents the saturation of injected water at distance X for time $T > 0$. Thus, the saturation of injected water into the native oil is expressed in powers of X and T . Equation (47) describes the analytical expression for the saturation of injected water for the instability phenomenon arising in fluid flow through homogeneous porous media.

VI. NUMERICAL AND GRAPHICAL SOLUTION

The BVPh is a Mathematica package for nonlinear boundary value problems which is a combination of the homotopy analysis method and the computational software Mathematica [10]. This package is used to obtain numerical

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presentation of equation (47). Table 1 indicates the numerical values of saturation of injected water for different distance X and time T .

Table – 1 Numerical values of the Saturation of injected water in homogeneous porous medium

		$T = 0.001$	$T = 0.002$	$T = 0.003$	$T = 0.004$	$T = 0.005$
$X = 0.1$	HAM Solution	0.100768907	0.101669626	0.102570344	0.103471061	0.104371777
	Exact Solution	0.1008	0.1016	0.1024	0.1032	0.1040
$X = 0.2$	HAM Solution	0.200545541	0.201346818	0.202148095	0.202949370	0.203750644
	Exact Solution	0.2006	0.2012	0.2018	0.2024	0.2030
$X = 0.3$	HAM Solution	0.300337897	0.301039574	0.301741251	0.302442925	0.303144598
	Exact Solution	0.3004	0.3008	0.3012	0.3016	0.3020
$X = 0.4$	HAM Solution	0.400153990	0.400755908	0.401357824	0.401959739	0.402561651
	Exact Solution	0.4002	0.4004	0.4006	0.4008	0.4010
$X = 0.5$	HAM Solution	0.500001851	0.500503850	0.501005847	0.501507842	0.502009835
	Exact Solution	0.5	0.5	0.5	0.5	0.5

Figure 2 represents the graph of $S_w(X, T)$ vs. distance X for time $T = 0.001, 0.002, 0.003, 0.004, 0.005$ and $c_0 = 0.0008$. Numerical values of Table 1 are used for Figure 2. The convergence of homotopy series is dependent upon the value of convergence control parameter c_0 [4]. So, we choose the proper value of the convergence control parameter $c_0 = 0.0008$.

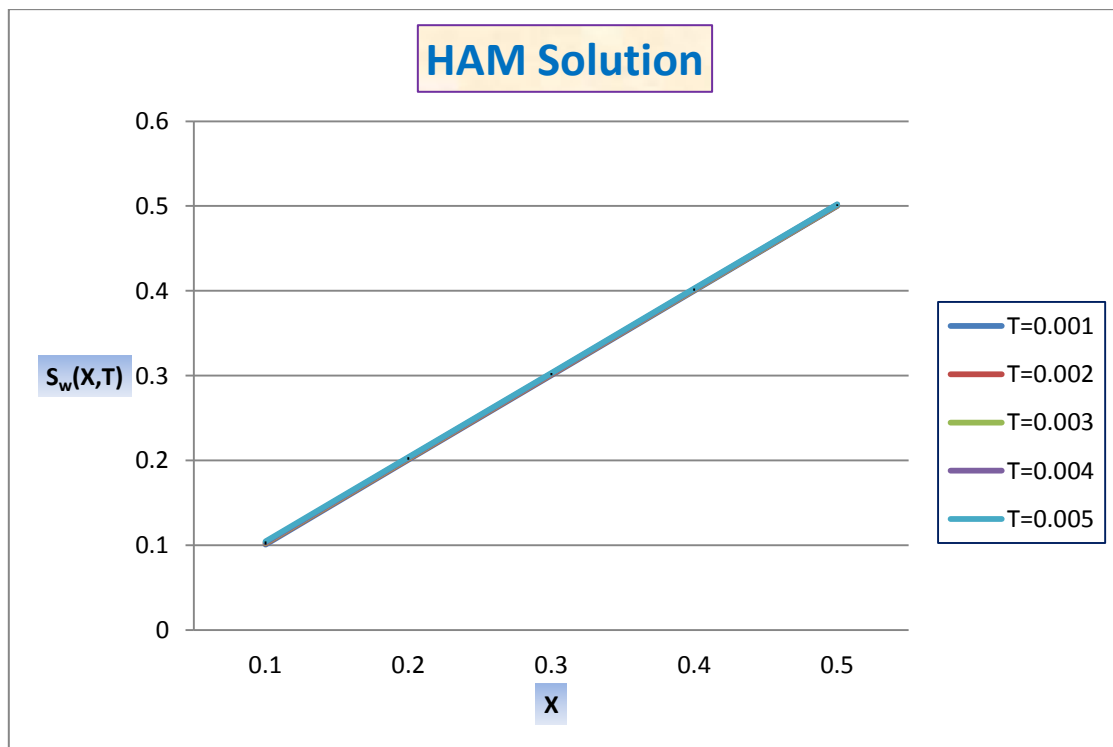


Figure 2: Saturation of injected water at distance X for time $T = 0.001, 0.002, 0.003, 0.004, 0.005$ with $c_0 = 0.0008$.

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Figure 3 represents the graph of $S_w(X, T)$ vs. distance X for time $T = 0.001, 0.002, 0.003, 0.004, 0.005$ for exact solution [15] and tabular values are used for the graphical representations of the saturation of injected water.

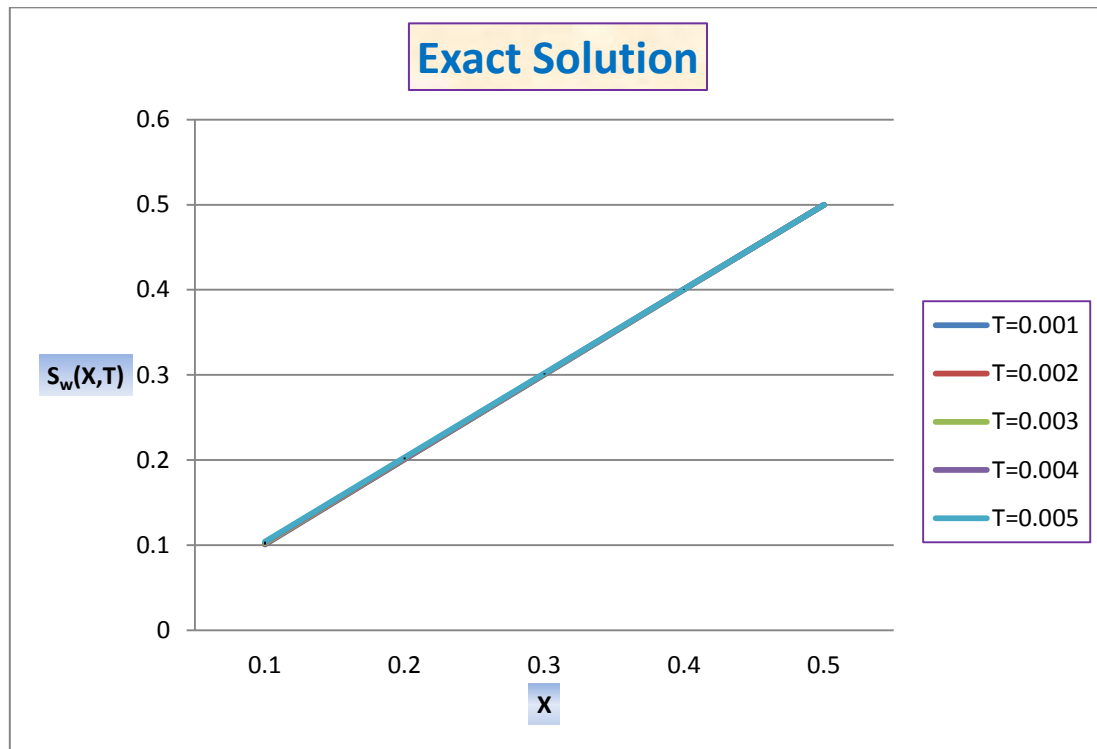


Figure 3: Saturation of injected water at distance X for time $T = 0.001, 0.002, 0.003, 0.004, 0.005$

VII. CONCLUSION

Equation (47) represents an approximate analytical solution of one dimensional nonlinear partial differential equation (23) arising in oil-water displacement process (instability) in a homogeneous porous medium. Comparison of the results obtained by using the homotopy analysis method with the numerical values of exact solution is given in Table 1. It is concluded that values of S_w in exact solution and solution by the HAM are much closer. Figure 2 and Figure 3 represent the graphs of $S_w(X, T)$ vs. distance X for time $T = 0.001, 0.002, 0.003, 0.004, 0.005$ by the HAM Solution for $c_0 = 0.0008$ and by the exact solution respectively which shows that saturation increases when X increases.

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