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Lucky Edge Labeling of Different Graphs

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ABSTRACT: Let G be a Simple Graph with Vertex set $V(G)$ and Edge set $E(G)$ respectively. Vertex set $V(G)$ are labeled arbitrary by positive integers and let $E(e)$ denote the edge label such that it is the sum of labels of vertices incident with edge e . The labeling is said to be **lucky edge labeling** if the edge set $E(G)$ is a proper coloring of G , that is, if we have $E(e_1) \neq E(e_2)$ whenever e_1 and e_2 are adjacent edges. The least integer k for which a graph G has a lucky edge labeling from the set $\{1, 2, \dots, k\}$ is the **lucky number** of G denoted by $\eta(G)$. A graph which admits lucky edge labeling is the **lucky edge labeled graph**. In this paper the lucky edge labeling of Bistar graph, Wheel graph and Fan graph are discussed.

KEYWORDS: Lucky edge labeling, Lucky number, Lucky edge labeled graph.[2]

I. INTRODUCTION

All the graphs considered are undirected simple graphs. The labeling of graphs trace their origin being introduced by Rosa[1] in 1967. A graph labeling is an assignment of integers to the vertices or edges, or both, subject to certain conditions[5]. For the basics of graph theory concepts and notations refer to Harary[4] and Douglas B. West[2].

Definition 1.1:

Let G be a Simple Graph with Vertex set $V(G)$ and Edge set $E(G)$ respectively. Vertex set $V(G)$ are labeled arbitrary by positive integers and let $E(e)$ denote the edge label such that it is the sum of labels of vertices incident with edge e . The labeling is said to be **lucky edge labeling** if the edge set $E(G)$ is a proper coloring of G , that is, if we have $E(e_1) \neq E(e_2)$ whenever e_1 and e_2 are adjacent edges.[3]

Definition 1.2:

The least integer k for which a graph G has a lucky edge labeling from the set $\{1, 2, \dots, k\}$ is the **lucky number** of G denoted by $\eta(G)$ [4].

Definition 1.3:

A graph which admits lucky edge labeling is the **lucky edge labeled graph**.

Definition 1.4:

The Bistar graph $B_{m,n}$ is the graph obtained from K_2 by joining m pendent edges to one end and n pendent edges to the other end of K_2 . [6]

Definition 1.5:

The wheel graph denoted by W_n is a graph with n vertices ($n \geq 4$), formed by connecting a single vertex to all vertices of an $(n-1)$ - cycle [6].

Definition 1.6:

The Fan graph denoted by F_n can be constructed by joining n copies of the cycle graph C_3 with a common vertex. The Fan graph F_n is a planar undirected graph with $2n+1$ vertices and $3n$ edges.

II. MAIN RESULTS

Theorem 1:-

The lucky number of the Bistar graph $B_{m,n}$ is given by

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$$\eta(B_{m,n}) = \begin{cases} m+2 & m \geq n \\ n+2 & n \geq m. \end{cases}$$

Proof:

Let the vertices of K_2 be v_1 and v_2 . Let the vertices joined to v_1 by the pendent edges be u_1, u_2, \dots, u_m and let the vertices joined to v_2 by the pendent edges be w_1, w_2, \dots, w_n . Label the vertices v_1 and v_2 with 1. [8] Label the vertices u_i by the labels $i+1$ ($i = 1$ to m) and label the vertices w_j by the labels $j+1$ ($j = 1$ to n). This induces a lucky edge labeling for $B_{m,n}$.

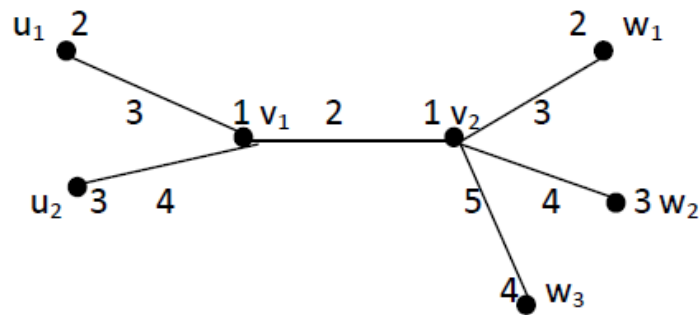


Fig. 1: $\eta(B_{2,3}) = 5$

Theorem 2:-

The lucky number of wheel graph W_n is $\eta(W_n) = 2n - 1$.

Proof:

Let the single vertex and the centre be v_1 and the other vertices of $(n-1) -$ cycle that are joined to the vertex v_1 be v_2, v_3, \dots, v_n . Label the vertices v_i by the label i ($i = 1$ to n). This induces a lucky edge labeling of W_n and $\eta(W_n) = 2n - 1$ [10].

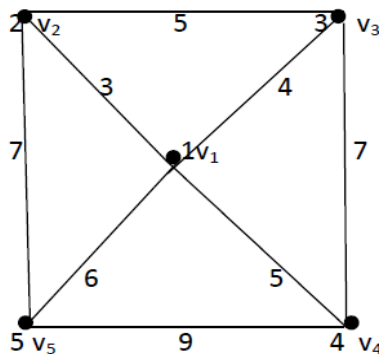


Fig. 2: $\eta(W_5) = 9$.

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Theorem 3:-

The lucky number of Fan graph F_n is $\eta(F_n) = 4n+1$.

Proof:

Let the middle vertex of the join of all the C_3 be v_1 and let the remaining vertices be v_2, v_3, \dots, v_n . Label the vertices v_i with labels i ($i = 1$ to n). [12] This induces a lucky edge labeling of the Fan graph F_n with lucky number given by $\eta(F_n) = 4n+1$.

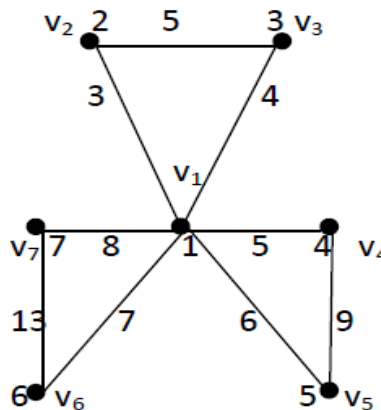


Fig.3: $\eta(F_3) = 13$.

III. CONCLUSION

In this paper the lucky numbers are found for three types of graphs – Bistar graph, Wheel graph and Fan graph. The study can be extended for other graphs.

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