

Application of Grey Possibility Degree in Comparing Poverty

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ABSTRACT: The poverty of a certain region is very much context dependent and is of great importance particularly for the rural areas. The nature of the attributes is complex and not easy to determine mathematically. A number of attempts have been raised towards this problem, but lack of consideration of inherent uncertainty draws back most of them. In this paper, the grey possibility degree has been applied to estimate and compare poverty of some regions. A case study is provided to illustrate the effectiveness of applying grey possibility degree. It can be applied to other decision making approaches too.

KEYWORDS: Poverty, Poverty estimation, Fuzzy Logic, Grey Numbers, Grey Possibility Theory.

I. INTRODUCTION

Poverty in rural areas is a fact of real concern to the policy makers. The traditional way of dealing with poverty is not worthy in this case. The attributes considered in their estimation are represented and aggregated using two valued logical algorithms. Uncertainty is present but not considered at all there. So the necessity of an approach that can deal with the inherent uncertainty must be placed. Otherwise the process remains ineffective. Estimating and comparing poverty in rural areas is really very critical and a concrete model of this problem satisfying all the socio economic parameters is somehow missing in the literature. Several authors represented their views on poverty from different angles both theoretically and practically. The main problem is that the theoretical possibilities face such type of problems that cannot be defined or structured in mathematical framework. So the error is there and we are unable to limit it. This work is a new one from the viewpoint of uncertainty. The various aspects of the status of a rural household are not completely deterministic. For example, consider the attribute: income. Normally in Indian rural areas the total income of an individual household varies in a certain range because of the unavailability of monetary engagements at different times. So when a household is asked to state their status corresponding to this attribute, it is quiet unnatural to have a numeric answer, rather we should seek for a linguistic termed answer, e.g., Very Poor, Poor, Medium Poor, Fair, Medium Good, Good, Very Good. The uncertainty is thus considered for each attribute.

Sen (1976) ([8], [9]) rolled over the problem of poverty index in two ways: poor household identification and aggregation of the attributes of the poor in an overall indicator which quantifies the poverty index. The transfer axiom proposed by Sen stipulates that a transfer of income from a poor to a rich person certainly increases the poverty degree when other attributes are fixed. Another axiom in this context is population monotonicity axiom which states that an addition of a person below the poverty line should increase the poverty degree and the addition of a person above the poverty line should decrease the degree when other attributes are fixed. The introduction of these two axioms leads to a certain anomaly regarding the possibility of any poverty index. The poverty index of a society certainly depends on the individual poverty degree of each household. So this problem has been taken into consideration with great importance. Some poverty models are also proposed on the basis of these methods, e.g., the logit model, conditional income distribution model, Fuzzy set theoretic model, etc.

In section II, some preliminaries on grey system are given. In section III, the methodology is illustrated and in section IV a real life example is demonstrated.

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II. GREY NUMBERS AND GREY POSSIBILITY DEGREE

Grey theory was first proposed by Deng in 1982. A grey system is a system presented by grey number and variables and it contains uncertain information.

An interval grey number $\otimes G$ is represented by $[\underline{G}, \bar{G}]$ where \underline{G} and \bar{G} are real numbers and may be infinite valued. According to Moore's dissertation ([6]) we state here some algebraic operational laws on interval grey numbers:

$$\otimes G_1 + \otimes G_2 = [\underline{G}_1 + \underline{G}_2, \bar{G}_1 + \bar{G}_2]$$

$$\otimes G_1 - \otimes G_2 = [\underline{G}_1 - \underline{G}_2, \bar{G}_1 - \bar{G}_2]$$

$$\otimes G_1 \times \otimes G_2 = [\min(\underline{G}_1 \underline{G}_2, \underline{G}_1 \bar{G}_2, \bar{G}_1 \underline{G}_2, \bar{G}_1 \bar{G}_2),$$

$$\max(\underline{G}_1 \underline{G}_2, \underline{G}_1 \bar{G}_2, \bar{G}_1 \underline{G}_2, \bar{G}_1 \bar{G}_2)]$$

For two intervals grey numbers $\otimes G_1$ and $\otimes G_2$ the possibility degree of $\otimes G_1 \leq \otimes G_2$ can be expressed as ([6]):

$$P\{\otimes G_1 \leq \otimes G_2\} =$$

$$\frac{\max(0, L(\otimes G_1) + L(\otimes G_2) - \max(0, \bar{G}_1 - \underline{G}_2))}{L(\otimes G_1) + L(\otimes G_2)}$$

$$\text{where } L(\otimes G) = \bar{G} - \underline{G}.$$

The above mentioned grey possibility degree has the following properties.

- i) If $\otimes G_1 = \otimes G_2$, then $P(\otimes G_1 \leq \otimes G_2) = 0.5$,
- ii) If $\underline{G}_2 > \bar{G}_1$, then $P(\otimes G_1 \leq \otimes G_2) = 1$,
- iii) If $\bar{G}_2 < \underline{G}_1$, then $P(\otimes G_1 \leq \otimes G_2) = 0$,
- iv) When there is some part in the intersection of $\otimes G_1$ and $\otimes G_2$, then $P(\otimes G_1 \leq \otimes G_2) < 0.5$ implies $\otimes G_2$ is smaller than $\otimes G_1$ and $P(\otimes G_1 \leq \otimes G_2) > 0.5$ implies $\otimes G_1$ is smaller than $\otimes G_2$.

We illustrate this by an example.

Consider a situation when two sets of grey numbers (G_1) and (G_2) are compared where (G_1) = {(1, 3), (2, 4), (5, 6)} and (G_2) = {(2, 3), (3, 5), (7, 9)}.

$$\text{Here } P\{(G_1) \leq (G_2)\} = \frac{1}{3} \left[\frac{2}{3} + \frac{3}{4} + 1 \right] = 0.80556.$$

Again consider another situation where two sets of grey numbers (G_1) and (G_3) are compared where (G_1) = {(1, 3), (2, 4), (5, 6)} and (G_3) = {(2, 3), (3, 5), (8, 10)}.

$$\text{Here } P\{(G_1) \leq (G_3)\} = \frac{1}{3} \left[\frac{2}{3} + \frac{3}{4} + 1 \right] = 0.80556.$$

Here we clearly observe that $P\{(G_1) \leq (G_2)\} = P\{(G_1) \leq (G_3)\}$, but (G_2) \neq (G_3).

III. COMPARISON OF POVERTY- A CASE STUDY

In this work, a case study is presented to compare the status of poverty in four districts of West Bengal, India. The four districts are Murshidabad (D_1), Malda (D_2), Birbhum (D_3) and Burdwan (D_4). The districts are compared according to the attributes considered. This consideration can be extended to any number of attributes and any suitable number of regions for the comparison. Nevertheless this is somehow equivalent to multi-criteria decision making in uncertain environment. Let us consider a discrete set of m Districts, $D = \{D_1, D_2, \dots, D_m\}$ and a set of n attributes, $A = \{A_1, A_2, \dots,$

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A_n }. These attributes are additively independent. Also consider $\otimes w = \{ \otimes w_1, \otimes w_2, \dots, \otimes w_n \}$ as an attribute weight vector of the attribute weights which are realized to be linguistic variables. The attributes we consider here in this example are

- A_1 : Housing Condition
- A_2 : Clothing Status
- A_3 : Educational Status
- A_4 : Average monthly income per capita
- A_5 : Food Security

Now these linguistic weights and attribute ratings can be expressed in grey numbers as shown in table 3.1.

TABLE 3.1: EXPRESSION OF LINGUISTIC TERMS IN GREY NUMBERS

Linguistic Attribute weights		Attribute ratings	
Linguistic terms	Grey numbers	Linguistic terms	Grey numbers
Very Low (VL)	[0.0,0.1]	Very Poor (VP)	[0,1]
Low (L)	[0.1,0.3]	Poor (P)	[1,3]
Medium Low (ML)	[0.3,0.4]	Medium Poor (MP)	[3,4]
Medium (M)	[0.4,0.5]	Fair (F)	[4,5]
Medium High (MH)	[0.5,0.6]	Medium Good (MG)	[5,6]
High (H)	[0.6,0.9]	Good (G)	[6,9]
Very High (VH)	[0.9,1.0]	Very Good (VG)	[9,10]

Step1: A group of k experts $E = \{E_1, E_2, \dots, E_k\}$ is formed. The attribute weight $\otimes w_j$ of the jth attribute A_j is calculated as

$$\otimes w_j = \frac{1}{k} \left[\otimes w_j^1 + \otimes w_j^2 + \dots + \otimes w_j^k \right]$$
 where $\otimes w_j^k$ is the attribute weight given by the kth Expert, described by the grey number $[\underline{w}_j^k, \bar{w}_j^k]$.

Here in this case study $E = \{E_1, E_2, E_3\}$, i.e., 3 distinct experts are considered to impose their decisions on different aspects in the decision making process.

Step2: The linguistic variables for the ratings to make an attribute rating value are denoted as $\otimes G_{ij}$, defined by the grey numbers $[\otimes \underline{G}_{ij}, \otimes \bar{G}_{ij}]$. Thus the Grey Decision Matrix (GDM) is established. This matrix is normalized and the

Normalized Grey Decision Matrix (NGDM) is constituted by the elements $\otimes G_{ij}' = \left[\frac{\otimes \underline{G}_{ij}}{G_j^{\max}}, \frac{\otimes \bar{G}_{ij}}{G_j^{\max}} \right]$ where

$G_j^{\max} = \max_{1 \leq i \leq m} \{ \otimes \bar{G}_{ij} \}$. Table 3.2 displays the status of the 4 districts with respect to the 5 attributes.

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TABLE 3.2: GREY DECISION MATRIX (GDM)

GDM	A ₁	A ₂	A ₃	A ₄	A ₅
D ₁	VP	P	MP	P	F
D ₂	F	MP	MP	F	MG
D ₃	P	VP	P	VP	MG
D ₄	MG	F	G	MG	VP

The weights of the attributes are gathered from the experts and displayed in table 3.3.

TABLE 3.3: ATTRIBUTE WEIGHT MATRIX (AWM)

$\otimes w_j$	D ₁	D ₂	D ₃	D ₄
A ₁	H	MH	M	MH
A ₂	ML	L	ML	ML
A ₃	L	ML	L	L
A ₄	VH	VH	VH	VH
A ₅	M	M	MH	M

Now the average attribute weights are calculated:

$$\otimes w_1 = [0.345, 0.325], \otimes w_2 = [0.45, 0.525], \otimes w_3 = [0.055, 0.15], \otimes w_4 = [0.87, 1], \otimes w_5 = [0.555, 0.625].$$

Using (3.2) and (3.3) we construct the Normalized Grey Decision Matrix as shown in table 3.4.

TABLE 3.4: NORMALIZED GREY DECISION MATRIX

NGDM	T ₁	T ₂	T ₃	T ₄	T ₅
D ₁	[0.11, 0.33]	[0.33, 0.44]	[0, 0.11]	[0.33, 0.44]	[0.33, 0.44]
D ₂	[0.33, 0.44]	[0.44, 0.55]	[0.44, 0.55]	[0.33, 0.44]	[0.55, 0.67]
D ₃	[0, 0.11]	[0.11, 0.33]	[0, 0.11]	[0.11, 0.33]	[0.44, 0.55]
D ₄	[0.44, 0.55]	[0.44, 0.55]	[0.55, 0.67]	[0.55, 0.67]	[0.11, 0.33]

Step3: Now each normalized elements are multiplied by their corresponding weights and we retrace the weighted NGDM by the elements $\otimes T_{ij} = \otimes G_{ij}' \times w_j$. The weighted normalized grey decision matrix is shown in table 3.5.

TABLE 3.5: WEIGHTED NORMALIZED GREY DECISION MATRIX

WNGDM	T ₁	T ₂	T ₃	T ₄	T ₅
D ₁	[0.047, 0.173]	[0.140, 0.231]	[0, 0.058]	[0.14, 0.231]	[0.14, 0.231]
D ₂	[0.049, 0.143]	[0.066, 0.179]	[0.066, 0.179]	[0.049, 0.143]	[0.082, 0.218]
D ₃	[0, 0.027]	[0.008, 0.082]	[0, 0.027]	[0.008, 0.082]	[0.033, 0.137]
D ₄	[0.396, 0.55]	[0.396, 0.55]	[0.495, 0.67]	[0.495, 0.67]	[0.099, 0.33]

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Step4: At the final stage of this method, a pseudo alternative, called Comparatively Richest District (CRD) is

constructed by $CRD = \left\{ \otimes G_1^{\max} = \left[\max_i T_{i1}, \max_i \bar{T}_{i1} \right], \dots, \otimes G_n^{\max} = \left[\max_i T_{in}, \max_i \bar{T}_{in} \right] \right\}$.

We use this pseudo alternative to evaluate the ordering. Each district is compared with the CRD by the grey-possibility

degree $P\{X_i \leq CRD\} = \frac{1}{n} \sum_{j=1}^n P\{\otimes T_{ij} \leq \otimes G_j^{\max}\}$.

The entries in the NGDM are multiplied by the corresponding attribute weights. Now using (3.4) we construct the WNGDM as:

From the above matrix, we first construct the 'Comparatively Richest District' CRD by using (3.5) as $CRD = \{[0.297, 0.46], [0.46, 0.57], [0.425, 0.77], [0.515, 0.67], [0.34, 0.331]\}$. Finally we use (3.6) to find the grey-possibility degrees:

$$P\{D_1 \leq CRD\} = 0.87,$$

$$P\{D_2 \leq CRD\} = 0.943,$$

$$P\{D_3 \leq CRD\} = 1,$$

$$P\{D_4 \leq CRD\} = 0.542.$$

The ranking is done according to this possibility value and higher possible alternative gets greater rank.

Based on this rank the Government or the policy makers can establish a poverty line separation algorithm. The next section exemplifies the methodology with an alternative proposition.

Now it is the responsibility of the policy makers to draw the line of separation. We propose here to divide the range [0, 1] of possibilistic values in some intervals like [0, 0.3), [0.3, 0.6), [0.6, 0.8), [0.8, 0.9) and [0.9, 1] and separately economic policies should be applied on them. We call the district individuals of these classes as Comparatively Richest, Comparatively Medium Rich, Comparatively Rich, Comparatively Poor and Comparatively Poorest respectively. In our example, the district D_4 belongs to the Comparatively Richest class and the districts D_1 , D_2 and D_3 belong to the Comparatively Poorest Class.

IV. CONCLUSION

In this work, the comparison of poverty status is surveyed with respect to some attributes using grey possibility degrees. The approach is novel and can be extended to any desired number of attributes. The process of applying grey possibility degree has been taken from Li et al.'s approach. The advantage of using this process lies in the fact that it considers uncertainty to some satisfactory extent. Thus the proposed approach is authentic and can be used for other multi attribute decision making problems also.

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