

Fixed Point in IFM-Space for Weakly - Compatible Maps

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ABSTRACT: The purpose of this paper is to obtain a common fixed point theorem for weakly compatible self-maps satisfying the property (E.A) using implicit relation on a non-complete Intuitionistic fuzzy metric space without involving the requirement of continuity of the maps. Our result generalize the result of Sedghi, S. [17] and many others.

KEYWORDS: Intuitionistic Fuzzy metric space, non compatible maps, E.A property, weakly compatible maps, common fixed point.

I. INTRODUCTION

The concept of fuzzy sets was introduced by Zadeh [13]. Following the concept of fuzzy sets, fuzzy metric spaces have been introduced by Kramosil and Michalek [9], George and Veeramani [6] modified the notion of fuzzy metric space with the help of continuous t-norms. As a generalization of fuzzy sets, Atanassov [14] introduced and studied the concept of intuitionistic fuzzy sets. Using the idea of intuitionistic fuzzy sets Park [11] defined the notion of intuitionistic fuzzy metric space with the help of continuous t norm and continuous t conorm as a generalization of fuzzy metric space, George and Veeramani [6] had showed that every metric induces an intuitionistic fuzzy metric and found a necessary and sufficient condition for an intuitionistic fuzzy metric space to be complete. Choudhary [15] introduced mutually contractive sequence of self maps and proved a fixed point theorem. Kramosil and michalek [9] introduced the notion of Cauchy sequences in an intuitionistic fuzzy metric space and proved the well known fixed point theorem of Banach [4]. Turkoglu et al [12] gave the generalization of Jungck's common fixed point theorem [19] to intuitionistic fuzzy metric space. Recently Aamri and Moutawakil [24] introduced the property (E.A) and thus generalized the concept of noncompatible maps. The results obtained in the metric fixed point theory by using the notion of noncompatible maps or the property (E.A) are very interesting. Question arises whether, by using the concept of noncompatibility or its generalized notion, that is, the property (E.A), can we find equally interesting results in intuitionistic fuzzy metric space also? Our answer is affirmative.

In this paper we obtain a common fixed point theorems for weakly compatible self-maps satisfying the property (E.A) using implicit relation on a non complete Intuitionistic fuzzy metric space without involving the requirement of continuity of the maps.

II. PRELIMINARIES

Definition 1 [7] A binary operation $*$: $[0,1] \times [0,1] \rightarrow [0,1]$ is called a continuous t-norm if

($[0,1], *$) is an abelian topological monoid with the unit 1 such that $a * b \leq c * d$ whenever

$a \leq c$ and $b \leq d$ for all $a, b, c, d \in [0,1]$.

Examples of t-norms are $a * b = ab$ and $a * b = \min \{a, b\}$.

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Definition 2 [7] A binary operation $\diamond: [0,1] \times [0,1] \rightarrow [0,1]$ is continuous t-conorm if \diamond is satisfying the following conditions :

- (a) \diamond is commutative and associate
- (b) \diamond is continuous
- (c) $\mathbf{a} \diamond \mathbf{0} = \mathbf{a}$ for all $\mathbf{a} \in [0,1]$
- (d) $\mathbf{a} \diamond \mathbf{b} \leq \mathbf{c} \diamond \mathbf{d}$ whenever $\mathbf{a} \leq \mathbf{c}$ and $\mathbf{b} \leq \mathbf{d}$ for all $\mathbf{a}, \mathbf{b}, \mathbf{c}, \mathbf{d} \in [0,1]$

Definition 3 [4] The 3-tuple $(X, M, *)$ is called a fuzzy metric space (FM-space) if X is an arbitrary set $*$ is a continuous t-norm and M is a fuzzy set in $X^2 \times [0, \infty]$ satisfying the following conditions for all $x, y, z \in X$ and $t, s > 0$.

- (a) $M(x, y, 0) > 0$
- (b) $M(x, y, t) = 1, \forall t > 0$ iff $x = y$
- (c) $M(x, y, t) = M(y, x, t)$,
- (d) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$
- (e) $M(x, y, .): [0, \infty] \rightarrow [0,1]$ is continuous.

Remark 1 Since $*$ is continuous, it follows from (2.3.4) that the limit of a sequence in FM-space is uniquely determined.

Definition 4 [16] A five –tuple $(X, M, N, *, \diamond)$ is said to be an intuitionistic fuzzy metric space if X is an arbitrary set $*$ is a continuous t – norm, \diamond is a continuous t-conorm and M, N are fuzzy sets on $X^2 \times (0, \infty)$ satisfying the following conditions for all $x, y, z \in X, s, t > 0$

- (a) $M(x, y, t) + N(x, y, t) \leq 1$
- (b) $M(x, y, t) > 0$
- (c) $M(x, y, t) = M(y, x, t)$
- (d) $M(x, y, t) * M(y, z, s) \leq M(x, z, t + s)$
- (e) $M(x, y, .): (0, \infty) \rightarrow (0,1)$ is continuous
- (f) $N(x, y, t) > 0$
- (g) $N(x, y, t) = N(y, x, t)$
- (h) $N(x, y, t) \diamond N(y, z, s) \geq N(x, z, t + s)$
- (i) $N(x, y, .): (0, \infty) \rightarrow (0,1)$ is continuous

Then (M, N) is called an intuitionistic fuzzy metric on X , the function $M(x, y, t)$ and $N(x, y, t)$ denote the degree of nearness and the degree of non- nearness between x and y with respect to t respectively.

Remark 2 Every fuzzy metric space $(X, M, *)$ is an intuitionistic fuzzy metric space form $(X, M, 1 - M, *, \diamond)$ such that t-norm $*$ and t-conorm \diamond are associated i.e.,

$$\mathbf{x} \diamond \mathbf{y} = 1 - ((1 - \mathbf{x}) * (1 - \mathbf{y})) \text{ for any } \mathbf{x}, \mathbf{y} \in [0,1] \text{ but the converse is not true.}$$

Example 1 Induced intuitionistic fuzzy metric space [1]

Let (X, d) be a metric space denote $\mathbf{a} * \mathbf{b} = \mathbf{ab}$ and $\mathbf{a} \diamond \mathbf{b} = \min \{1, \mathbf{a} + \mathbf{b}\}$ for all $\mathbf{a}, \mathbf{b} \in [0,1]$ and let M , and N be fuzzy sets on $X^2 \times (0, \infty)$ defined as follows:

$$M(x, y, t) = \frac{t}{t + d(x, y)}$$

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$$\text{and } N(x, y, t) = \frac{d(x, y)}{t + d(x, y)}$$

Then $(X, M, N, *, \diamond)$ is an Intuitionistic fuzzy metric space. We call this intuitionistic fuzzy metric induced by a metric 'd' the standard intuitionistic fuzzy metric.

Remark 3 Note that the above example holds even with the t-norm $a * b = \min \{a, b\}$ and the t-conorm $a \diamond b = \max \{a, b\}$.

Example 2 Let $X = \mathbb{N}$ define $a * b = \max \{0, a + b - 1\}$ and $a \diamond b = a + b - ab$ for all $a, b \in [0, 1]$ and let M and N be fuzzy sets on $X^2 \times (0, \infty)$ as follows :

$$M(x, y, t) = \begin{cases} \frac{x}{y}, & \text{if } x \leq y \\ \frac{y}{x}, & \text{if } y \leq x \end{cases}$$

$$\text{and } N(x, y, t) = \begin{cases} \frac{y-x}{y}, & \text{if } x \leq y \\ \frac{x-y}{x}, & \text{if } y \leq x \end{cases} \quad \text{for all } x, y \in X$$

and $t > 0$, then $(X, M, N, *, \diamond)$ is an intuitionistic fuzzy metric space.

Remark 4 Note that in above example, t-norm * and t-conorm \diamond are not associated, and there exist no metric 'd' on X satisfying

$$M(x, y, t) = \frac{t}{t + d(x, y)}$$

$$\text{and } N(x, y, t) = \frac{d(x, y)}{t + d(x, y)}$$

where $M(x, y, t)$ and $N(x, y, t)$ are defined in above example, also note that the above functions (M, N) is not an intuitionistic metric with the t-norm and t-conorm defined as $a * b = \min \{a, b\}$, $a \diamond b = \max \{a, b\}$.

Definition 5 [1] Let $(X, M, N, *, \diamond)$ be an intuitionistic fuzzy metric space then

(a) A sequence $\{x_n\}$ in X is said to be convergent x in X if for each $\varepsilon > 0$ and each $t > 0$ there exists $n_0 \in \mathbb{N}$ such that

$$M(x_n, x, t) > 1 - \varepsilon$$

$$\text{and } N(x_n, x, t) < \varepsilon, \text{ for all } n \geq n_0.$$

(b) An intuitionistic fuzzy metric space in which every Cauchy sequence is convergent is said to be complete.

Definition 6 [19] Let A and B maps from a Intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ into itself. The maps A and B are said to be compatible (or asymptotically commuting) if for all $t > 0$

$$\lim_{n \rightarrow \infty} M(ABx_n, BAx_n, t) = 1$$

$$\text{and } \lim_{n \rightarrow \infty} N(ABx_n, BAx_n, t) = 0 \text{ whenever } \{x_n\} \text{ is a sequence in } X \text{ such that}$$

$$\lim_{n \rightarrow \infty} Ax_n = \lim_{n \rightarrow \infty} Bx_n = z \text{ for some } z \in X.$$

Definition 7 [21] Two mappings A and B of a intuitionistic fuzzy metric space $(X, M, N, *, \diamond)$ into itself are R-weakly commuting provided there exists some real number R such that

$$M(ABx, BAx, t) \geq M(Ax, Bx, t/R)$$

$$\text{and } N(ABx, BAx, t) \leq N(Ax, Bx, t/R) \text{ for each } x \in X \text{ and } t > 0.$$

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Definition 8 [20] Let f and g be two self mappings of an intuitionistic fuzzy metric space $(X, M, N, *, \emptyset)$. We say that f and g satisfy the property (E.A) if there exists a sequence $\{x_n\}$ such that $\lim_{n \rightarrow \infty} f x_n = \lim_{n \rightarrow \infty} g x_n = t$ for some $t \in X$.

Lemma 1 [1] In intuitionistic fuzzy metric space X , $M(x, y, \cdot)$ is non decreasing and $N(x, y, \cdot)$ is non increasing for all $x, y \in X$

Lemma 2 [1] Let $(X, M, N, *, \emptyset)$ be an intuitionistic fuzzy metric space if there exist $k \in (0, 1)$ s.t.

$$M(x, y, kt) \geq M(x, y, t)$$

and $N(x, y, kt) \leq N(x, y, t)$ for all $x, y \in X$,

Then $x = y$.

Proof: $\because M(x, y, kt) \geq M(x, y, t)$
and $N(x, y, kt) \leq N(x, y, t)$

Then we have,

$$M(x, y, t) \geq M(x, y, \frac{t}{k})$$

and $N(x, y, t) \leq N(x, y, \frac{t}{k})$

By repeated application of above inequality as we have

$$M(x, y, t) \geq M(x, y, \frac{t}{k}) \geq M(x, y, \frac{t}{k^2}) \geq \dots \geq M(x, y, \frac{t}{k^n}) \geq \dots$$

and

$$N(x, y, t) \leq N(x, y, \frac{t}{k}) \leq N(x, y, \frac{t}{k^2}) \leq \dots \leq N(x, y, \frac{t}{k^n}) \leq \dots$$

for $n \in \mathbb{N}$ which tends to 1 and 0 as $n \rightarrow \infty$ respectively.

Thus, $M(x, y, t) = 1$
and $N(x, y, t) = 0$ for all $t > 0$

Therefore, we get $x = y$.

III. MAIN RESULTS

Let Φ be the sets of increasing and continuous and Ψ be the sets of decreasing and continuous function, $\Phi, \Psi: [0, 1] \rightarrow [0, 1]$ such that $\Phi(t) > t$ and $\Psi(t) < t$ for all $t \in (0, 1)$.

Theorem 1 Let f and g be two weak compatible self maps of an IFM-space $(X, M, N, *, \emptyset)$ satisfying the property (E. A) and

(i) $fX \subset gX$,

(ii) $M(fx, fy, kt) \geq M(gx, gy, t)$ and $N(fx, fy, kt) \leq N(gx, gy, t), k \geq 0$

(iii) $M(fx, ffx, t) > \varphi(\min \left\{ \begin{aligned} &M(ggx, gfx, t), \sup_{t_1+t_2=\frac{t}{k}} \min\{M(fx, gx, t_1), M(gx, f^2x, t_2)\}, \\ &\sup_{t_3+t_4=\frac{t}{k}} \max\{M(f^2x, gfx, t_3), M(fx, gfx, t_4)\} \end{aligned} \right\})$

and

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$$N(fx, ffx, t) < \Psi \left(\max \left\{ \begin{array}{l} N(ggx, gfx, t), \sup_{t_1+t_2=\frac{2}{k}t} \min\{N(fx, gx, t_1), N(gx, f^2x, t_2)\}, \\ \sup_{t_3+t_4=\frac{2}{k}t} \max\{N(f^2x, gfx, t_3), N(fx, gfx, t_4)\} \end{array} \right\} \right)$$

whenever $ffx \neq f^2x$, for all $x, y \in X, t > 0$ and for some $1 \leq k < 2$. If the range of f or g is a complete subspace of X , then f and g have a common fixed point.

Proof: Since f and g satisfy the property (E, A) there exists a sequence $\{x_n\}$ in X such that $fx_n, gx_n \rightarrow z$ as $n \rightarrow \infty$, for some $z \in X$. Since $z \in fX$ and $fX \subset gX$, therefore there exist some point u in X such that $z = gu$, where $gx_n \rightarrow z$, as $n \rightarrow \infty$

If $fu \neq gu$.

Then $M(fx_n, fu, kt) \geq M(gx_n, gu, t)$ and $N(fx_n, fu, kt) \leq N(gx_n, gu, t)$.

Taking limit as $n \rightarrow \infty$, we get,

$$M(gu, fu, kt) \geq M(gu, gu, t) \text{ and } N(gu, fu, kt) \leq N(gu, gu, t)$$

Therefore by lemma 2.2, $fu = gu$.

Since f and g are weakly compatible so $fgu = gfu$ and therefore $fgu = ffu = gfu = ggu$

If $ffu \neq fu$, then by contractive condition we have,

$$\begin{aligned} M(fu, ffu, t_0) &> \varphi \left(\min \left\{ \begin{array}{l} M(ggu, gfu, t_0), \sup_{t_1+t_2=\frac{2}{k}t_0} \min\{M(fu, gu, t_1), M(gu, f^2u, t_2)\}, \\ \sup_{t_3+t_4=\frac{2}{k}t_0} \max\{M(f^2u, gfu, t_3), M(fu, gfu, t_4)\} \end{array} \right\} \right) \\ &= \varphi \left(\min \left\{ \begin{array}{l} M(ffu, ffu, t_0), \min\{M(fu, fu, \varepsilon), M(fu, f^2u, \frac{2}{k}t_0 - \varepsilon)\}, \\ \max\{M(f^2u, gfu, \varepsilon), M(fu, ffu, \frac{2}{k}t_0 - \varepsilon)\} \end{array} \right\} \right) \end{aligned}$$

and

$$\begin{aligned} N(fu, ffu, t_0) &< \Psi \left(\max \left\{ \begin{array}{l} N(ggu, gfu, t_0), \sup_{t_1+t_2=\frac{2}{k}t_0} \min\{N(fu, gu, t_1), N(gu, f^2u, t_2)\}, \\ \sup_{t_3+t_4=\frac{2}{k}t_0} \max\{N(f^2u, gfu, t_3), N(fu, gfu, t_4)\} \end{array} \right\} \right) \\ &= \Psi \left(\max \left\{ \begin{array}{l} N(ffu, ffu, t_0), \min\{N(fu, fu, \varepsilon), N(fu, f^2u, \frac{2}{k}t_0 - \varepsilon)\}, \\ \max\{N(f^2u, gfu, \varepsilon), N(fu, ffu, \frac{2}{k}t_0 - \varepsilon)\} \end{array} \right\} \right) \end{aligned}$$

$$\forall \varepsilon \in (0, \frac{2}{k}t_0)$$

As $\varepsilon \rightarrow 0$, it follows

$$M(fu, ffu, t_0) > \varphi \{M(fu, ffu, \frac{2}{k}t_0 - \varepsilon)\} > M(fu, ffu, \frac{2}{k}t_0)$$

and

$$N(fu, ffu, t_0) < \Psi \{N(fu, ffu, \frac{2}{k}t_0 - \varepsilon)\} < N(fu, ffu, \frac{2}{k}t_0) \text{ which is a contradiction.}$$

Therefore $fu = ffu$ and $fu = fgu = gfu = ggu$.

Hence fu is a common fixed point of f and g .

The case when fX is a complete subspace of X is similar to the above since $fX \subset gX$, this completes the proof of the theorem.

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Note: In the next theorem we will show that if we take non-compatible maps in place of (E, A) property we can show in addition that the mappings are discontinuous at the common fixed point

Theorem 2 Let f and g be two non-compatible, weakly compatible self mappings of an IFM-space $(X, M, N, *, \phi)$ such that

(i) $fX \subset gX$,

(ii) $M(fx, fy, Kt) \geq M(gx, gy, t)$ and $N(fx, fy, Kt) \leq N(gx, gy, t)$, $k \geq 0$

(iii) $M(fx, ffx, t) > \phi(\min \left\{ \begin{array}{l} M(ggx, gfx, t), \sup_{t_1+t_2=\frac{2}{k}t} \min\{M(fx, gx, t_1), M(gx, f^2x, t_2)\}, \\ \sup_{t_3+t_4=\frac{2}{k}t} \max\{M(f^2x, gfx, t_3), M(fx, gfx, t_4)\} \end{array} \right\})$

and

$$N(fx, ffx, t) < \Psi(\max \left\{ \begin{array}{l} N(ggx, gfx, t), \sup_{t_1+t_2=\frac{2}{k}t} \min\{N(fx, gx, t_1), N(gx, f^2x, t_2)\}, \\ \sup_{t_3+t_4=\frac{2}{k}t} \max\{N(f^2x, gfx, t_3), N(fx, gfx, t_4)\} \end{array} \right\})$$

whenever $fx \neq f^2x$, for all $x, y \in X, t > 0$ and for some $1 \leq k < 2$. If the range of f or g is a complete subspace of X , then f and g have a common fixed point and the fixed point is the point of discontinuity.

Proof : Since f and g are non-compatible maps there exists a sequence $\{x_n\}$ in X such that

$$\lim_{n \rightarrow \infty} fx_n = \lim_{n \rightarrow \infty} gx_n = z \quad \dots(1)$$

for some z in X but either $\lim_{n \rightarrow \infty} M(fgx_n, gfx_n, t) \neq 1$ and $\lim_{n \rightarrow \infty} N(fgx_n, gfx_n, t) \neq 0$ or the limit does not exist.

Since $z \in fX$ and $fX \subset gX$, there exists some point u in X such that $z = gu$ where $gx_n \rightarrow z$ as $n \rightarrow \infty$.

We claim that $fu = gu$. Suppose that $fu \neq gu$ then,

$$M(fx_n, fu, Kt) \geq M(gx_n, gu, t) \text{ and } N(fx_n, fu, Kt) \leq N(gx_n, gu, t)$$

Taking limit $n \rightarrow \infty$ we get, $M(gu, fu, Kt) \geq M(gu, gu, t)$ and $N(gu, fu, Kt) \leq N(gu, gu, t)$

Hence $fu = gu$.

Since f and g are weakly compatible so $fgu = gfu$ and therefore $fgu = ffu = gfu = ggu$.

If $ffu \neq fu$, then by the contractive condition

$$M(fu, ffu, t_0) > \phi(\min \left\{ \begin{array}{l} M(ggu, gfu, t_0), \sup_{t_1+t_2=\frac{2}{k}t_0} \min\{M(fu, gu, t_1), M(gu, f^2u, t_2)\}, \\ \sup_{t_3+t_4=\frac{2}{k}t_0} \max\{M(f^2u, gfu, t_3), M(fu, gfu, t_4)\} \end{array} \right\})$$

$$= \phi(\min \left\{ \begin{array}{l} M(ffu, ffu, t_0), \min \left\{ M(fu, fu, \varepsilon), M \left(fu, f^2u, \frac{2}{k} t_0 - \varepsilon \right) \right\}, \\ \max \left\{ M(f^2u, gfu, \varepsilon), M \left(fu, ffu, \frac{2}{k} t_0 - \varepsilon \right) \right\} \end{array} \right\})$$

and

$$N(fu, ffu, t_0) < \Psi(\max \left\{ \begin{array}{l} N(ggu, gfu, t_0), \sup_{t_1+t_2=\frac{2}{k}t_0} \min\{N(fu, gu, t_1), N(gu, f^2u, t_2)\}, \\ \sup_{t_3+t_4=\frac{2}{k}t_0} \max\{N(f^2u, gfu, t_3), N(fu, gfu, t_4)\} \end{array} \right\})$$

$$= \Psi(\max \left\{ \begin{array}{l} N(ffu, ffu, t_0), \min \left\{ N(fu, fu, \varepsilon), N \left(fu, f^2u, \frac{2}{k} t_0 - \varepsilon \right) \right\}, \\ \max \left\{ N(f^2u, gfu, \varepsilon), N \left(fu, ffu, \frac{2}{k} t_0 - \varepsilon \right) \right\} \end{array} \right\})$$

$$\forall \varepsilon \in \left(0, \frac{2}{k} t_0 \right).$$

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As $\varepsilon \rightarrow 0$, it follows

$$M(fu, ffu, t_0) > \varphi\{M(fu, ffu, \frac{2}{k} t_0 - \varepsilon)\} > M(fu, ffu, \frac{2}{k} t_0)$$

and

$$N(fu, ffu, t_0) < \Psi\{N(fu, ffu, \frac{2}{k} t_0 - \varepsilon)\} < N(fu, ffu, \frac{2}{k} t_0)$$
 which is a contradiction.

Therefore $fu = ffu$ and $u = ffu = fgu = gfu = ggu$.

Hence fu is a common fixed point of f .

The case when fX is a complete subspace of X is similar to the above since $fX \subset gX$.

We now show that f and g are discontinuous at the common fixed point $= fu = gu$.

If possible suppose f is continuous, then considering the sequence $\{x_n\}$ of (1) we get,

$$\lim_{n \rightarrow \infty} ffx_n = fz = z.$$

Since f and g are weakly compatible, so $ffu = gfu$ so $fz = gz$ or $ffx_n = gfx_n$.

Taking limit $n \rightarrow \infty$, we get $fz = \lim_{n \rightarrow \infty} gfx_n$ or $z = \lim_{n \rightarrow \infty} gfx_n$

This in turn yields, $\lim_{n \rightarrow \infty} M(fgx_n, gfx_n, t) = 1, \lim_{n \rightarrow \infty} N(fgx_n, gfx_n, t) = 0$ which contradicts the fact that $\lim_{n \rightarrow \infty} M(fgx_n, gfx_n, t) \neq 1$ and $\lim_{n \rightarrow \infty} N(fgx_n, gfx_n, t) \neq 0$.

Hence f is discontinuous at the fixed point.

Similarly we can prove that g is discontinuous at the fixed point which completes the proof.

IV. CONCLUSION

The result on a common fixed point is successfully obtained for weakly compatible maps satisfying the property (E.A.) using implicit relation on a non complete Intuitionistic Fuzzy Metric space which also generalizes the result of Sedghi, S.[17] and many others.

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