

Evaluation of Cable Tension Using Vibration Based Methodologies for Health Monitoring Of Structures

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ABSTRACT: Structural Health Monitoring (SHM) provides a way to evaluate the safety and durability of a structure during its service life, to ensure its serviceability and sustainability. Structural Health Monitoring systems are developed and installed in several long span bridges, to monitor the health condition of the bridges. In structural health monitoring of the cable supported bridges, estimation procedure for the cable tension influences the accuracy of monitoring system. As cables are crucial element for the overall structural safety of cable supported bridges the accurate estimation of tension force plays an important role in the construction and the maintenance stages of the bridges. Incorporating smart sensors with vibration-based tension force estimation methodologies, proves to be an efficient means of autonomous long-term monitoring of cable tensions. The accelerations from the cable vibrations have embedded frequencies which can be extracted using Fast Fourier Transform and processed using computer aided software. After the accelerations are processed and the frequencies are extracted and organized, the cable tension can be calculated using dynamic cable theory and cable properties.

Numerical studies were carried out on a cable with known tensile force using ABAQUS, to determine the natural frequencies. A series of laboratory tests were conducted on a prestressed steel cable with initial cable sag and applied tension forces. The accelerometer and the strain gauges are bonded on the surface of the cable, to determine the acceleration and strain data. The frequency of the cable are extracted from the acceleration data using Fast Fourier Transforms. The various vibration based methods were used to assess the cable force from the extracted frequency. A MATLAB script was developed to automate the estimation of tension force.

KEYWORDS: Structural Health Monitoring, Cable tension, Frequency, FFT, Accelerometer, MATLAB

I. INTRODUCTION

In these days, constructing long span bridges is a growing trend, and the record-breaking long span bridges are frequently introduced. These accomplishments are attributed to modern advances in material, analysis, and construction technology. These innovative long span bridges are exposed to harsh conditions, such as heavy traffic loads, strong winds, and severe earthquakes. This presents many challenges to professionals, such as how they can ensure these structures function properly during their long service life and how to prevent them from damage and failure under extreme events. The emergence of high strength cables plays an important role in winning a great popularity of cable supported bridges. Since cables are a crucial element for the overall structural safety of such structures, the accurate estimation of tension force is of major importance on both the construction and the maintenance stages of the cables

Many methods are available for measuring the tension in a bridge cable. During construction, cable tension is closely monitored using load cells or pressure meters at one end of the cable (Wang et al., 1999; Kim and Park, 2007). Another approach is by measuring the elongation as hydraulic jack is tensioning the cable (Casas, 1994). After construction, direct measurement of cable tensile stress can be made using elastomagnetic (EM) stress sensors (Wang et al., 2005).

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However, one difficulty associated with EM sensors is the need to wrap the EM sensor solenoid around the cable which can be laborious and costly. Indirect methods of cable tension measurement based on the dynamic properties of a cable have been widely used (Zui et al., 1996; Russell and Lardner, 1998). This approach is easy to employ and cost-effective because it requires only an accelerometer to measure the vibration of the cable during ambient loading.

The general procedures of the vibration based methods are, to extract the natural frequency from the measurements of acceleration signal and enter the extracted data into an equation that shows the relationship between the natural frequency and tension, and thus the cable tension can be estimated indirectly. A piezoelectric accelerometer that employs the piezoelectric effect is used in this study to measure the dynamic changes in the cable. For the purpose of comparison, the tension force of the cable is directly measured using strain gauges, which is connected to the upper and lower end of the cable.

In this study the performance of the existing vibration-based methods are investigated through numerical and experimental studies. To achieve the objective, the following tasks are performed. First, the available vibration-based tension estimation methods are introduced and their application are reexamined. Second, a set of numerical study is carried out using ABAQUS, FEA software to determine the natural frequencies and mode shapes of the cable. Third, the performance of the methods has been examined through a set of laboratory experiments. A suspended steel cable instrumented with piezoelectric accelerometers and wireless sensors is exposed to broad-band excitations. The wireless sensors record the cable responses and the extracted cable frequencies is used to estimate cable tension. Finally, A MATLAB script file is developed to automate the processing of the various vibration based methodologies to estimate the tension force through the extracted natural frequencies.

II. TENSION ESTIMATION METHODS

Many dynamic methods have been developed considering the dynamic characteristics and physical properties of cables, to estimate the tensile force for the cables. Depending on the sag-extensibility and bending stiffness parameter, the existing vibration methods may be classified as the following five categories.

The flat taut string theory for cables neglects both sag-extensibility and flexural rigidity. Eqn 1, shows the relationship between the frequency and tension force. Where f_n denotes the n^{th} natural frequency in Hz. The terms T , m , and L denote tension force, mass density, and length of cable, respectively. Given the measured frequency and the mode number, the computation of tension force is straightforward and it is helpful for the first approximation of tension force. However, the application of this formula is strictly limited to a flat long slender cable because it does not consider both sag-extensibility and bending stiffness of cables.

$$T = 4mL^2 \left(\frac{f_n}{n} \right)^2 \quad (1)$$

The second category utilizes the following frequency formula of an axially loaded beam that considers the bending stiffness but neglects the sag-extensibility (Shimada et al., 1989). Where EI denotes the flexural rigidity of a cable. With respect to the unknown tension force and flexural rigidity, the linear regression procedures are applied to the measured frequencies and the corresponding mode numbers. Hence, the flexural rigidity and tension can be simultaneously estimated. Although the estimated results are very reliable for a short thick single cable, the estimated flexural rigidity is unreliable for the high sagged cables. Furthermore, the utilization of higher modes provides better accuracy while the higher modes of a large cable are hardly excited by the ambient sources. Despite those limitations, this approach is often used in the field due to its simplicity and speediness.

$$T = 4mL^2 \left(\frac{f_n}{n} \right)^2 - \frac{EI\pi^2 n^2}{L^2} \quad (2)$$

The practical formula to estimate the cable tension is proposed in a simply explicit form, in which the bending stiffness of the cable is included, and the sag effect is neglected for simplicity by using the frequencies relative to anti symmetric or higher vibration modes of the cable (Fang et al., 2012). The sag effect on cable tension measurement was discussed by Zui et al. (1996) with introduction of parameters ξ and Γ

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$$T = 4\pi^2 mL^2 \left(\frac{f_n}{\gamma_n} \right)^2 - \frac{EI}{L^2} \gamma_n^2 \quad (3)$$

Where γ_n is the circular frequency of the cable, Zui et al. 1996 concluded that only the first-order mode can be used when $\Gamma \geq 3$, for cables with $\Gamma < 3$, the second or higher order mode should be used for the measurement of cable tension. The proposed formula is the unified expression covering both cases from beam theory and string theory, and has sufficient accuracy for cable tension estimation.

The fourth category includes both sag-extensibility (Γ) and bending stiffness effect (ξ). (Zui et al., 1996): The criteria for the use of the below mentioned formulas are given as a). When $\Gamma > 3$, vibrations of first-order mode are used because the effects of cable sag and inclination are negligibly small even for first order mode b). When $\Gamma \leq 3$, the effects of cable sag are large, namely, the values of Γ are small, it is desirable to use second-order mode for the measurement of cable force.

In the case that a cable has small sag ($3 \leq \Gamma$);

$$T = 4m(f_1 L)^2 \left[1 - 2.2 \frac{\xi}{f_1} - 0.55 \left(\frac{\xi}{f_1} \right)^2 \right] \quad (17 < \xi) \quad (4a)$$

$$T = 4m(f_1 L)^2 \left[0.865 - 11.6 \left(\frac{\xi}{f_1} \right)^2 \right] \quad (6 \leq \xi \leq 17) \quad (4b)$$

$$T = 4m(f_1 L)^2 \left[0.828 - 10.5 \left(\frac{\xi}{f_1} \right)^2 \right] \quad (0 \leq \xi \leq 6) \quad (4c)$$

In the case that a cable has large sag ($\Gamma \leq 3$)

$$T = m(f_2 L)^2 \left[1 - 4.40 \frac{\xi}{f_2} - 1.10 \left(\frac{\xi}{f_2} \right)^2 \right] \quad (60 \leq \xi) \quad (5a)$$

$$T = m(f_2 L)^2 \left[1.03 - 6.33 \frac{\xi}{f_2} - 1.58 \left(\frac{\xi}{f_2} \right)^2 \right] \quad (17 \leq \xi \leq 60) \quad (5b)$$

$$T = m(f_2 L)^2 \left[0.882 - 85.0 \left(\frac{\xi}{f_2} \right)^2 \right] \quad (0 \leq \xi \leq 17) \quad (5c)$$

The determination of tension is straight forward for the given measured modes, the sag-to span ratio of cable, the flexural rigidity, and axial rigidity. A prior information regarding the axial and flexural rigidities of the target cable system is important for the proper use of this practical method. However, the flexural rigidity of cable is often neither available nor valid because the shear and bending mechanisms of a cross section of cable could be different from those of beam. For the sag-to-span ratio is often not available in practice because measuring the static shape of cable needs high cost.

The empirical formulas to estimate cable tension forces by using cable fundamental frequency are proposed by Wei-Xin et al (2004), where the cable sag and bending stiffness are taken into account respectively. The applicability of proposed formulas depends on the sizes of two non-dimensional characteristic cable parameters that are λ^2 characterizing the cable sag effect and ξ reflecting the cable bending stiffness effect. The formulas are simple in form and they are very convenient for practical engineers to fast evaluate the cable tensions.

$$T = 4m^2 f^2 l^2 ; (\lambda^2 \leq 0.17) \quad (7a)$$

$$T = \sqrt[3]{ml^4(4f^4 T^2 - 7.569mEA)} ; (0.17 \leq \lambda^2 \leq 4\pi^2) \quad (7b)$$

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$$T = m^2 f^2 l^2; (4\pi^2 < \lambda^2) \quad (7c)$$

$$T = 3.432m^2 f^2 l^2 - 45.191 \frac{EI}{l^2}; (0 \leq \xi \leq 18) \quad (8a)$$

$$T = m \left(2lf - \frac{2.263}{l} \sqrt{\frac{EI}{m}} \right)^2; (18 < \xi \leq 210) \quad (8b)$$

$$T = 4m^2 f^2 l^2; (210 < \xi) \quad (8c)$$

III. NUMERICAL INVESTIGATIONS

Consider a suspended cable with two hinged ends as shown in Fig. 1. The nominal diameter and span of the cable are 7mm, 4m respectively. The mass density per unit length of the cable (m) is 0.3 kg/m. The moment of inertia and the cross sectional area of the cable are $1.176 \times 10^{-10} \text{ m}^4$, $3.85 \times 10^{-5} \text{ m}^2$ respectively. The ratio of the span to dip of the cable is assumed as 10%. The FEM analysis of the suspended cable is carried out using ABAQUS software. The cable is modelled using 10 beam elements of type B32 (2-node, quadratic beam in space). The FEM model of the cable with applied load is shown in Fig.2

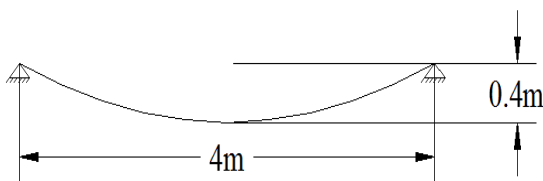


FIG: 1 SUSPENDED CABLE MODEL

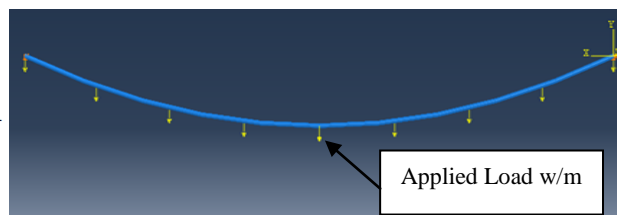


FIG: 2 FEM MODELLING WITH APPLIED
LOAD

TABLE: 1 FREQUENCIES FOR THE APPLIED LOAD

Applied Load (N)	Cable Tensile force(N)	Natural Frequencies (Hz)		
		n=1	n=2	n=3
1000	1028	7.207	14.66	22.59
1500	1541	8.54	17.26	26.38
2000	2054	9.85	19.84	30.30
2500	2555	11	22.11	33.56

The vibration analysis is performed to calculate the natural frequencies and the mode shapes of the cable. First to identify the parameters a geometric nonlinear finite element analysis under the applied load is performed to calculate the static sag and distribution of the tension force. Modal analysis is carried out to determine the vibrational characteristics. Load is applied in the increments of 500N. Table.1 shows the first three frequencies of the cable for the applied tensile force.

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IV. MATLAB SCRIPT

The MATLAB script is been developed to automate the estimation of tension force from the measured frequencies.

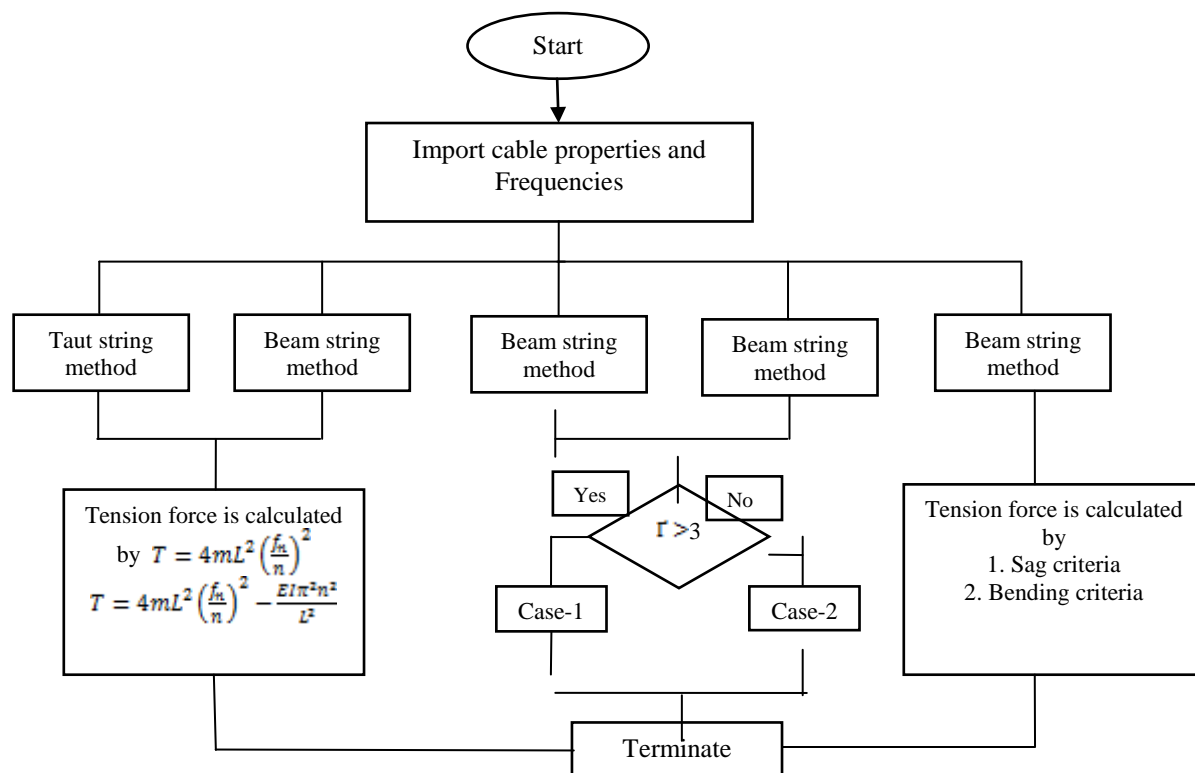


FIG: 3 FLOW CHART FOR CALCULATING TENSILE FORCE

For the extracted natural frequencies, the existing five vibration based methodologies are applied to estimate the tension forces through the developed MATLAB script. Taut string and beam string theories utilizes the symmetric mode of frequency to determine the tension force in all the four load cases. In case of Fang and Zui's theories, the two condition parameters (Γ and ξ) are taken into account to include the effect of sag and bending stiffness. Since $\Gamma < 3$ in the considered load cases the 1st anti-symmetric frequency is considered for tension force calculation. As per Wei-Xin's theory the sag parameter (λ^2) of the cable varies from moderate sag to high sag (1.083 to 0.084) and the appropriate tension equation is selected for the estimation of cable force.

The tension force evaluated by adopting various vibration based methodologies are listed in table: 2. The first row indicates the applied tensile force using line load feature in ABAQUS. The parenthesis in table: 2 shows the selected formulae and mode of frequency for the estimation of tensile force. % of error in tension force estimation is about 15%. For the practical formulae proposed by Zui et al [1], the ranges of error are also around 10%. Furthermore the resulting tension force tends to be underestimated. The last category by Wei-Xin produces least error of about 4% except for load case 2, where the cable experiences a moderate sag ($\lambda^2 = 0.388$) but the initial value of T depends on the tension force estimated by taut string theory. The taut string theory is limited to the cables with low sag-extensibility and bending stiffness. The % of error increases with increase in the sag to span ratio, however this theory can be used to calculate the initial value of T (7b).

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TABLE: 2 EVALUATED TENSILE FORCE

Load case	Applied Load(N)	Cable Tensile force (N)	Taut String Method	Beam String Method	Zhi Fang Method	Zui's method	Wei-Xin Method
1	1000	1028	995(1,f1)	980(2,f1)	926.31(3,f2)	964.7(5b,f2)	983.6(7b,f1)
2	1500	1541	1400(1,f1)	1385(2,f1)	1339(3,f2)	1380(5b,f2)	1410(7b,f1)
3	2000	2054	1862(1,f1)	1842(2,f1)	1874.2(3,f2)	1822(5b,f2)	1969.8(7a,f1)
4	2500	2555	2232.9(1,f1)	2314.6(2,f1)	2254(3,f2)	2329.9(5b,f2)	2459(7a,f1)

Note :() is the selected formulae and mode of frequency

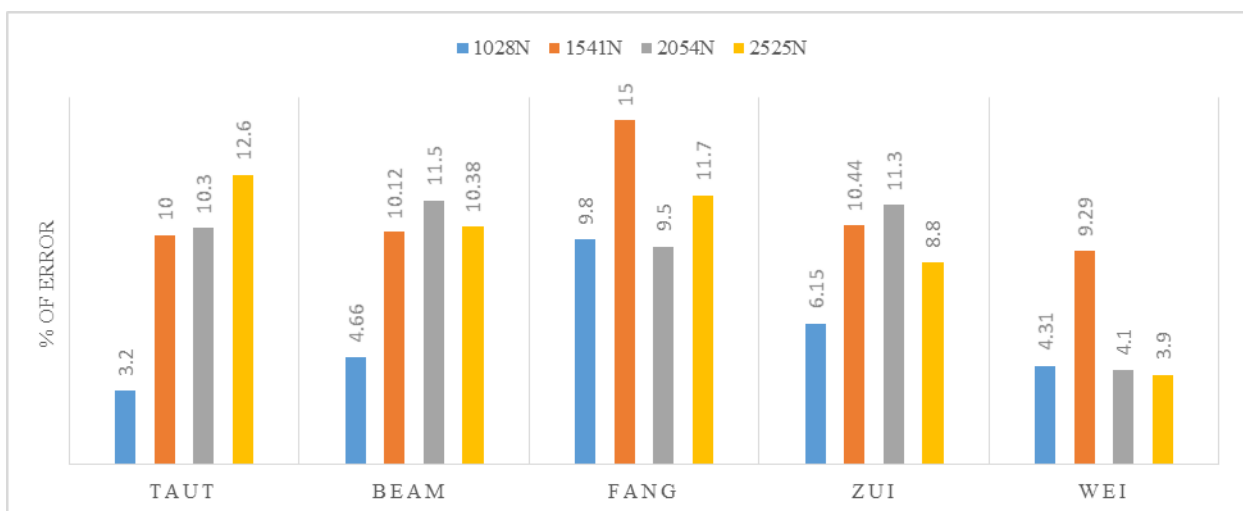


FIG: 3 PERCENTAGE OF ERROR WITH RESPECT TO THE APPLIED TENSILE FORCE

V. EXPERIMENTS FOR VALIDATION

Experimental Setup

To verify the proposed approach, an experimental verification task is conducted for a laboratory model that scales a main cable of the suspension bridge. A prestressing cable of 7mm with a span of 2000mm was used. The mass, cross sectional area and the moment of inertia of the cable are 0.3 kg/m, $3.84 \times 10^{-5} \text{m}^2$ and $1.176 \times 10^{-10} \text{m}^4$ respectively. The cable was fixed on a self-straining reaction frame as shown in Fig: 4 .Tensile force was applied to the cable by using

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dead weights at different points as shown in Fig.5. The cable was suspended with an initial dip of 0.2m ($1/10^{\text{th}}$ of span). The cable is excited by a hammer and given initial disturbance to record the free vibrational characteristics. The acceleration time history is recorded by an accelerometer installed on the cable. The piezoelectric accelerometer of range $\pm 7g$ is used. Strain gauges at mid span and quarter span were fixed to measure the applied tensile force and free vibrational characteristics.



FIG:4 SUSPENDED CABLE WITH INTIAL SAG



FIG:5 CABLE WITH APPLIED LOADS

The strain gauge and accelerometer are connected to the dynamic data acquisition system. Four vibration tests are conducted with various levels of applied load. For example, the cable of Test 1 has an applied load of 268N (26.8 kgf). On exciting the cable by hammer, the acceleration time histories are collected through the data acquisition system. A typical acceleration time history of Test 1 is shown in Fig. 6.

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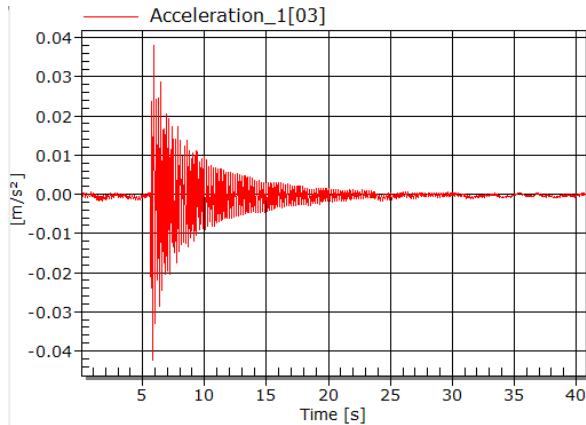


FIG:6 TIME HISTORY OF ACCELEROMETER

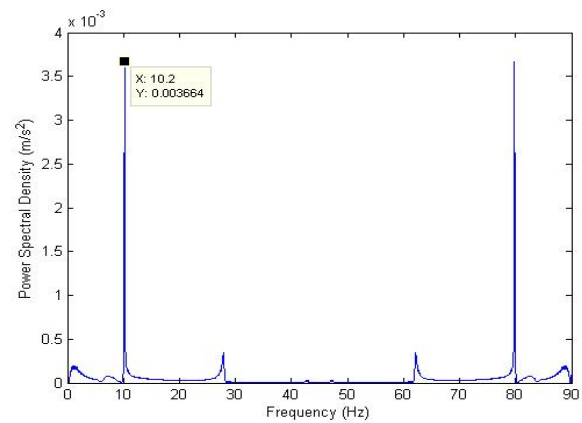


FIG:7 FOURIER SPECTRA AND NATURAL
FREQUENCIES

TABLE: 3 COMPUTED FREQUENCIES FROM ACCELERATION DATA

Applied Load (N)	Cable Tensile force(N)	Natural Frequencies (Hz)		
		Trial-1	Trial-2	Trial-3
268	550	10.2	10.16	10.4
784	1511	17.43	17.65	17
1576	2163	21.26	20.14	19.84
2610	4314	28.2	28.5	29.2

Using the Welch's averaged periodogram method with the Hanning windows, the power spectral densities of the accelerations are calculated and are shown in Fig. 7. It is seen that only a few of the lower modes are excited. Table: 3 shows the extracted frequencies of the cable with respect to the developed tensile force. From the extracted natural frequencies, the different vibration methods are applied to estimate the applied tensile force using the MATLAB script developed.

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TABLE: 4 EVALUATED TENSILE FORCE

Load case	Applied Load(N)	Cable Tensile force (N)	Taut String method	Beam String Method	Wei-Xin's method	Zui's Method
1	268	550	534	480	512.3	460
2	784	1511	1562	1507	1607	1404.6
3	1576	2163	2085	2031	2145	1889.35
4	2610	4314	4176	4122	4298	3800

For the measured natural frequencies, existing methods are applied to estimate the tension forces. The % error in each method with respect to the developed tensile force is listed in Fig: 8.

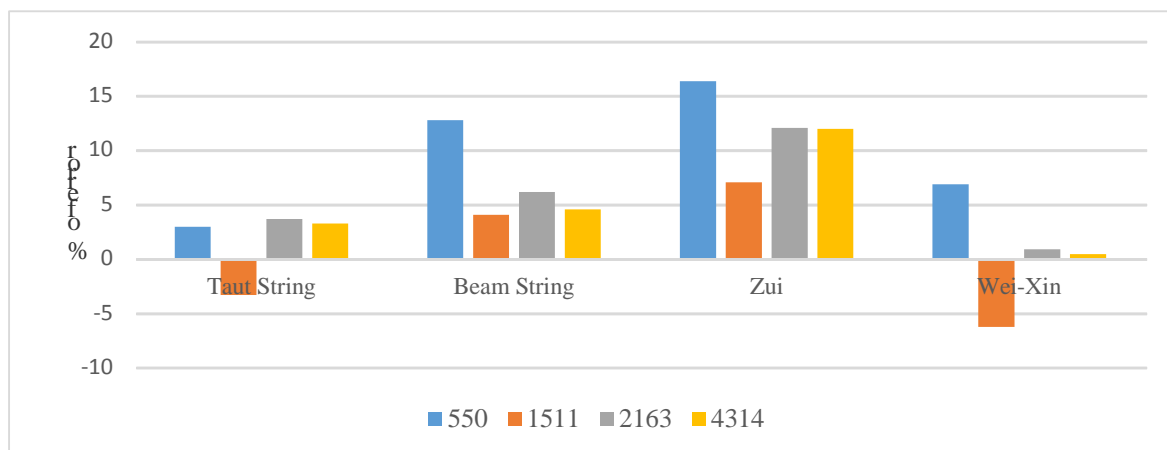


FIG: 8 PERCENTAGE OF ERROR WITH RESPECT TO THE APPLIED TENSILE FORCE

As per the results flat taut string theory has the minimum error for all cases, since the effect of sag and bending stiffness is not predominant in this cable system. Hence, the taut string theory is acceptable for the cable system tested here and the estimated results by the taut string theory could be used as the good maximum bounds of applied tensile forces. In addition, the empirical formula by Wei-Xin and the string theory tend to show the lower and the upper bounds. % of error was approximately around 15%.

VI. SUMMARY AND CONCLUSIONS

The existing vibration-based tension force estimation methods were classified as five categories, and their theoretical backgrounds were revisited. Numerical studies were carried out on 7mm diameter cable of 4m, 0.4m span and dip

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respectively using ABAQUS software by varying the tensile force. Analysis was carried out to determine the vibrational characteristic such as the frequencies of the cable. The proposed approach was experimentally verified using a laboratory model. From the extracted frequencies the tension force was evaluated. MATLAB script was developed to automate the estimation of tension force from the extracted frequencies. A comparative study of the existing methods were carried out and the corresponding percentage of errors were computed, the maximum percentage of error was around 15%. Based on results and interpretations, the following three conclusions are made. 1) the string theory is a good tool for cables with low sag and bending stiffness. 2) string theory is used for the first level of approximation due to its simplicity. 3) the empirical formulas by Wei-Xin has the most wide application range with reasonable accuracy. 4) the percentage of errors evaluated by various methodologies were around 10%.

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