

Product Cordial Labeling of Some Cycle Related Graphs

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ABSTRACT: A product cordial labeling of a graph G is a function $f: V(G) \rightarrow \{0, 1\}$ such that vertices with label 1 & label 0 differ by at most 1 and edges with label 1 & label 0 also differ by at most 1. In this paper we have derived product cordial labeling of ringsum of different graphs.

KEY WORDS: product cordial labeling, ring Sum.

AMS SUBJECT CLASSIFICATION NUMBER: 05C78.

I. INTRODUCTION

Throughout this paper, a graph $G = (V, E)$ is a undirected, finite, connected, and simple graph with vertex set V and edge set E . For different notations and terminology we follow Gross and Yellen[2].

A survey on different graph labeling techniques is given by Gallian[1].

The product cordial labeling was introduced by Sundaram et al.[3] and they proved that trees, unicyclic graphs of odd order, triangular snakes, dragons, helms and union of two path graphs are product cordial. They have also established that a graph with p vertices and q edges with $p \geq 4$ is product cordial then $q < (p^2 - 1)/4$. The graphs obtained by joining apex vertices of k copies of stars, shells and wheels to a new vertex are product cordial is proved in Vaidya and Dani[4] while the product cordial labeling for some cycle related graphs is reported in

Vaidya and Kanani[5]. In the same paper they have investigated product cordial labeling for shadow graph of cycle C_n . Vaidya and Barasara[6] have proved that the cycle with one chord, the cycle with twin chords, the friendship graph and the middle graph of path admit product cordial labeling while the same authors in [7] have obtained some new results on product cordial labeling. Product cordial labeling in the context of tensor product of some graphs is discussed by Vaidya and Vyas[8].

DEFINITION 1: Ring sum of two graphs $G_1 = (V_1, E_1)$ and $G_2 = (V_2, E_2)$, denoted by $G_1 \oplus G_2$, is the graph, $G_1 \oplus G_2 = ((V_1 \cup V_2), ((E_1 \cup E_2) - (E_1 \cap E_2)))$.

DEFINITION 2: A product cordial labeling of a graph G with vertex set V is a function f from V to $\{0, 1\}$ such that if each edge uv is assigned the label $f(u)f(v)$, the number of vertices labeled with 0 and the number of vertices labeled with 1 differ by at most 1, and the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1. A graph with a product cordial labeling is called a product cordial graph.

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II. MAIN RESULTS

THEOREM 1: $C_n \oplus K_{1,n}$ is a product cordial graph for all $n \in \mathbb{N}$.

PROOF: Let $V(G) = V_1 \cup V_2$, where $V_1 = \{u_1, u_2, \dots, u_n\}$ be the vertex set of C_n and $V_2 = \{v = u_1, v_1, v_2, \dots, v_n\}$ be the vertex set of $K_{1,n}$. Here $v_1, v_2, \dots, v_n$ are pendent vertices. Also $|V(G)| = |E(G)| = 2n$.

We define labeling $f: V(G) \rightarrow \{0, 1\}$ as follows.

for $n \equiv 0, 1 \pmod{2}$
 $f(u_i) = 1; 1 \leq i \leq n$
 $f(v_j) = 0; 1 \leq j \leq n$

Thus in each cases we have $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence $C_n \oplus K_{1,n}$ is a product cordial graph for all $n \in \mathbb{N}$.

EXAMPLE 1: product cordial labeling of the graph $C_5 \oplus K_{1,5}$ is shown in Fig. 1 as an illustration for the Theorem 1

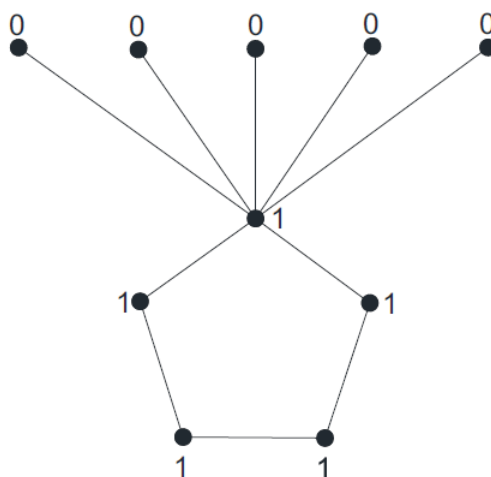


FIG. 1: product cordial labeling of the graph $C_5 \oplus K_{1,5}$.

THEOREM 2: $G \oplus K_{1,n}$ is a product cordial graph, where G is cycle with one chord and chord forms a triangle with two edges of the cycle, for all $n \in \mathbb{N}$.

PROOF: Let G be the cycle C_n with one chord. Let $V = V_1 \cup V_2$, where V_1 is the vertex set of G and V_2 be the vertex set of $K_{1,n}$. And let $u_1, u_2, \dots, u_n$ be consecutive vertices of cycle C_n and $e = u_1u_3$ be a chord of cycle C_n . The vertices u_1, u_2, u_3 forms a triangle with chord e . Let $v_1, v_2, \dots, v_n$ be the pendent vertices, v be the apex vertex of $K_{1,n}$ and take $v = u_1$. We define labeling $f: V(G) \rightarrow \{0, 1\}$ as follows.

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for $n \equiv 0, 1 \pmod{2}$
 $f(u_i) = 1; 1 \leq i \leq n$
 $f(v_j) = 0; 1 \leq j \leq n$

In view of above defined labeling pattern $v_f(0) = v_f(1) = n$ and $e_f(0) = n, e_f(1) = n + 1$.

Thus in each cases we have $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence $G \oplus K_{1,n}$ is a product cordial graph, where G is cycle with one chord and chord forms a triangle with two edges of the cycle, for all $n \in \mathbb{N}$.

Example 2. product cordial labeling of ring sum of the graph cycle C_6 with one chord and $K_{1,6}$ is shown in Fig. 2 as an illustration for the Theorem 2.

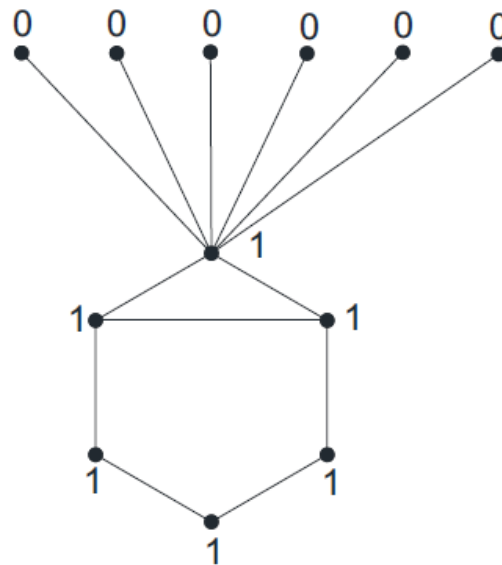


FIG. 2: product cordial labeling of ring sum of the graph cycle C_6 with one chord and $K_{1,6}$

THEOREM 3: $G \oplus K_{1,n}$ is product cordial graph, where G is cycle with twin chord and chords forms two triangles and one cycle C_{n-2} , for all $n \in \mathbb{N}$.

PROOF: Let G be the cycle C_n with twin chords, where chords form two triangles and one cycle C_{n-2} .

Let $V = V_1 \cup V_2$, where V_1 is the vertex set of G and V_2 be the vertex set of $K_{1,n}$. And let $u_1, u_2, \dots, u_n$ be successive vertices of G and $e_1 = u_n u_2$ and $e_2 = u_n u_3$ be the chords of cycle C_n . Let $v_1, v_2, \dots, v_n$ be the pendent vertices, v be the apex vertex of $K_{1,n}$ and take $v = u_1$.

We define labeling $f: V(G) \rightarrow \{0, 1\}$ as follows.

for $n \equiv 0, 1 \pmod{2}$

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$$\begin{aligned} f(u_{n-1}) &= 0, f(v_1) = 1 \\ f(u_i) &= 1; 1 \leq i \leq n \\ f(v_j) &= 0; 2 \leq j \leq n \end{aligned}$$

In view of above defined labeling pattern $v_f(0) = v_f(1) = n$ and $e_f(0) = e_f(1) = n + 1$.

Thus in each cases we have $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

Hence $G \oplus K_{1,n}$ is product cordial graph, where G is cycle with twin chord and chords forms two triangles and one cycle C_{n-2} , for all $n \in \mathbb{N}$.

Example 3. Product cordial labeling of ring sum of the graph the graph cycle C_7 with twin chords and $K_{1,7}$ is shown in Fig. 3 as an illustration for the Theorem 3.

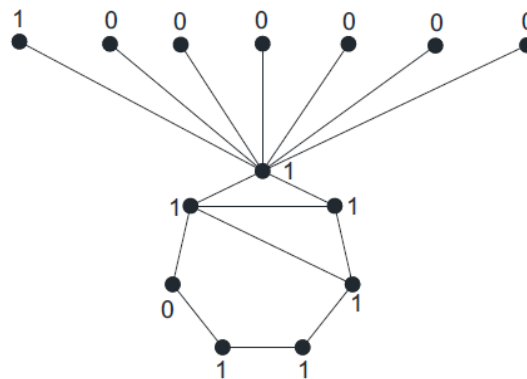


FIG. 3: product cordial labeling of ring sum of the graph cycle C_7 with twin chords and $K_{1,7}$

THEOREM 4: $G \oplus K_{1,n}$ is a product cordial graph, where G is cycle with triangle and chords forms three triangles and one cycle C_{n-5} , for all $n \in \mathbb{N}$.

PROOF: Let G be cycle with triangle $C_n(1, 1, n - 5)$. Let $V = V_1 \cup V_2$, where V_1 is the vertex set of G and V_2 be the vertex set of $K_{1,n}$. And Let $u_1, u_2, \dots, u_n$ be successive vertices of G .

Let u_1, u_3 and u_5 be the vertices of triangle formed by edges $e_1 = u_1u_3$, $e_2 = u_3u_5$ and $e_3 = u_1u_5$.

Let $v_1, v_2, \dots, v_n$ be the pendent vertices, v be the apex vertex of $K_{1,n}$ and take $v = u_1$.

We define labeling $f: V(G) \rightarrow \{0, 1\}$ as follows.

$$\begin{aligned} \text{for } n &\equiv 0, 1 \pmod{2} \\ f(u_n) &= 0, f(v_1) = 1 \\ f(u_i) &= 1; 1 \leq i \leq n \\ f(v_j) &= 0; 2 \leq j \leq n \end{aligned}$$

In view of above defined labeling pattern $v_f(0) = v_f(1) = n$ and $e_f(0) = n + 1, e_f(1) = n + 2$.

Thus in each cases we have $|v_f(0) - v_f(1)| \leq 1$ and $|e_f(0) - e_f(1)| \leq 1$.

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Hence $G \oplus K_{1,n}$ is a product cordial graph, where G is cycle with triangle and chords forms three triangles and one cycle C_{n-5} , for all $n \in \mathbb{N}$.

Example 4. product cordial labeling of ring sum of the graph cycle C_8 with triangle and $K_{1,8}$ is shown in Fig. 4 as an illustration for the Theorem 4.

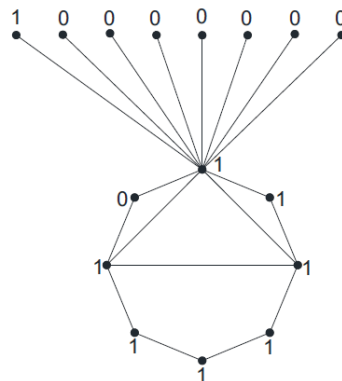


FIG. 4: product cordial labeling of ring sum of the graph cycle C_8 with triangle and $K_{1,8}$

III. CONCLUSION

We have proved that the graph $C_n \oplus K_{1,n}$, $G \oplus K_{1,n}$, where G is cycle with one chord, $G \oplus K_{1,n}$, where G is cycle with twin chord, $G \oplus K_{1,n}$, where G is cycle with triangle are product cordial graphs.

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