

Flow and Heat Transfer of Viscous Fluid between Two Parallel Walls When One of Them Moves with a Time Variable Velocity

A.C.Sahoo¹ and T.Biswal²

Lecturer, Dept. of Mathematics, Temple City Institute of Tech.& Engg. (TITE), Bhubaneswar, Orissa India¹

Professor, Dept. of Mathematics, Vivekananda Institute of Technology, Bhubaneswar, Orissa, India²

ABSTRACT: The two dimensional unsteady laminar flow of viscous fluid with heat transfer between two parallel walls, one of them is at rest and the other is moving in its own plane with a time variable velocity is studied in this paper. The governing equations are solved by series solution method. The effect of pressure gradient, Prandtl number and Brinkman number on the unsteady velocity is discussed. It is depicted in different graphs and tables.

KEY WORDS: Time variable velocity, unsteady laminar flow, series solution method, Prandtl number, Brinkman number.

I.INTRODUCTION

The flow and heat transfer of viscous fluid has a very wide industrial utility and involves in multilayered fluid flow situations and attracted many researchers in recent years due to its astounding applications. In engineering and other technical field, the influence of heat transfer characteristics of two dimensional laminar flow of incompressible fluid between two parallel walls are of great industrial importance. Present research activities in fluid dynamics have attracted many researchers due to its multiphase application such as in petroleum engineering, chemical engineering and heat exchangers. Lighthill (1954) studied the response of boundary layer to the fluctuation in the main stream velocity and opened a new era for further investigation. Stuart (1955) studied the above problem and considered the influence of porosity in the flow pattern. Messiha (1966) extended this problem to a variable suction. Liou et.al (1970) investigated free and forced convection flows adjacent to an isothermal vertical plate. They have used series solution method to solve the problem. Mori et.al (1974) have studied the forced convection heat transfer in a curved channel with square cross section. Its basic knowledge is essential to know the process of cooling of hollow conductors in cryogenic engineering. Pallath Chandran et.al (1998) discussed the unsteady hydro magnetic free convection flow with heat flux and accelerated boundary motion. Raju et.al (2005) studied the quasi steady state solution of an unsteady ionized hydro magnetic flow and heat transfer between two parallel walls in a rotating system.

Above mentioned studies is related to single fluid flow configuration. But most of the problem related to the geophysical fluid dynamics, plasma physics and many such areas involves multilayer fluid flow situations. The researchers such as Shail (1973), Lielausis (1975), Chan (1979), Gherson et.al. (1984), Serizawa et.al (1990), Attia et.al (1996), Ramdan et.al, (1999) and Linga Raju et.al (2006) have made wide contribution in both theoretical and experimental aspects of magneto hydrodynamics two layer fluid flow. Attia (2004) considered the unsteady Hartmann flow with heat transfer of viscoelastic fluid. Also Attia (2005) investigated the unsteady laminar flow of an incompressible viscous fluid and heat transfer between two parallel plates in the presence of a uniform suction and injection. Ganesh et.al (2007) studied the unsteady stokes flow of viscous fluid between two parallel porous plates. Linga Raju et.al (2009) investigated the unsteady two fluid flow and heat transfer of conducting fluid in channels under transverse magnetic field. Sajid et.al (2010) considered the fully developed mixed convection flow of a visco-elastic

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fluid between parallel vertical plates. An exact solution is obtained for the fluid velocity and temperature distribution. Rushikumar et.al (2012) analyzed the influence of heat and mass transfer characteristics of two dimensional steady laminar free convective flow of a viscous incompressible fluid between two parallel porous walls. Lingaraju et.al (2013) discussed the unsteady two layered fluid flow and heat transfer of conducting fluids in a channel between parallel porous plates. The unsteady two dimensional laminar flow of viscous fluid between two parallel porous plates was analyzed by Ganesh et.al (2014). The problem was reduced to a third order nonlinear differential equation depending on a suction Reynolds number and Hartmann number.

In this paper we have used the series expansion method of Sparrow and Gregg (1960) and the works of Ganesh et.al. (2014) for studying unsteady two dimensional laminar flow of viscous fluid between parallel porous plates to consider the inertia effect of unsteady relative motion of one wall with respect to the other. Our assumptions are that the two walls are parallel and are of infinite extent. One of the walls moves with a time variant velocity $U(t)$ which is assumed to be a continuously differentiable function of time.

II.THE GOVERNING EQUATIONS:

We consider two parallel walls separated by a distance '2d'. One of them is at rest and the other is moving in its own plane with a velocity $U(t)$, which varies with time in the direction of positive x-axis. The fixed wall is at a constant temperature T_0 and the moving one is at T_1 . Let the x-axis to be at the midway between the walls and y-axis is taken perpendicular to either of them and defined by the planes $y = d$ and $y = -d$, the positive sign corresponds to the moving wall and the negative sign corresponds to the fixed wall. We further assume that the flow is due to the motion of the moving wall and the pressure gradient is also in the same direction.

The only non-vanishing component of the velocity 'u' which acts in the x-direction in this case satisfies the equation of continuity.

$$\frac{\partial u}{\partial x} = 0 \quad [1]$$

With the given assumptions, the governing equation of motion is expressed as

$$\frac{\partial u}{\partial t} - \nu \frac{\partial^2 u}{\partial y^2} = -\frac{1}{\rho} \frac{\partial p}{\partial x} \quad [2]$$

The pressure function p must also satisfy the equation

$$-\frac{1}{\rho} \frac{\partial p}{\partial y} = 0 \quad [3]$$

In view of equations [1], [2] and [3], it would be proper to suggest the solutions of the type

$$\left. \begin{aligned} u &= u(y, t) \\ p &= p(x, t) \end{aligned} \right\} \quad [4]$$

$$\text{It is obvious that } \frac{\partial u}{\partial t} - \nu \frac{\partial^2 u}{\partial y^2} = -\frac{1}{\rho} \frac{\partial p}{\partial x} = A(t) \quad [5]$$

The energy equation in the present case reduces to

$$\rho c \frac{\partial T}{\partial t} = k \frac{\partial^2 T}{\partial y^2} + \mu \left(\frac{\partial u}{\partial y} \right)^2 \quad [6]$$

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Where $\mu \left(\frac{\partial u}{\partial y} \right)^2$ is the expression for dissipated heat.

The boundary conditions for the fluid motion and on temperature are considered as

$$\left. \begin{aligned} y = -d, u = 0, T = T_0 \\ y = d, u = U(t), T = T_1 \end{aligned} \right\} \quad [7]$$

III.SOLUTION OF THE EQUATIONS:

The equation [5] and [6] with the boundary condition [7] are non dimensionalised using the following dimensionless quantities.

$$\left. \begin{aligned} \xi^* &= \frac{yU}{\nu}, \xi_d^* = \frac{dU}{\nu} \\ \frac{u}{U} &= f(\xi^*, \beta_r) = f_0 + \beta_1 f_1 + \beta_2 f_2 + \beta_3 f_3 + \dots \\ \frac{d^2 A(t)}{2\nu U} &= A_0 + \beta_1 A_1 + \beta_2 A_2 + \beta_3 A_3 + \dots \\ \frac{T - T_0}{T_1 - T_0} &= Q(\xi^*, \beta_r) = Q_0(\xi^*) + \beta_1 Q_1(\xi^*) + \beta_2 Q_2(\xi^*) + \dots \end{aligned} \right\} \quad [8]$$

$$\text{Where } \beta_r = \frac{d^r D^r U}{U^{r+1}}, D^r U = \frac{d^r U}{dt^r}$$

With the help of these transformations form, the flow equation and temperature equations, by equating the coefficient functions of $\beta_1, \beta_2, \beta_3$ and free terms separately to zero, and neglecting the asterisks, we get;

$$f_0'' = -\frac{A_0}{\xi_d} \quad [9]$$

$$f_1'' - \frac{\xi f_0'}{\xi_d} - \frac{f_0}{\xi_d} = -\frac{A_1}{\xi_d} \quad [10]$$

$$f_2'' - \frac{1}{\xi_d} f_1 = -\frac{A_2}{\xi_d} \quad [11]$$

$$\theta_0'' = -P_r E f_0'^2 \quad [12]$$

$$\theta_1'' = P_r \frac{\xi}{\xi_d} \theta_0' - 2P_r E f_0' f_1' \quad [13]$$

$$\theta_2'' = \frac{P_r}{\xi_d} \theta_1 - 2P_r E f_0' f_2' \quad [14]$$

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Here in the above expressions, the dashes denote the differentiation with respect to ξ and

$$P_r = \frac{\mu_c}{k}, E = \frac{U^2}{c(T_1 - T_0)} = \text{Ekert number and}$$

$$B_r = P_r \cdot E \text{ is the Brinkman number.} \quad [15]$$

The boundary condition are now reduces to

$$\left. \begin{aligned} \xi = -\xi_d, f_0 = 0, f_1 = 0, f_2 = 0, \theta_0 = 0, \theta_1 = 0, \theta_2 = 0 \\ \xi = +\xi_d, f_0 = 1, f_1 = 0, f_2 = 0, \theta_0 = 1, \theta_1 = 0, \theta_2 = 0 \end{aligned} \right\} \quad [16]$$

The above set of equations is solved in succession in order of their occurrence. The solutions are

$$f_0 = \frac{1}{2} \left(1 + \frac{\xi}{\xi_d} \right) + \frac{1}{2} A_0 \xi_d \left(1 - \frac{\xi^2}{\xi_d^2} \right) \quad [17]$$

$$f_1 = \frac{\xi_d}{24} \left(1 - \frac{\xi^2}{\xi_d^2} \right) \cdot \left\{ 12A_1 - 3A_0 \xi_d \left(1 - \frac{\xi^2}{\xi_d^2} \right) - 2 \left(3 + \frac{2\xi}{\xi_d} \right) \right\} \quad [18]$$

$$f_2 = \frac{\xi_d}{720} \left(1 - \frac{\xi^2}{\xi_d^2} \right) \cdot \left\{ 360A_2 - 30A_1 \xi_d \left(5 - \frac{\xi^2}{\xi_d^2} \right) + 3A_0 \xi_d^2 \left(11 - \frac{4\xi^2}{\xi_d^2} + \frac{\xi^4}{\xi_d^2} \right) + \xi_d \left(75 + \frac{14\xi}{\xi_d} - \frac{21\xi^3}{\xi_d^3} \right) \right\} \quad [19]$$

$$\theta_0 = -B_r \left[\frac{A_0 \xi_d}{12} \left(\frac{\xi}{\xi_d} \right)^2 - \frac{A_1 \xi_d}{6} \left(\frac{\xi}{\xi_d} \right)^3 + \frac{1}{8} \left(\frac{\xi}{\xi_d} \right)^2 + \frac{\xi}{\xi_d} \left(\frac{A_0 \xi_d}{6} - \frac{1}{2} B_r \right) - \frac{1}{2} \left\{ \frac{(A_0 \xi_d)^2}{6} + \frac{1}{B_r} + \frac{1}{4} \right\} \right] \quad [20]$$

$$\begin{aligned} \theta_1 = \frac{1}{30} B_r (\xi_d)^2 \left(\frac{\xi}{\xi_d} \right)^3 \xi \left(\frac{P_r}{3} + 1 \right) + \frac{1}{40} B_r (A_0 \xi_d) \left(\frac{\xi}{\xi_d} \right)^4 \xi (P_r + 3) + \frac{B_r}{12} \left(\frac{\xi}{\xi_d} \right)^3 \\ \left[-\frac{P_r \xi}{4} + A_0 \xi_d \xi - 2A_0 \xi_d A_1 \xi \frac{\xi}{\xi_d} - \frac{\xi}{2} \right] \\ - \frac{B_r}{6} A_1 \xi_d \left(\frac{\xi}{\xi_d} \right)^2 \left[-P_r \left(\frac{A_0 \xi_d}{6} - \frac{1}{2B_r} \right) \xi - \frac{5}{6} A_0 \xi_d \xi - \frac{\xi}{2} + A_1 \xi_d \frac{\xi}{\xi_d} \right] \end{aligned} \quad [21]$$

$$\begin{aligned} B_r \xi \left[\frac{1}{12} \frac{\xi}{\xi_d} - (P_r + 3) \frac{A_0 \xi_d}{40} \right] + \frac{B_r}{36} A_0 \xi_d \xi (P_r + 5) + \frac{\xi}{12} (B_r - P_r) + \frac{B_r}{6} A_1 \xi_d \left(A_0 \xi_d - \frac{\xi}{\xi_d} \right) + \\ \frac{(A_0 \xi_d)^2}{30} B_r \xi_d \left(1 + \frac{P_r}{3} \right) + \frac{B_r}{12} \xi_d \left[\frac{P_r}{4} - A_0 \xi_d - (A_0 \xi_d)^2 \right] - \frac{B_r}{24} \xi_d \end{aligned}$$

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and

$$\begin{aligned} \theta_2 = & \left(\frac{P_r^3 EA_0^2}{90 \xi_d^4} - \frac{P_r^2 EA_0^2}{30 \xi_d^4} - \frac{P_r EA_0^2}{20 \xi_d^4} \right) \frac{\xi^8}{56} + \left(\frac{P_r^2 EA_0}{40 \xi_d^4} + \frac{3P_r^2 EA_0}{40 \xi_d^4} + \frac{P_r EA_0}{20 \xi_d^4} + \frac{P_r EA_0}{40 \xi_d^4} \right) \frac{\xi^7}{42} \\ & + \left(\frac{P_r^3 E}{48 \xi_d^4} + \frac{P_r^2 EA_0}{12 \xi_d^3} + \frac{P_r^2 EA_0^2}{12 \xi_d^2} - \frac{P_r^2 EA_0 A_1}{6 \xi_d^3} - \frac{P_r^2 E}{24 \xi_d^4} + \frac{P_r EA_0^2}{6 \xi_d^2} - \frac{P_r EA_0 A_1}{3 \xi_d^3} - \frac{P_r E}{24 \xi_d^4} \right) \frac{\xi^6}{30} \\ & + \left(\frac{P_r^3 EA_0}{36 \xi_d^2} + \frac{P_r^2}{13 \xi_d^2} - \frac{5P_r^2 EA_0^2}{36 \xi_d^3} + \frac{P_r^2 E}{6 \xi_d^3} - \frac{P_r^2 E}{6 \xi_d^2} - \frac{P_r^2 E}{12 \xi_d^3} - \frac{P_r E}{12 \xi_d^2} - \frac{P_r EA_0}{12 \xi_d^2} - \frac{P_r EA_1}{6 \xi_d^3} \right) \frac{\xi^5}{20} \\ & + \left(\frac{P_r^2 E}{12 \xi_d^2} - \frac{P_r EA_0^2}{8} + \frac{P_r EA_0}{4 \xi_d} - \frac{P_r E}{12 \xi_d^2} \right) \frac{\xi^4}{12} \\ & + \left(\frac{P_r^3 EA_0}{40 \xi_d^4} - \frac{3P_r^2 EA_0}{40 \xi_d^4} + \frac{P_r^3 EA_0}{36} + \frac{P_r^2}{12 \xi_d} + \frac{5P_r^2 EA_0}{36} + \frac{P_r^2 EA_1}{12 \xi_d} - \frac{P_r^2 E}{6 \xi_d} - \frac{P_r EA_0 A_1}{2 \xi_d} \right) \frac{\xi^3}{6} \\ & - \left(\frac{P_r EA_0}{\xi_d^2} + \frac{P_r EA_0}{16} + \frac{P_r E}{8 \xi_d} - \frac{P_r EA_1}{4 \xi_d} + \frac{P_r EA_3}{2 \xi_d} \right) \frac{\xi^3}{6} \\ & + \left(\frac{P_r^3 EA_0^2 \xi_d^2}{90} + \frac{P_r^2 EA_0^2 \xi_d^2}{90} + \frac{P_r^3 E}{48} - \frac{P_r^2 EA_0^2 \xi_d}{12} - \frac{P_r^2 EA_0^2 \xi_d^2}{12} + \frac{P_r^2 EA_0^2 \xi_d}{12} \right) \frac{\xi^2}{2} \\ & + \left(\frac{P_r^3 EA_0}{40 \xi_d} - 3 \frac{P_r^3 EA_0}{40 \xi_d} - \frac{P_r EA_0}{20 \xi_d} - \frac{P_r EA_0}{40 \xi_d^4} \right) \frac{\xi_d^6 \xi}{42} \\ & + \left(\frac{P_r^3 EA_0^2}{36 \xi_d^2} - \frac{P_r^2}{12 \xi_d^3} + \frac{P_r EA_0}{12 \xi_d^2} + 5 \frac{P_r^2 EA_0}{36 \xi_d^2} + \frac{P_r^2 E}{12 \xi_d^3} - \frac{P_r^2 E}{6 \xi_d^3} + \frac{P_r EA_0}{6 \xi_d^2} + \frac{P_r E}{12 \xi_d^3} - \frac{P_r EA_1}{6 \xi_d^3} \right) \frac{\xi_d^4 \xi}{20} \\ & \left(- \frac{P_r^3 EA_0}{40 \xi_d^4} + \frac{3P_r^2 EA_0}{40 \xi_d^4} - \frac{P_r^3 EA_0}{36} + \frac{P_r^2}{12 \xi_d} - \frac{5P_r^2 EA_0}{36} - \frac{P_r^2 E}{12 \xi_d} + \frac{P_r^2 EA_1}{6 \xi_d} - \frac{P_r EA_0 A_1}{2 \xi_d} \right) \frac{\xi_d^2 \xi}{6} \\ & + \left(\frac{P_r EA_0 A_1}{\xi_d^2} - \frac{P_r EA_0}{16} - \frac{P_r E}{8 \xi_d} + \frac{P_r EA_0}{4 \xi_d} - \frac{P_r EA_3}{2 \xi_d} \right) \frac{\xi_d^2 \xi}{6} + \left(\frac{P_r^3 EA_0^2}{90 \xi_d^4} + \frac{P_r^2 EA_0^2}{30 \xi_d^4} + \frac{P_r^2 EA_0^2}{20 \xi_d^4} \right) \frac{\xi_d^8}{56} \\ & + \left(\frac{P_r^3 E}{48 \xi_d^4} - \frac{P_r^2 EA_0}{12 \xi_d^3} - \frac{P_r^2 EA_0^2}{12 \xi_d^2} + \frac{P_r^2 EA_0 A_1}{6 \xi_d^3} + \frac{P_r^2 E}{24 \xi_d^4} - \frac{P_r EA_0^2}{6 \xi_d^2} + \frac{P_r EA_0 A_1}{3 \xi_d^3} + \frac{P_r E}{24 \xi_d^4} \right) \frac{\xi_d^6}{30} \\ & + \left(- \frac{P_r^2 E}{12 \xi_d^2} + \frac{P_r EA_0^2}{8} - \frac{P_r EA_0}{4 \xi_d} - \frac{P_r E}{12 \xi_d^2} \right) \frac{\xi_d^4}{12} \\ & + \left(- \frac{P_r^2 EA_0^2 \xi_d^2}{90} - \frac{P_r^2 EA_0^2 \xi_d^2}{30} - \frac{P_r^3 E}{48} + \frac{P_r^2 EA_0 \xi_d}{12} + \frac{P_r^2 EA_0^2 \xi_d^2}{12} - \frac{P_r^2 EA_0 \xi_d}{12} \right) \frac{\xi_d^2}{2} \end{aligned} \quad [22]$$

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IV.SKIN FRICTION

The skin friction coefficients are given by

$$\frac{\mu}{\rho U^2} \left(\frac{\partial u}{\partial y} \right)_{y=\pm h} = f(\pm \xi_d) \quad [23]$$

Here in the above expression the positive sign corresponds to the value at the upper wall and the negative sign to the lower one. The first term in the expansion of $f'(\xi_d)$, which is $f'_0(\xi_d)$ corresponds to the quasi-steady state at the upper wall and is designated as Z_{qs}^{up} . We have neglected the terms involving β_2 and succeeding members down the sequence for a reasonable assessment of the establishment of the flow when the strictly decreasing sequence of parameters β_x rapidly converges. The instantaneous value of the skin friction is designated as

$$\tau_{inst}^{up} = \tau_{qs}^{up} \left[1 + \beta_1 \frac{f'_1(\xi_d)}{f'_0(\xi_d)} \right] \quad [24]$$

For any prescribed value of velocity function;

$$U(t) = 100(1 - e^{-t}) \quad [25]$$

It is easy to calculate

$$\beta_1 = \frac{U(t)}{U(t)^2} \quad [26]$$

And ascertain an approximate value of the time, after which the flow would, became effectively established near the upper wall. The time of establishment of the flow near the lower wall may be calculated in the same way.

The greater of the two values is taken as the time after which the entire flow between the plates would get established.

V.NUSSELT NUMBER

If we denote the Nusselt Numbers for the instantaneous and the quasi-steady conditions at the upper wall then

$$\frac{Nu_{inst}(\xi_d)}{Nu_{qs}(\xi_d)} = 1 + \frac{\dot{U}}{U^2} \frac{\theta'_1(\xi_d)}{\theta'_0(\xi_d)} + \frac{\ddot{U}}{U^3} \frac{\theta'_2(\xi_d)}{\theta'_0(\xi_d)} \quad [27]$$

For any prescribed velocity function, one would similarly know the time after which, the heat transfer becomes effectively established. In a similar manner, we can calculate these values at the lower wall.

VI.CONCLUSION

The calculations were made for different values of pressure gradient parameters A_0, A_1, A_2, ξ_d , and the Reynolds number, which is based on the channel width. Here f_0 represents the steady parabolic velocity of the generalized Coquette flow. The unsteadiness in the flow is represented by the succeeding terms, f_1, f_2 of the series for f . The pressure gradient parameters, occurs in the solutions in combination with the Reynolds number. From the graph-2, it is observed that the deviation from the steady state value has attained a parabolic distribution in the mid stream. It is also observed that in a positively accelerated flow ($\beta_1 > 0$), the unsteady velocity tends to attain the established values from figure-2. Initially, with an increase in the pressure gradient from zero value, the establishment is delayed more and more but beyond a certain value, it is fastened when the pressure gradient increases. In the absence of pressure gradient, it is seen that f_0 linearly increases with ξ from zero to one, while f_1 negatively attains its

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maximum at $\xi = 0.46$. It is also observed that for a positively rising value of pressure gradient A_0 , causes the flow deviated more and more from the established state. But for a positive constant, the pressure gradient f_1 is always negative. Figure-3 depicts the value of f_2 for different values of ξ . Here we see that the values of f_2 becomes parabolic for different values of $A_0\xi_d$

In studying the temperature functions, it is observed that the heat transfer gets established sooner on increasing the pressure gradient parameter, $A_0\xi_d$, but in general much later than the flow established (Table-2) From table-3 and table-4, it is observed that if the Brinkman number is fixed, Prandtl number has little influence on θ_0 , but the unsteadiness increases with greater values of Prandtl number, but when the Prandtl number is unchanged as increase in Brinkman number increases the steady temperature as well as the unsteadiness in the heat transfer.

Table-1

Sl. No.	$A_0\xi_d$	$A_1\xi_d$	$A_2\xi_d$	Time of establishment of flow at lower plate	Time of establishment of flow at upper plate	Time of establishment of flow
1	-2	-2	-2	1.71	1.50	1.71
2	-1	-2	-2	2.95	1.95	2.95
3	0	-2	-2	2.95	3.36	3.36
4	1	-2	-2	1.70	3.36	3.36
5	2	-2	-2	1.32	1.95	1.95

Table-2

Sl. No.	$A_0\xi_d$	$A_1\xi_d$	$A_2\xi_d$	P_r	B_r	Time of establishment of flow at lower plate	Time of establishment of flow at upper plate	Time of establishment of flow
1	1	1	1	0.1	1	11.87	17.30	17.30
2	1	1	1	1	1	5.95	7.39	7.39
3	1	1	1	10	1	13.52	34.5	34.5
4	1	1	1	0.1	5	15.81	25.85	25.85
5	1	1	1	1	5	31.95	27.65	31.95
6	2	1	1	0.1	1	7.95	13.20	13.20
7	2	1	1	0.1	5	4.04	31.3	31.3
8	2	1	1	10	5	6.24	28.70	28.70
9	2	1	1	10	5	6.24	28.70	28.70

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Table-3

$P_r = 0.1$			$P_r = 1$		$P_r = 10$	
ξ	θ_0	θ_1	θ_0	θ_1	θ_0	θ_1
-1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.9	0.1308	0.0308	0.1308	0.0574	0.1309	0.3239
-0.8	0.2421	0.0434	0.2421	0.0869	0.2422	0.5217
-0.7	0.3365	0.0445	0.3365	0.0974	0.3366	0.6263
-0.6	0.4165	0.0389	0.4165	0.0957	0.4165	0.6637
-0.5	0.4846	0.0300	0.4846	0.0867	0.4844	0.6539
-0.4	0.5424	0.0203	0.5424	0.0741	0.5422	0.6121
-0.3	0.5920	0.0114	0.5920	0.0603	0.5919	0.5497
-0.2	0.6351	0.0040	0.6351	0.0468	0.6352	0.4749
-0.1	0.6737	-0.0015	0.6737	0.0344	0.6736	0.3939
0	0.7084	-0.0051	0.7084	0.0236	0.7083	0.3111
0.1	0.7407	-0.0071	0.7407	0.0144	0.7406	0.2299
0.2	0.7713	-0.0078	0.7713	0.0669	0.7712	0.1531
0.3	0.8008	-0.0074	0.8008	0.0008	0.8009	0.0833
0.4	0.8393	-0.0065	0.8393	-0.0038	0.8392	0.0228
0.5	0.8595	-0.0052	0.8595	-0.0070	0.8594	-0.0258
0.6	0.8886	-0.0037	0.8886	-0.0088	0.8885	-0.0598
0.7	0.9177	-0.0022	0.9177	-0.0090	0.9176	-0.0765
0.8	0.9463	-0.0010	0.9463	-0.0075	0.9462	-0.0733
0.9	0.9740	-0.0001	0.9740	-0.0045	0.9739	-0.0483
1	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000

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Table-4

$P_r = 0.1$					$P_r = 1$			
$B_r = 1$			$B_r = 5$		$B_r = 1$		$B_r = 5$	
ξ	θ_0	θ_1	θ_0	θ_1	θ_0	θ_1	θ_0	θ_1
-1	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000	0.0000
-0.9	0.1308	0.0308	0.4546	0.1482	0.1308	0.0574	0.4546	0.2302
-0.8	0.2423	0.0434	0.8111	0.2076	0.2423	0.0869	0.8111	0.3386
-0.7	0.3367	0.0445	1.0830	0.2107	0.3367	0.0974	1.0830	0.3680
-0.6	0.4164	0.0389	1.2828	0.1817	0.4164	0.0957	1.2828	0.3503
-0.5	0.4843	0.0300	1.4212	0.1375	0.4843	0.0867	1.4212	0.3086
-0.4	0.5423	0.0203	1.5110	0.0905	0.5423	0.0741	1.5110	0.2587
-0.3	0.5918	0.0114	1.5596	0.0479	0.5918	0.0603	1.5596	0.2106
-0.2	0.6353	0.0040	1.5762	0.0136	0.6353	0.0468	1.5762	0.1700
-0.1	0.6737	-0.0015	1.5676	-0.0109	0.6737	0.0344	1.5676	0.1392
0	0.7084	-0.0051	1.5418	-0.0257	0.7084	0.0236	1.5418	0.1181
0.1	0.7405	-0.0071	1.5028	-0.0322	0.7405	0.0144	1.5028	0.1052
0.2	0.7713	-0.0078	1.4562	-0.0324	0.7713	0.0069	1.4562	0.0984
0.3	0.8008	-0.0074	1.4046	-0.0281	0.8008	0.0008	1.4046	0.0951
0.4	0.8393	-0.0065	1.3511	-0.0212	0.8393	0.0038	1.3511	0.0929
0.5	0.8595	-0.0052	1.2969	-0.0133	0.8595	0.0070	1.2969	0.0898
0.6	0.8887	-0.0037	1.2428	0.0056	0.8887	0.0088	1.2428	0.0841
0.7	0.9177	-0.0022	1.1880	0.0007	0.9177	0.0090	1.1880	0.0741
0.8	0.9463	-0.0010	1.1310	0.0048	0.9463	0.0075	1.1310	0.0583
0.9	0.9738	-0.0001	1.0697	0.0053	0.9738	0.0045	1.0697	0.0347
1	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000	1.0000	0.0000

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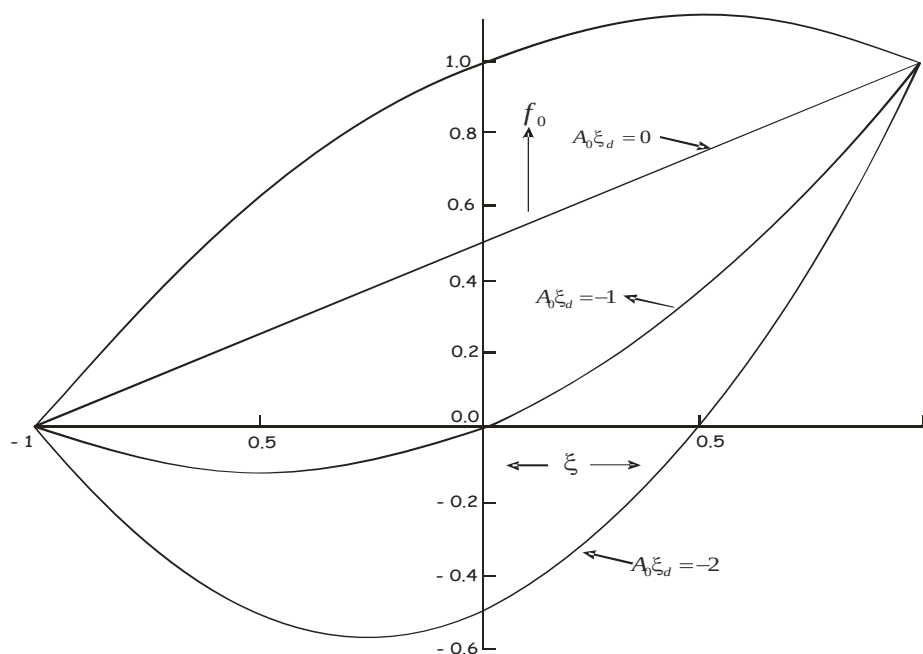


Figure-1: First flow functions for different values of $A_0 \xi_d$

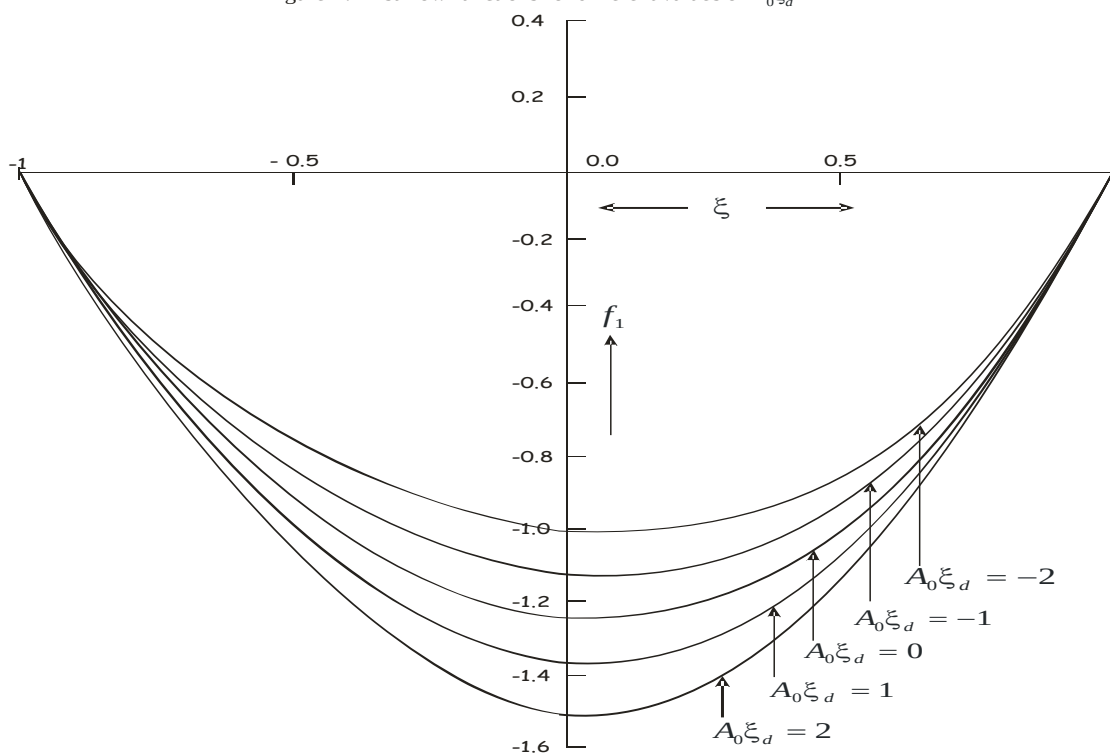


Figure-2: Second flow functions for different values of $A_0 \xi_d$

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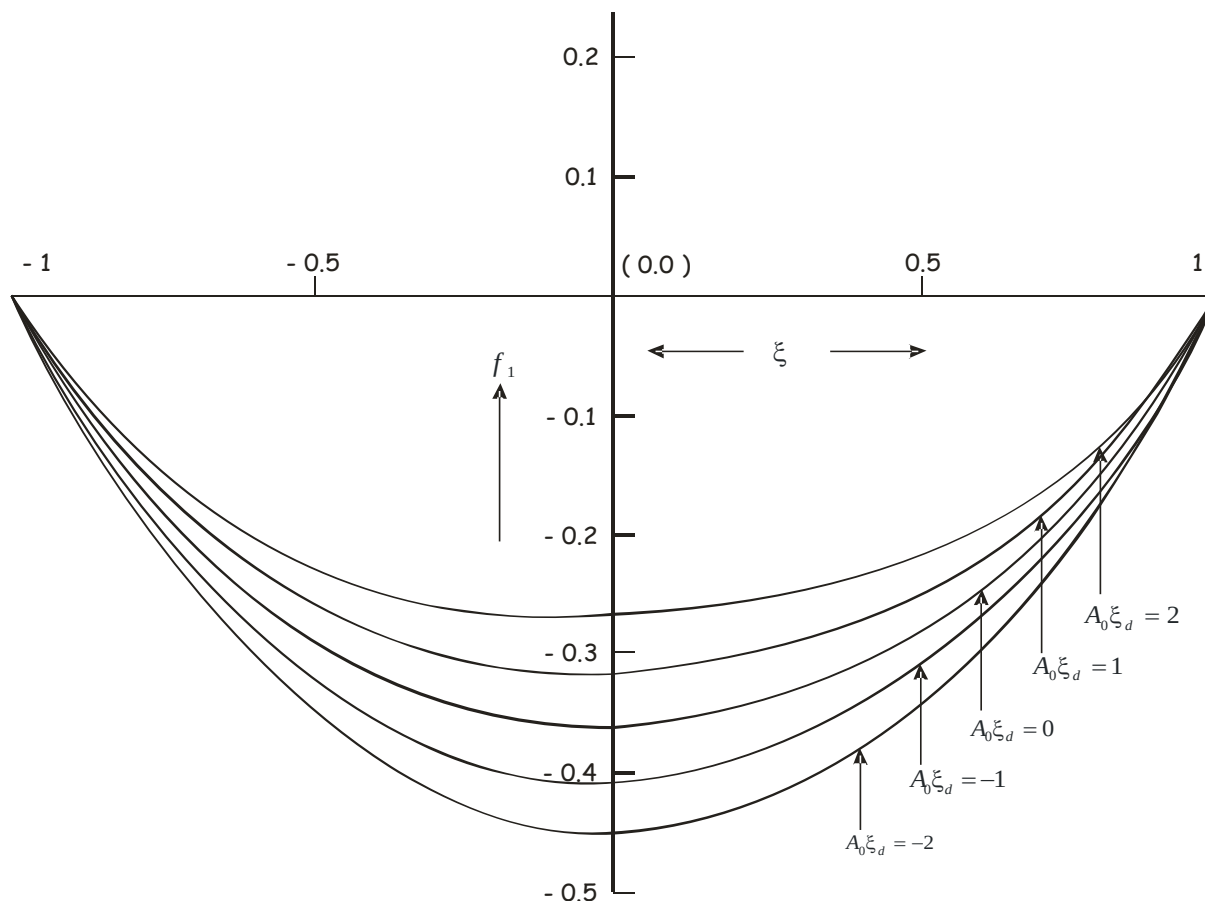


Figure-3: Third flow functions for different values of $A_0 \xi_d$

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