Fuzzy Regression Model and Its Application: A Review

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ABSTRACT: Fuzzy regression model has been widely used in recent years throughout the globe. In view of this, an attempt has been made in this research paper to present the review of fuzzy regression model for better estimation and prediction. The regression analysis is statistical tool used for prediction. As we know that the regression analysis follows Gaussian assumptions, sometimes dataset is too small and vague. In this situation fuzzy regression gives better results as compared to conventional tools. The present review is providing the information literature and recent methods developed by different Author. Also this review highlight on the work done by different authors in different field of linear programming problem on fuzzy least square & fuzzy interval estimation. The fuzzy regression models were providing better accuracy than conventional regression model. In view of this an attempt has been made in this paper to review recent work done in the field of fuzzy regression. The review has shown that majority of studies on fuzzy regression models with linear programming approach using different fuzzy numbers.

KEYWORDS: Fuzzy regression, Literature Review.

I. INTRODUCTION

Regression analysis is powerful technique for modelling and forecasting in conventional regression analysis. In this analysis, data should be crisp and should follow Gaussian assumption. If data set is too small, uncertainty and vagueness occurs in this situation fuzzy regression model is appropriate and gives better results. If the data set is crisp and fuzzy then we have to fit the Fuzzy regression model. Whereas conventional regression is appropriate for only crisp data. In this review, authors were focuses on fuzzy regression model used in linear and nonlinear programming problem. The fuzzy regression model is applied to price mechanism of prefabricated houses, evaluating functional relationship between dependent and independent variables in fuzzy environment, applied to various problems such as forecasting engineering, concept of maximizing decision making problems(Bellman and Zadehet.al 1970), a fuzzy approach to multi-objective linear programming problems(Zimmermann,1983), fuzzy linear programming problem with fuzzy coefficients (Negoiita,1970), Tanaka and Asai was proposed a formulation of fuzzy linear programming problem with fuzzy constraints and given methods for its solution on inequality relation between fuzzy numbers( Tanaka and Asai 1984).

II. FUZZYMODEL USING FUZZY REGRESSION LINEAR AND NONLINEAR PROGRAMMING

Fuzzy regression analysis used in linear programming and nonlinear programming problem to minimize spread of triangular fuzzy numbers and gives better prediction than the conventional regression analysis. Review of Fuzzy regression model developed by different authors is given as follows.
Tanaka et al.[1982] applied the fuzzy linear function to linear regression model and established a methods of system representation for a vague phenomenon. The vagueness of the evaluation system was represented by the fuzzy linear function.
Tanaka et al.[1989] used a linear systems as a model for analyzed the given data. This methods was applied to conventional data obtain fuzzy parameters.
Dragan A. Savic et al. [1991] proposed a two-stage construction of a linear regression model in an enhancement of a minimal vagueness criterion. They said that use of Fuzzy regression was highly influenced by its purpose and the nature of the practical application.

Georg Peters [1994] introduced a new class of fuzzy linear regression models based on Tanaka’s approach. He demonstrated that proposed model was varied to generalization of the model of Tanaka and also applied for real-life applications.

Kwang Jae Kim et.al.[1996] investigated fuzzy linear regression as viable alternative to statistical linear regression in estimated regression parameters when data set was insufficient to support statistical regression analysis.

Ping-Teng Chang et. al.[1996] proposed fuzzy regression model based on weighted least square methods. They had developed method in three stages viz, the goodness of fit of the model to the data was tested by using the formal statistical procedures such as the forward, backward, or Stepwise procedures. Furthermore, the fuzzy regression analysis was extended to fuzzy-random regression analysis where the randomness of the data was considered. They observed that weighted least square method has reduced the sensitivity to outliers tremendously.

Byungjoon Kim et. al. [1998] proposed, fuzzy regression models were evaluated on the basis of the comparison of the observed membership function and estimated membership function. Rather than they used the vagueness of model as likely as do fuzzy linear regression and fuzzy linear least square regression, the error of fitting of the estimated fuzzy number to the observed fuzzy number has used to evaluate the models as likely as do the conventional regression models.

K.K. Yen et .al. [1999] developed fuzzy linear model used symmetric triangular fuzzy numbers as coefficients such that the inflexibility of the earlier fuzzy linear models. They extended the fuzzy linear model as used for non-symmetric fuzzy number coefficients without changing the original formulation. Also they introduced skew factor as the ratio between the unequal spreads of the two sides of the non-symmetric triangle. They established the relationships among skew factor, spread and error for the fuzzy linear model.

Hsiao-Fan Wang et al.[2000] proposed fuzzy linear regression model with crisp input and fuzzy output described by Tanaka was considered. They modified fuzzy least square method for the solution. They showed that modified least square methods has better performance than both Tanaka’s methods and the conventional fuzzy least square method.

Yun-Shiow Chen [2001] investigated fuzzy liner regression model with non fuzzy input and fuzzy output data type to handle the outlier in the problem. He introduced a pre-assigned k-limiting value of these value must be determined based on the conditions of the current problem. He observed that k-limiting values were effective in detecting abnormal values in used data.

Hisao Ishibuchi et al.[2001], Proposed fuzzy linear regression models with symmetric triangular fuzzy coefficients were several limitations. They extended symmetric triangular fuzzy coefficients to asymmetric triangular and trapezoidal fuzzy numbers increased the flexibility of the linear fuzzy models also extended of the linear fuzzy regression models to non-linear model was increased the flexibility of fuzzy regression model.

Jing-Rung Yu et. al.[2001], proposed fuzzy piecewise regression analysis with automatic change-point detection. They introduced possibility and necessity regression models were utilized and also the function behaved differently in different parts of the range of crisp input variables. They observed that fuzzy piecewise regression model with detecting change-points was derived.

Miin-Shen Yang et al.[2002], proposed fuzzy regression model was used in evaluating the functional relationship between the dependent and independent variables in a fuzzy environment. Most fuzzy regression models were considered to be fuzzy outputs and parameters button-fuzzy (crisp) inputs. They formulated multi objective programming methods for the model estimation along with a linear-programming problem. They developed two estimation methods along with a fuzzy least-squares and interval distance fuzzy least square. They suggested that interval distance fuzzy least square was efficient alternative procedure to estimate parameters for fuzzy linear regression models with fuzzy outputs, fuzzy parameters and also fuzzy inputs.

Liem Tran et. al [2002], developed A multi-objective fuzzy regression model (MOFR). They combined central tendency and possibility properties of statistical and fuzzy regressions also overcomes several shortcomings of these two methods. They introduced distance measure for two intervals that take into account all the points in lower and upper bounds intervals was introduced. They concluded that both the central tendency and possibilistic properties of statistical and fuzzy regressions had capable of exhibiting the good characteristics. Also they effective for measured of the distance between lower and upper bounds intervals.
Chiang Kao et al. [2003] proposed classical least square methods to handle fuzzy regression observation in regression analysis. This method, based on extension principle, the membership function of the sum of squared error was constructed. They were applied classical least square methods in three cases such as crisp input-fuzzy output, fuzzy input-fuzzy output, and non-triangular observation. They observed that classical least square methods was better performance than Sakawa-Yano least square methods.

Y. Chen et al. [2004] proposed fuzzy linear regression with symmetric triangular fuzzy coefficients was considered to model the relational functions between engineering characteristics and customer requirement. They extended symmetric triangular fuzzy coefficient to non-symmetric triangular fuzzy coefficients. They introduced design budget and customer expectations among the competitors was taken in the quality function development for product planning and also proposed quality improvement problem for an emulsification dynamite packing machine. They compared symmetric and non-symmetric triangular fuzzy number with the help of simulation analysis finally they concluded that the non-symmetric fuzzy number was better performance than the symmetric fuzzy number.

M. Modarres et al. [2005] developed fuzzy linear regression model was applied for mathematical programming. They assumed that the dependent variables were crisp while the independent variable was a symmetric fuzzy number. They concluded that fuzzy linear regression model was applied mathematical programming had better performance than Tanaka’s method, Diamond least square method and Kim-Bishu methods.

Kyung-Bin Song et al. [2005] proposed fuzzy linear regression method for the short-term hourly loads forecasting of the holidays. They introduced fuzzy linear regression model was based on Tanaka’s approach, and used fuzzy arithmetic operations where both input data and output data was fuzzy numbers. They conclude that fuzzy linear regression algorithm showed good accuracy for predication of load holiday.

Wen-Liang Hung, et al. [2006] proposed the detection of outliers in fuzzy linear regression models. They observed that the disadvantage in several of the procedures for detected outliers was a lack of defining cut-offs for outliers. They had overcome the drawbacks in existing methods, and also used an omission approach to detect outliers. They concluded that an omission approach for outlier detection was better than the Tanaka’s methods.

Hong-Wei Ge et al. [2007] proposed fuzzy linear regression model used on non-symmetric fuzzy triangular coefficients based on the support vector regression. They extended fuzzy linear regression model to its regularized version and interpreted as the corresponding equivalent maximum a posterior problem. They observed that the optimal choice of the parameter was actually dependent on the variance of the input data. They concluded that regular fuzzy linear regression model used non-symmetric triangular coefficients was very helpful for the practical applications than both fuzzy linear regression model and regular fuzzy linear regression model used symmetric and non-symmetric fuzzy triangular coefficients.

Chi-Yo Huang, et al. [2008], proposed a forecast methodology for predicting both product life time and non-linear product life cycle, based upon a two-stage, fuzzy, piecewise regression analysis model. Indifferent to traditional time-based forecast methodology, a generation-based approach was applied, predicts product life cycle by deriving the annual fuzzy regression lines, based upon the annual shipments of earlier generation products. Proposed methodology was applied to forecast other multiple-generation products like PCs, semiconductor processes, etc. The results of a forecast methodology were applied as a foundation for policy foresight and strategic definitions for each stage of the product life cycle.

Nimet Yapıcı Pehliv, et al. [2010], developed fuzzy regression analysis used fuzzy radial basis function network. They observed that fuzzy radial basis function network had higher flexibility and a wider application field than the linear programming based method and fuzzy least square regression methods. They showed that the fuzzy regression analysis used fuzzy radial basis function network model was better performance than linear programming method and fuzzy least square regression methods.

S. Arulchinnappan, et al. [2011], demonstrated fuzzy linear regression model for estimation of reverse osmosis permeating parameters conditions. They observed that fuzzy regression model was effectively taken on non-crisp, fuzzy and crisp data. They examined the variables that contribute to the deterioration of membrane. They concluded that the fuzzy linear regression model had accuracy with efficient prediction model.

A. Azadeh, et al. [2011], proposed fuzzy regression–analysis of variance (ANOVA) algorithm to estimated and predicted electricity consumption in uncertain environment. They compared the fuzzy regression–analysis of variance algorithm with sixteen fuzzy regression models. They observed that there was no clear cut as to which of the recent fuzzy regression model was suitable for given set of actual data with respect to electricity consumption and also difficult to model uncertain behavior of electricity consumption with conventional time series and proper fuzzy
regression could be an ideal substitute for such cases. They suggested that the fuzzy regression–analysis of variance algorithm used to identify the optimum model with lowest error.

M. R. Taghizadeh, et al.[2011], investigated fuzzy linear regression model was applied for transport energy demand forecasting used socio-economic and transport related indicators. They examined fuzzy linear regression model was based on Gross Domestic Product (GDP), population and the number of vehicles as three inputs in the model. They had taken Energy data from 1993 to 2005 were used to estimate each of the three inputs. They had designed fuzzy linear regression (FLR) models, a multi-level fuzzy linear regression model and also predicted energy demand.

Lazim Abdullah et. al.[2012], developed the fuzzy regression model used for road accident in Malaysia over the period of 1974 to 2007 used three predictors. They introduced the threshold level \( h=0.5, 0.9 \) were accounted and also model structured was the road accident uses registered vehicles, population and road length as variables based on one response variable was road accident. They showed that the variables of registered vehicles and population provide higher impact to the number of road accident.

Hande Gunay Akdemir, et al[2013], studied the effect of the variation of metrological conditions on the total burned area in hectares, by used fuzzy linear regression analysis based on Tanaka’s fuzzy regression model. They observed that temperature, wind speed and rainfall were significant on total burned area.

Ali Azadeh, et al.[2013] implemented, fuzzy regression model was applied to gas consumption in Iran. They compared with conventional regression methods and fuzzy regression model to the crisp data. They observed that fuzzy algorithm was better performance than conventional regression analysis.

Xu Jialu & Lu Qiujun, et al. [2014] demonstrated that the outlier effect in predicting GDP growth on the basis of fuzzy linear regression model. They suggested that fuzzy linear regression was better performance than conventional linear regression model.

Souhir Charfeddin, et al.[2014], introduced fuzzy linear regression analysis based on the Tanaka’s method. They extended to build trapezoidal fuzzy sets for the estimated variable and also predicted air transport demand.

Muhammad Ammar Shafi, et al. [2015], introduced fuzzy regression model to determined the Tumor size in colorectal cancer in Malaysia. The study was conducted colorectal cancer patients from Malaysia. They compared multiple regression analysis and different fuzzy regression analysis was used. They suggested that Yongshen Ni Fuzzy linear regression model best for predicting stages of colorectal cancer patients.

S. Muzzioli et. al.[2015] investigated the linear fuzzy regression methods of Tanaka et al., Savic, Pedrycz, Ishibuchi and Nii and applied on finance problem and estimated of the smile function. They evaluated both the in-sample pricing performance of the estimation methods and the out-of-sample forecasting performance of the moments, by using as a benchmark the standard cubic spine interpolation. They suggested that in sample, by using fuzzy regression was better estimation than the classical approach based on cubic spines.

Fangning Chen et. al.[2015], proposed the optimized threshold \( h \) value for fuzzy linear regression analysis used with minimum fuzziness criteria with symmetric triangular coefficient. They extended to asymmetric triangular coefficient and also determined optimal \( h \) value to maximize the system reliability of fuzzy regression model with asymmetric triangular fuzzy coefficient. Finely they suggested that threshold \( h \) value should not be more than 0.5 when fuzzy linear regression was used with symmetric triangular fuzzy numbers or asymmetric triangular fuzzy number.

### III. FUZZY REGRESSION MODEL USING LEAST SQUARE

Fuzzy regression model used as least square methods, input fuzzy output, fuzzy input crisp output, crisp input crisp output and fuzzy input fuzzy output etc. fuzzy least square was Developed by different authors review summarized below.

Phil Diamond,[1988], proposed several models for simple least squares fitting of fuzzy valued data. He introduced fuzzy data sets was fitted to the model, and similar of normal equation was derived. Finally he had fitted the model depending on the nature of data.

Pierpaolo D’Urso, et al.[2000], introduced new approach of fuzzy linear regression analysis. They had developed doubly linear adaptive fuzzy regression model, based on two linear models such as center regression model and a spread regression model. The first one was explained the centers of the fuzzy observations, while the second one was for their spreads. They had observed that doubly linear adaptive fuzzy regression analysis had alternative methods for fuzzy linear regression analysis.
Ruoning Xu, et. al.[2001], developed fuzzy multivariable linear regression by least squares and also the methods was similar to the traditional least-squares method. They had introduced a fuzzy analogue by a distance defined on a fuzzy number space, and fuzzy multivariable least square linear regression model. They showed that the model had a unique solution and the solution was given in an analytic expression.

Yun-His, et. al.[2001]. Developed hybrid fuzzy least-squares regression. They were used weighted fuzzy-arithmetic mean and least-squares fitting criterion. They had compared hybrid regression with the ordinary regression and other fuzzy regression methods. They suggested that hybrid fuzzy regression model satisfied a limiting behaviour that the fuzziness decreases, the equations were similar to the results of the ordinary regression.

Volker Krätschmer [2006] developed new fuzzy linear regression models. He had generalized the type of single ordinary equation in linear regression models by incorporated the physical vagueness of the involved items in the form of fuzzy data for the variables. He had suggested that ordinary least-squares method was greater flexibility for modelling and estimation.

Seung Hoe Choi, et al.[2008], proposed least absolute deviation estimators to construct fuzzy regression models. They had compared a fuzzy regression model and used the least absolute deviation estimators and another fuzzy regression model used the least square method. They were shown that the fuzzy least absolute deviation model was better performance than the fuzzy regression based on least squares method in case the data contains fuzzy outliers.

Furkan Baseretet. al.[2010] investigated hybrid fuzzy least squares regression analysis was applied to predict future claim costs by used the concept of London Chain Ladder (LCL) method. They had suggested that the hybrid fuzzy least-squares regression model was taken both randomness and fuzziness type of uncertainty into a regression model.

Fatemeh Torfi ,et. al.[2011], investigated a fuzzy least-squares linear regression (FLLRs) to estimate the stochastic demands in the vehicle routing problem (VRP). They had developed approximate- distance fuzzy least-squares (ADFL) estimator and also applied to given data. They suggested that approximate- distance fuzzy least-squares method was realistic and efficient estimator to face the stochastic demand in vehicle routing problem.

B.Pushpa, et. al.[2013] developed least absolute multiple fuzzy regression model. They had introduced functional dependence of crisp inputs-fuzzy outputs variables used the generalized Hausdorff- metric between fuzzy numbers as well as linear programming problem. They suggested that least absolute fuzzy regression model used shape preserving operation was more effective than Hung and Yang method.

IV. FUZZY REGRESSION MODEL USING LEFT RIGHT FUZZY NUMBER

Fuzzy regression model used as left right fuzzy numbers. Used as fuzzy and crisp data developed model better accuracy than conventional regression model. The different researches review as summarize below.

Ralf Korner, et al.[1998], discussed Zadeh's extension principle to classical crisp estimated to the least squares method. They were developed linear estimation theory. Unfortunately, since fuzzy sets fail to constitute a linear space w.r.t, addition and scalar multiplication and also modified linear estimates often lead to better estimates than by use of the extension principle. Finally they used least squares approximation principle for fuzzy data was investigated better performance than the extension principal.

H. hassanpour, et. al.[2009], proposed fuzzy linear regression model and also modified Kim and Bish’s fuzzy linear regression analysis model. They observed that Kim and Bish’s method often does not find acceptable fuzzy linear regression coefficients. Finally they suggested that modified fuzzy linear regression model was better performance than the Kim and Bish’s method.

V. FUZZY REGRESSION MODEL USED INTERVAL ESTIMATON

Fuzzy regression model used for fuzzy data and crisp data to estimated parameters and Minimizing residual by the different authors in the different field proposed methods were better accuracy than conventional regression analysis. Review summarized below.

J.J. Boreux, et. al.[1998], investigated fuzzy linear regression model was used in dendroecology to model empirically tree growth as a function of a bioclimatic index representing the water stress. They showed that fuzzy linear regression was given more precise expected growth was obtained for high water stress than ordinary least squares and orthogonal least squares methods.
Chiang Kao et. al.[2002] proposed two-stage fuzzy linear regression model. In first stage, the fuzzy observations was de-fuzzified so that the traditional least-squares method was applied to find a crisp regression line showed the general trend of the data. In the second stage, the error term of the fuzzy regression model was represents the fuzziness of the data and also determined to given the regression model was best explanatory power for the data. Finally they suggested that two-stage method had better performance than the Kim–Bushu’s and Diamond fuzzy linear regression model.

Hsien-Chung Wu [2003] proposed fuzzy estimates of regression parameters with the help of “Resolution Identity”. He said that the fuzzy estimates was constructed from the alpha level least-squares estimates used the alpha level real-valued data. Finally he had developed two computational procedures to solve the optimization problems.

Kyung, Kim, et. al.[2005] proposed fuzzy least absolute deviation method to construct fuzzy linear regression model with fuzzy input and fuzzy output. They had suggested that the fuzzy least absolute deviation method was more effective than the least square method used the in the fuzzy regression analysis.

Rajan Alex [2006] introduced fuzzy regression and fuzzy inference. He was applied the two kinds of information resources, quantitative and qualitative information and also used simultaneously in practical prediction. Finally he suggested that fuzzy regression and fuzzy inference had better performance than pure regression or pure inference model.

Hye-YoungJung, et. al.[2015]. Proposed an rank transformation method. They investigated a method to obtain a predicted output with respect to a specific target value. They suggested that the rank transformation method in fuzzy regression model was better performs than the Chen and Hsuehand Diamond fuzzy regression models.

VLCONCLUSION

This review has undertaken on fuzzy regression model using linear and non linear programming problem, fuzzy regression analysis using least square method, fuzzy regression analysis used left and right fuzzy numbers and fuzzy regression model used in interval estimation. Some authors was suggested that fuzzy regression model in different field for prediction purpose was better performance than conventional regression analysis. The researchers developed various algorithms for predicting fuzziness and vague phenomenon with linear programming, fuzzy least square, interval estimation and left right fuzzy numbers suitable for predication.

REFERENCES


