

# Model Analysis of Variation of Taper Angle for Cantilever and Simply Supported Beam

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**ABSTRACT:** Beams are very common types of structural components and it can be classified according to beams and can meet special functional requirements in architecture, aeronautics, robotics, their geometric configuration as uniform or taper and slender or thick. If practically analyzed, the non-uniform beams provide a better distribution of mass and strength than uniform and other innovative engineering applications. Design of such structures is important to resist dynamic forces, such as wind and earthquakes. It requires the basic knowledge of natural frequencies and mode shapes of those structures. In this research work, the equation of motion of a double tapered cantilever Euler beam is derived to find out the natural frequencies of the structure. Finite element formulation has been done by using Weighted residual and Gale kin's method. Natural frequencies and mode shapes are obtained for different taper ratios. The effect of taper ratio on natural frequencies and mode shapes are evaluated and compared.

**KEYWORDS:** Text detection, Inpainting, Morphological operations, Connected component labelling.

## I. INTRODUCTION

It is well known that beams are very common types of structural components and can be classified according to their geometric configuration as uniform or tapered, and slender or thick. It has been used in many engineering applications and a large number of studies can be found in literature about transverse vibration of uniform isotropic beams. But if practically analysed, the non-uniform beams may provide a better or more suitable distribution of mass and strength than uniform beams and therefore can meet special functional requirements in architecture, aeronautics, robotics, and other innovative engineering applications and they have been the subject of numerous studies. Non-prismatic members are increasingly being used in diversities as for their economic, aesthetic, and other considerations.

Design of such structures to resist dynamic forces, such as wind and earthquakes, requires a knowledge of their natural frequencies and the mode shapes of vibration. The vibration of tapered beam linearly in either the horizontal or the vertical plane finds wide application for electrical contacts and for springs in electromechanical devices.

For the tapered beam vibration analysis Euler beam theory is used. Free vibration analysis that has been done in here is a process of describing a structure in terms of its natural characteristics which are the frequency and mode shapes. The change of modal characteristics directly provides an indication of structural condition based on changes in mode shapes and frequencies of vibration.

There have been many methods developed yet now for calculating the frequencies and mode shapes of beam. Due to advancement in computational techniques and availability of software, FEA is quite a less cumbersome than the conventional methods. Prior to development of the Finite Element Method, there existed an approximation technique for solving differential equations called the Method of Weighted Residuals (MWR). This method is presented as an introduction, before using a particular subclass of MWR, the Gale kin's Method of Weighted Residuals, to derive the element equations for the finite element method. These formulations have displacements and

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rotations as the primary nodal variables; to satisfy the continuity requirement each node has both deflection and slope as nodal variables. Since there are four nodal variables for the beam element, a cubic polynomial function is assumed. The clamped free beam that is being considered here is assumed to be homogeneous and isotropic. Beam of rectangular cross-section area with width and depth varying linearly are taken and elemental mass and stiffness matrices are being derived. The effects of taper ratio on the fundamental frequency and mode shapes are shown in with comparison for clamped-free and simply supported beam via graphs and tables. These results are then compared with the available analytical solutions.

### II. NUMERICAL ANALYSIS

For numerical analysis a taper beam is considered with the following properties:

a) Geometrical properties

Width of the beam =  $b_0$  (fixed end) =  $2.54 \times 10^{-2}$  m,  $b_1$  (free end) =  $2.54 \times 10^{-2}$  m Depth of the beam =  $h_0$  (fixed end) =  $5.715 \times 10^{-2}$  m,  $h_1$  (free end) =  $3.51 \times 10^{-2}$  m  $\beta = b_0 / b_1$  and  $\alpha = h_0 / h_1$ . Length of the beam = 0.762 m

b) Material properties

Elastic modulus of the beam =  $2.109 \times 10^{11}$  N/m<sup>2</sup> Density = 7995.74 Kg/m<sup>3</sup>.

Fundamental frequencies for free vibration analysis is calculated by using the equation of motion described in the previous section for two end conditions of beam given below

- 1) One end fixed and one end free (cantilever beam), which is shown in Fig.1.
- 2) Simply supported beam, which is shown in Fig. 2.

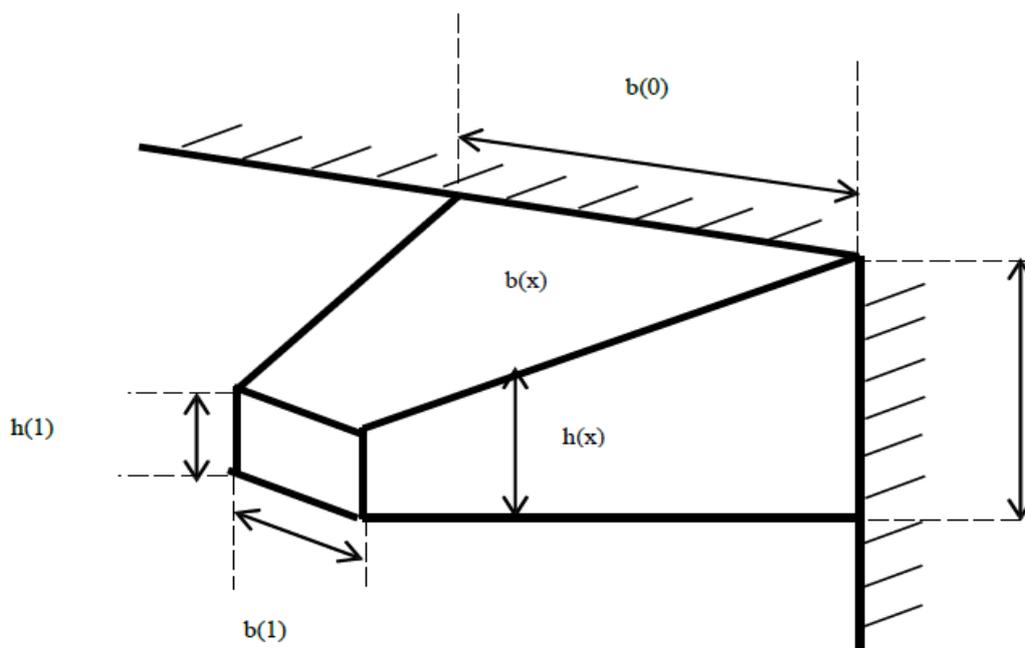


Figure-1 Tapered cantilever beam with linearly variable width and depth.

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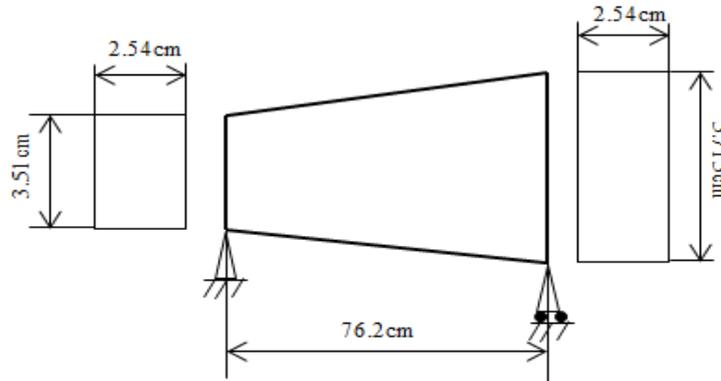


Figure-2 simply supported tapered beam with linearly variable width and depth.

## a) Calculation of frequency of cantilever beam:

After discretizing to 10 numbers of elements, natural frequencies of the tapered beam are calculated using ANSYS program and shown in figure-3.

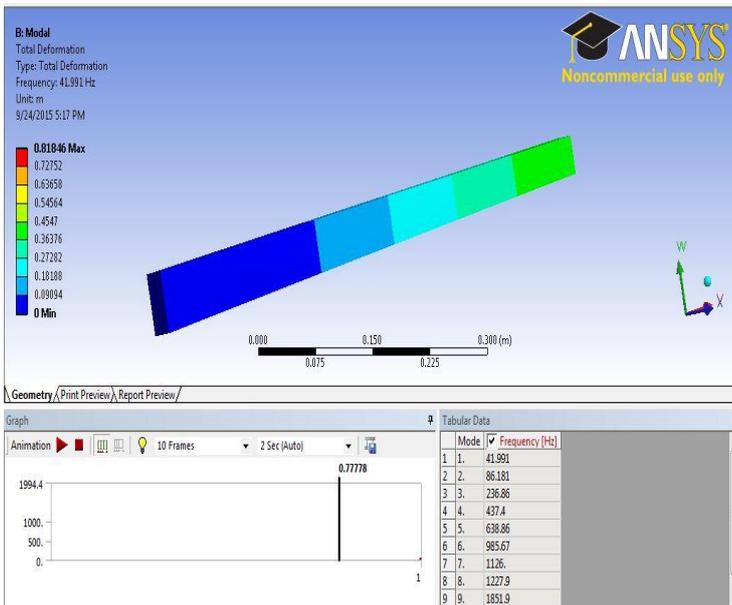


Figure-3 natural frequencies of the tapered cantilever beam.

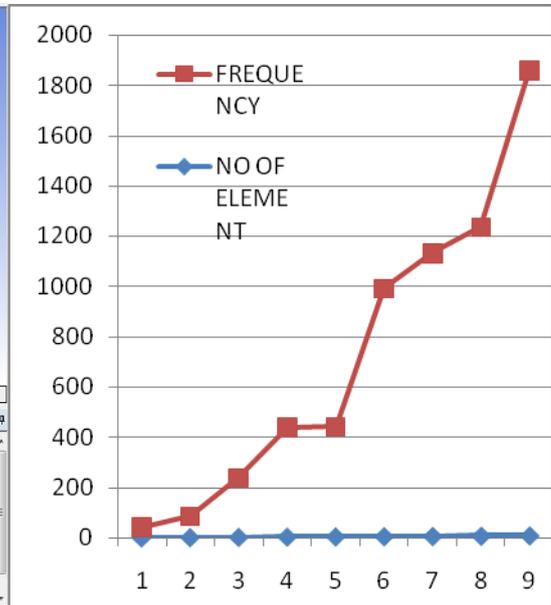


Figure-4 frequencies vs. no of element graph of The tapered cantilever beam.

As we can see in Fig.-3 the frequency converges after discretizing to only four elements. The method described in here is less cumbersome than the conventional methods. By using ansys we calculated frequencies of cantilever beam. As the number of element increases the frequencies of the tapered cantilever beam also increases.

Figure-4 indicates frequencies vs. no of element graph of the tapered cantilever beam. By the graph we can easily conclude that the natural frequencies of the tapered cantilever beam increases when the number of element increases.

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### b) Calculation of frequency of simply supported beam:

The same procedure is applied also for the simply supported beam and natural frequencies are calculated and shown in Fig-5. The convergence of frequency with number of elements is shown in Fig.-

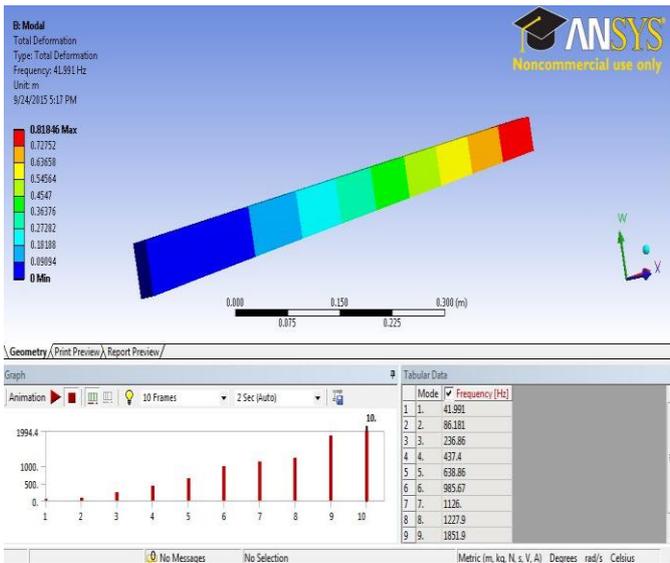


Figure-5 Frequencies of linearly tapered simply supported beam.

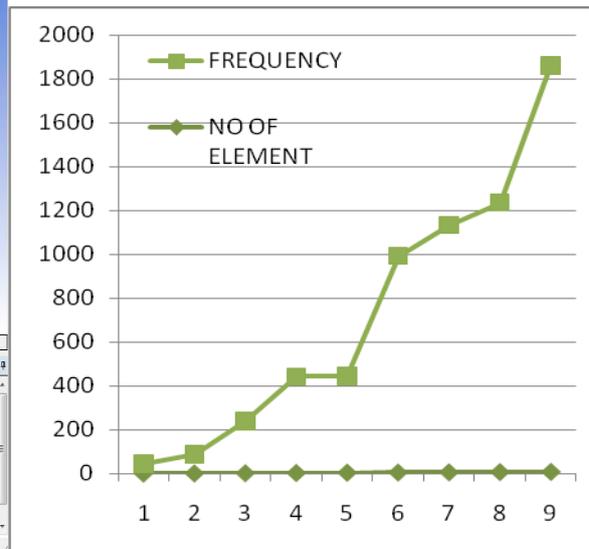


Figure-6 frequencies vs. no of element graph of the tapered cantilever beam.

As we can see in Fig.-5 the frequency converges after discretizing to only four elements. The method described in here is less cumbersome than the conventional methods. By using ANSYS tool we calculated frequencies of simply supported beam. As the number of element increases the frequencies of the tapered simply supported beam also increases.

Figure-6 indicates frequencies vs. no of element graph of the tapered simply supported beam. By the graph we can easily conclude that the natural frequencies of the tapered simply supported beam increases when the number of element increases.

## II. EXPERIMENTAL RESULTS

### A. Effect of taper ratio on the frequency by theoretical analysis for cantilever beam.

For determining the effect of taper ratio on the frequency, different values of  $\alpha$  (depth variation) varying from 1 to 2 are taken in consideration and shown in Table 1.

$\alpha$	F1	F2	F3	F4	F5
1	52.05	341.3	958.5	1893.6	3143.02
1.2	52.27	363.76	1044.9	2075.2	3450.9
1.4	52.14	385.32	1128.2	2249.5	3745.3
1.6	53.03	406.16	1209.3	2417.74	4030.45
1.8	53.62	426.43	1288.51	2580.77	4309.72
2	54.46	446.22	1366.2	2739.26	4585.43

Table-1 Frequencies of Linearly Tapered Cantilever Beam for different value of  $\alpha$ .

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### B. Effect of taper ratio on the frequency by model analysis by ANSYS for cantilever beam.

By taking a model of cantilever beam and varying the taper angle of that model by using ANSYS. And determined the effect of taper ratio on the frequency, different values of  $\alpha$  (depth variation) varying from 1 to 2 are taken in consideration and shown in Table-2.

$\alpha$	F1	F2	F3	F4	F5
1	36.347	50.168	226.62	311.34	629.42
1.2	38.433	64.452	230.62	352.87	633.42
1.4	40.257	72.953	234.02	393.24	636.94
1.6	41.889	84.739	237.09	433.03	640.15
1.8	43.35	96.68	239.82	472	643.09
2	44.674	108.78	242.75	510.32	645.42

Table-2 Frequencies of Linearly Tapered Cantilever Beam for different value of  $\alpha$  in model analysis.

### C. Effect of taper ratio on the frequency by theoretical analysis for simply supported beam.

For determining the effect of taper ratio on the frequency, different values of  $\alpha$  (depth variation) varying from 1 to 2 are taken in consideration and shown in Table-3. Table-3 indicates the Frequencies of Linearly Tapered simple supported Beam for different value of  $\alpha$ .

$\alpha$	F1	F2	F3	F4	F5
1	184.05	757.6	1704.4	3037.17	4770.4
1.2	184.13	757.7	1705	3045	4797.9
1.4	184.45	757.8	1706.1	3059.6	4860.7
1.6	184.86	758.2	1708.5	3095.3	5275.7
1.8	185.31	762.5	1730.1	3388.3	5352.2
2	186.34	768.2	1913.2	3515.6	6154

Table-3 Frequencies of Linearly Tapered simple supported Beam for different value of  $\alpha$  by theoretical analysis.

$\alpha$	F1	F2	F3	F4	F5
1	230.3	315.88	628.74	855.22	1217.2
1.2	230.24	315.85	628.79	933.04	1217.4
1.4	229.99	374.37	628.55	1006.2	1217.4
1.6	229.63	401.99	628.18	1076.1	1217.2
1.8	229.21	428.59	627.71	1142.5	1216.9
2	228.76	454.36	627.18	1206.1	1216.5

Table-4 Frequencies of Linearly Tapered simple supported Beam for different value of  $\alpha$  in model analysis.

### D. Effect of taper ratio on the frequency by model analysis by ANSYS for simply supported beam.

By taking a model of simply supported beam and varying the taper angle of that model by using ANSYS. And determined the effect of taper ratio on the frequency, different values of  $\alpha$  (depth variation) varying from 1 to 2 are taken in consideration and shown in Table-4. From the table we conclude that by varying the taper angle the frequencies of simply supported beam also increases.

### E. Length- Frequency chart for cantilever beam at different taper angle in theoretical analysis.

By taking different frequencies for different taper angle for cantilever beam and calculating the different frequencies for different length we plot the graph between frequency and length. figure 7 indicates that frequency will be more for increasing in taper angle along with the length.

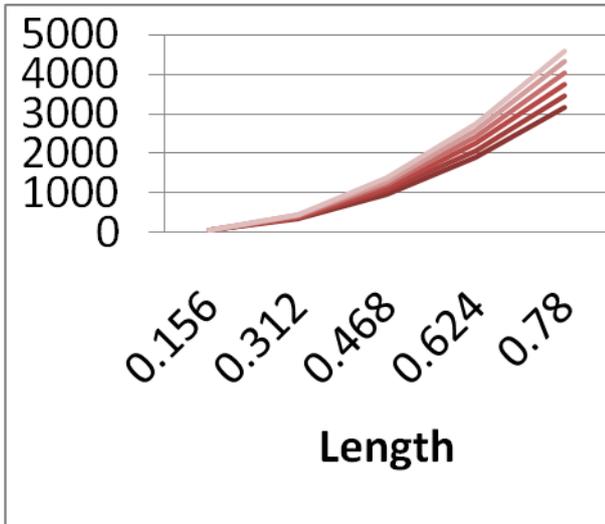


Figure-7 Length- Frequency chart for cantilever beam at different taper angle in theoretical analysis.

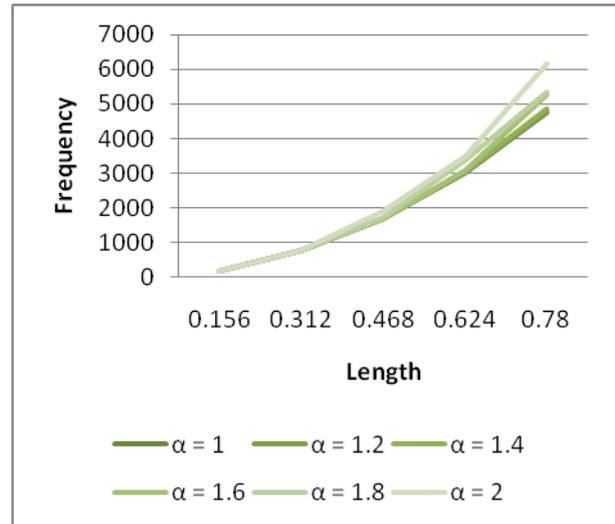


Figure-8 Length- Frequency chart for simply supported Beam at different taper angle in theoretical analysis.

**F. Length- Frequency chart for simply supported beam at different taper angle in theoretical analysis.**

By taking different frequencies for different taper angle for simply supported beam and calculating the different frequencies for different length we plot the graph between frequency and length. Figure 8 indicates that frequency will be more for increasing in taper angle along with the length.

**G. Length- Frequency chart for cantilever beam at different taper angle in model analysis.**

By taking different frequencies for different taper angle for cantilever beam and calculating the different frequencies for different length by using ANSYS tool, we plot the graph between frequency and length. Figure 9 indicates that frequency will be more for increasing in taper angle along with the length.

**H. Length- Frequency chart for simple supported beam at different taper angle in model analysis**

By taking different frequencies for different taper angle for simply supported beam and calculating the different frequencies for different length by using ANSYS tool, we plot the graph between frequency and length. Figure 10 indicates that frequency will be more for increasing in taper angle along with the length.

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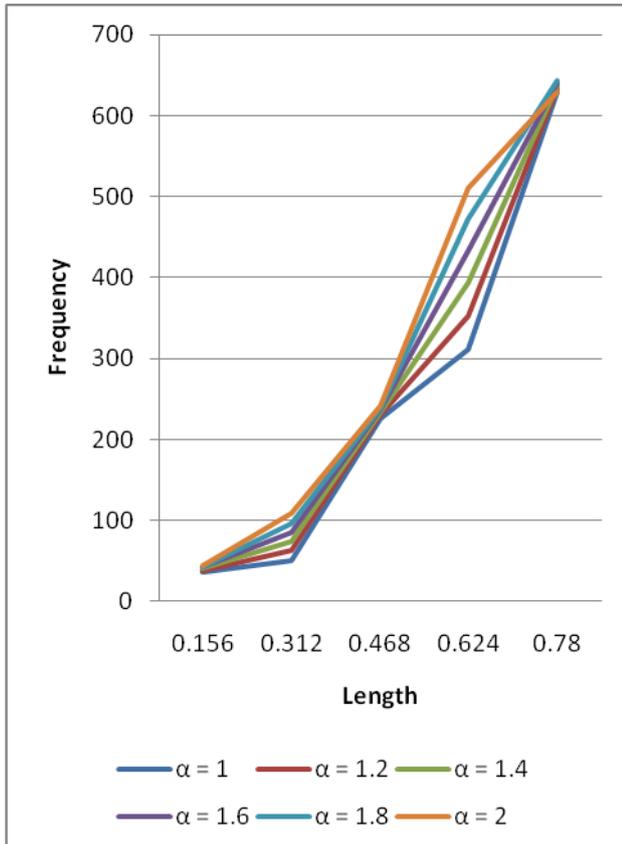


Figure-9 Length- Frequency chart for cantilever beam at different taper angle in model analysis.

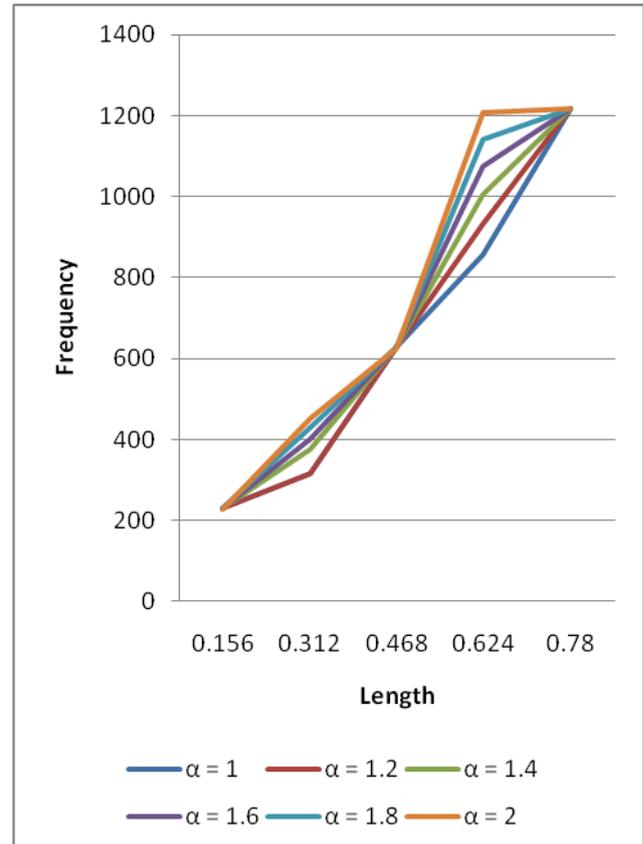


Figure-10 Length- Frequency chart for simply supported beam at different taper angle in theoretical analysis.

### III.CONCLUSION

- 1.This research work presents a simple procedure to obtain the frequency tapered beam of rectangular cross section.
- 2.The value of the natural frequency converges after dividing into smaller number of elements.
- 3.It has been observed that by increasing the taper ratio ' $\alpha$ ' the fundamental frequency increases.
- 4,Mode shapes for different taper ratio has been plotted.
- 5.Taper ratio ' $\alpha$ ' is more effective in decreasing the amplitude in vertical plane.
- 6.The above result is of great use for structural members where high strength to weight ratio is required

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- [10] P. Henri Gavin, "Structural element stiffness matrices and mass matrices," Department of Civil and Environmental Engineering CEE 541 Structural Dynamics, 2012. Fig. 2 Cantilever Beam of rectangular cross-section area with width and depth varying linearly.