

Logarithmic Corrections to (2+1)-dimensional Schwarzschild de Sitter Entropy

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ABSTRACT: Logarithmic corrections to thermodynamic entropy arise in all thermodynamic systems when small stable fluctuations around equilibrium are taken into account. The stability condition is equivalent to the specific heat being positive, so that the corresponding canonical ensemble is stable. I compute leading order corrections to the entropy of 3-dimensional Schwarzschild de-Sitter system due to small statistical fluctuations around equilibrium.

KEYWORDS: Schwarzschild, de Sitter, Entropy, Hawking.

1. INTRODUCTION

We begin with one of our favourite equations

$$S = \frac{A}{4G} \quad (1)$$

This is the Bekenstein-Hawking area-entropy law, which says that the entropy S associated with an event horizon is its area A divided by $4G$, where G is Newton's constant [1,2]. This is a macroscopic formula. It describes how properties of event horizons in general relativity change as their parameters are varied. This behaviour can be succinctly summarized by ascribing to them an entropy given by (1). One of the surprising and impressive features of this formula is its universality. It applies to all kinds of black holes with all kinds of charges, shapes and rotation, as well as to black strings and to all of the strange new objects we've found in string theory. It also applies to cosmological horizons, like the event horizon in de Sitter space [3].

In some ways cosmological horizons are much more puzzling than black hole horizons because in the black hole case one may expect that the black hole is a localized object with some quantum microstates. Then if you could provide the correct description of that localized object, you would be able to count those microstates and compare your result to the Bekenstein-Hawking formula and see that they agree. In some stringy cases this agreement has been achieved. On the other hand in de Sitter space the event horizon is observer dependent, and it is difficult even to see where the quantum microstates that we would like to count are supposed to be.

While the importance of understanding de Sitter quantum gravity has been evident for decades, it has recently been receiving more attention. One reason for this is the recent astronomical observations which indicate that the cosmological constant in our universe is positive [6-9]. A second reason is that recent progress in string theory and black holes provides new tools and suggests potentially fruitful new angles.

II. SCHWARZSCHILD DE-SITTER (SdS)

The simplest generalization of the de Sitter space solution is Schwarzschild-de Sitter, which we abbreviate as SdS[10]. This solution represents a black hole in de Sitter space. In d dimensions in static coordinates the SdS metric takes the form

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$$ds^2 = -\left(1 - \frac{2m}{r^{d-3}} - r^2\right) dt^2 + \frac{1}{1 - \frac{2m}{r^{d-3}} - r^2} dr^2 + r^2 d\Omega_{d-2}^2, \quad (2)$$

where m is a parameter related to the black hole mass (up to some d -dependent normalization constant). In general there are two horizons, one of which is the black hole horizon and the other of which is the de Sitter horizon. Note that the two horizons approach each other as m is increased, so that there is a maximum size black hole which can fit inside de Sitter space before the black hole horizon hits the de Sitter horizon.

It will be convenient to focus on the three dimensional Schwarzschild-de Sitter solution

$$ds^2 = -(1 - 8GE - r^2) dt^2 + \frac{1}{1 - 8GE - r^2} dr^2 + r^2 d\phi^2, \quad (3)$$

where we have normalized the energy E of the Schwarzschild black hole appropriately for three dimensions. In three dimensions there is only one horizon, at $r_H = \sqrt{1 - 8GE}$, and as E goes to zero this reduces to the usual horizon in empty de Sitter space. The fact that there is only a de Sitter horizon and not a black hole horizon is not surprising in light of the fact that in three dimensional flat space there are no black holes.

The temperature associated with the 3-dimensional Schwarzschild-de Sitter solution is

$$T_{SdS} = \frac{\sqrt{1 - 8GE}}{2\pi}. \quad (4)$$

Then we arrive at the entropy for Schwarzschild-de Sitter

$$S_{SdS} = \frac{A_H}{4G} = \frac{\pi}{2G} \sqrt{1 - 8GE}. \quad (5)$$

III. GENERAL LOGARITHMIC CORRECTIONS TO SCHWARZSCHILD dESITTER ENTROPY

Logarithmic corrections to thermodynamic entropy arise in all thermodynamic systems when small stable fluctuations around equilibrium are taken into account. The stability condition is equivalent to the specific heat being positive, so that the corresponding canonical ensemble is stable. The master formula of logarithmic corrections to black holes entropy as given by [11]

$$S = \ln \rho = S_0 - \frac{1}{2} \ln (CT^2) + \dots \quad (6)$$

Where ρ is the density of states, C is the specific heat and T is the temperature of black hole.

For dimensional correctness, it is understood that the quantity within the logarithm is divided by k_B^2 . This logarithmic correction to the entropy formula is applicable to all the objects having positive specific heat.

For 3-dimensional Schwarzschild de-Sitter (assuming SdS as a canonical ensemble) $T = TSdS$ and $S_0 = SSdS$. The first law in this case is:

$$dM = T_H dS_0 \quad (7)$$

M being mass of the SdS. Since there are no work terms, the specific heat in this case is simply a total derivative:

$$C = \frac{dM}{dT_H} = S_0 = \frac{G}{\pi^2} T \quad (8)$$

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Putting values in eqn. (6), we get

$$S = S_0 - \frac{1}{2} \ln(S_0 S_0^2) + \dots$$

$$= S_0 - \frac{3}{2} \ln(S_0) + \dots (9)$$

We have ignored the constant term because it is independent of black hole parameters and can be ignored for $S_0 \gg 1$. Equation (9) gives the leading order corrections to 3-dimensional SdS entropy.

IV. CONCLUSION

In the there have been many attempts to explain the black hole entropy and its corrections[12-15] but till now none have attempted to explain the corrections in de-Sitter entropy. In this paper, we assumed that aSdS is a thermodynamic system in equilibrium at Hawking temperature, and systematically analyzed its small statistical fluctuations around equilibrium. These fluctuations give rise to non-trivial multiplicative factor in front of the expression for the density of states, whose logarithm gives the corrected entropy formula.

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