

Extended Kalman Filter Modelling for Tracking Radar with Missing Measurements

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ABSTRACT: The theory of kinematic state estimation using Kalman Filter has gained wide popularity in the world of modern tracking systems. Here a first order Extended Kalman Filter is designed and applied for a tracking radar with missing measurements. There arise the problem of losing the target information due to a measurement miss which may occur owing to temporary sensor failures and network congestions. The missing measurements are modeled by a series of mutually independent random variables obeying Bernoulli distribution. The desired filter parameters are obtained by modifying the Riccati difference equations that are of a recursive form. It makes use of three target models viz., a slow moving balloon, a satellite trajectory and an accelerating launch vehicle for quantifying the filter performance for varied rates of state updation. The target dynamics and the measurement noise are accurately modelled to provide a general solution to the target tracking problem. The filter algorithm guarantees that the filter error covariance lies within the acceptable limit for targets of different dynamics. The filter results are compared to those obtained in a general filtering scenario where the filter is reinitialized for every measurement miss.

KEYWORDS: Kalman Filter, Extended Kalman Filter, missing measurements, Bernoulli distribution.

1. INTRODUCTION

Filtering has gained considerable importance in communication and engineering systems. A good filtering algorithm removes the noise from electromagnetic signals and retains the useful information. It is a recursive filter algorithm that minimizes the variance of the estimation error. Kalman filters are widely implemented in control systems in order to find an accurate estimate of the process variables. It is originally developed to be used in spacecraft navigation, but is found to be useful for many applications. It is mainly used to estimate system states that can only be observed indirectly or inaccurately by the system itself. The Kalman filter is a tool that is capable of estimating the control variables of a wide range of processes in linear systems. A linear system is simply a process which can be described by the following two equations:

State equation:

$$\mathbf{x}_{k+1} = \mathbf{A}\mathbf{x}_k + \mathbf{B}\mathbf{u}_k + \mathbf{w}_k \quad (1)$$

Output equation:

$$\mathbf{y}_k = \mathbf{C}\mathbf{x}_k + \mathbf{z}_k \quad (2)$$

In the above equations A, B, and C are matrices; k is the time index; x is called the state of the system; u is a known input to the system; y is the measured output; and w and z are the noise parameters. The variable w is called the process noise, and z is called the measurement noise. Each of these quantities is a vector and therefore contains more than one element. The vector x contains all of the information about the present state of the system, but we cannot measure x directly. Instead we measure y , which is a function of x that is corrupted by the noise z . We can use y to help us obtain an estimate of x , but we cannot necessarily take the information from y at face value because it is corrupted by noise.

But in most of the networks, the measurement goes on missing owing to sensor failures, packet dropouts, fading or congestions in the network. Bernoulli distribution can be used to model the measurement miss by assigning 1 for presence of signal and 0 for miss or noise. Most existing results concerning the measurement miss have concentrated

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 9, September 2015

on linear systems. Nonlinearity is a pervasive feature in almost all practical systems, and the occurrence of the nonlinearity necessarily cheapens the system performance and even leads to unreliability. Hence nonlinear filtering is required to solve the problem. It is there the use of EKF arises.

Paper is organized as follows. Section II describes the literature review undergone for understanding the earlier works. The basic Kalman Filter theory and algorithm is given in Section III. Section IV embodies the description of existing system and problems associated with it. Section V presents the proposed EKF modelling and it follows the simulation results in Section VI. Finally Section VII gives the conclusion and future scope of the proposed system.

II. RELATED WORK

The past few decades have seen a surge of research interest on the filtering or state estimation theories due to their extensive applications in a variety of practical areas including weather forecasting, economics, radar tracker and global positioning system [1]. According to [8], Kalman filtering is one of the most popular estimation approaches. This filtering approach assumes that both the state equation and output measurement are subjected to stationary Gaussian noises. The applications of the Kalman filtering theory may be found in a large spectrum of different fields ranging from various engineering problems to biology, geoscience, economics, and management, etc. If the error behavior of the discrete-time extended Kalman filter for nonlinear systems with intermittent observations is analyzed, modelling the arrival of the observations as a random process, given a certain regularity of the system, the estimation error remains bounded if the noise covariance and the initial estimation error are small enough [4]. Among the various filter algorithms, the traditional Kalman filter has been shown to be an optimal one in the sense of minimum variance for the linear systems, and the Extended Kalman Filter has been developed to serve as an effective way for handling the nonlinear estimation problems.

Most traditional filter design approaches rely on the assumption that the measurement signals are perfectly transmitted. Such an assumption, however, is conservative in many engineering practice presented with unreliable communication channels. For example, due to temporal sensor failures or network congestions, the system measurements may contain noise only at certain time points and the true signals are simply missing [1]. In general, two main approaches are popular for modeling the uncertainties. One is the stochastic parameter method. The second approach is to use a discrete-time linear system with Markovian jumping parameter to represent the random uncertainties. We shall refer to this approach as the Markov chain approach. A common way for modeling the data missing is to introduce a random variable satisfying the Bernoulli binary distribution taking values on either 1 or 0, where 1 is for the perfect signal delivery and 0 represents the measurement missing [2]. Most of the aforementioned results have been based on the hypothesis that all sensors have identical failure characteristics. However, such a hypothesis may not be true in the case that the signals are observed by multiple sensors and each individual sensor may have different failure rate.

Most existing results regarding the missing measurements have concentrated on linear systems. It is well known that the nonlinearity is a ubiquitous feature in almost all practical systems, and the occurrence of the nonlinearity inevitably degrades the system performance and even leads to instability [3]. During the last four decades, the Kalman filter and the EKF have been widely used for state estimation within communication and aerospace applications. It is well known that the optimality of the Kalman filter holds only when the system model is accurate. The Kalman Filter minimizes the mean-square prediction error when the random processes are Gaussian. The Kalman Filter can be formulated for target motion in one, two or three dimensions in polar, Cartesian or Earth-centered coordinates and for three-dimensional, two-dimensional or one-dimensional radar measurements [5]. Kalman filtering is so popular for its simplicity, tractability, optimality and robustness. The theory of kinematic state estimation using Kalman filter has gained wide popularity in the world of modern tracking systems [7].

When either the system state dynamics or the observation dynamics is nonlinear, the conditional probability density functions that provide the minimum mean-square estimate are no longer Gaussian. The optimal non-linear filter propagates these non-Gaussian functions and evaluates their mean, which represents a high computational burden. A non-optimal approach to solve the problem, in the frame of linear filters, is the Extended Kalman filter (EKF). The EKF implements a Kalman filter for a system dynamics that results from the linearization of the original non-linear

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 9, September 2015

filter dynamics around the previous state estimates. Radar can provide measurement of target position in range and in two angular coordinates, relative to the radar location. The angular coordinates are usually defined as elevation angle relative to the local horizontal, and azimuth, which may be measured relative to true north, or for phased arrays, relative to array broadside azimuth. Radar measurement data may be smoothed to increase the measurement accuracy, estimate target trajectory parameters, and predict future target position [6].

III. KALMAN FILTER THEORY AND ALGORITHM

Suppose we have a linear system model described by (1) and (2). We want to use the available measurements y to estimate the state of the system x . We know how the system behaves according to the state equation, and we have measurements of the position, so now we have to determine the best estimate of the state x . We want an estimator that gives an accurate estimate of the true state even though we cannot directly measure it. Our estimator should satisfy certain criteria.

There are mainly two requirements. First, we want the average value of the state estimate to be equal to the average value of the true state. That is, the estimate should not be biased. Mathematically, we would say that the expected value of the estimate should be equal to the expected value of the state. Second, we want a state estimate that varies from the true state as little as possible. That is, not only do we want the average of the state estimate to be equal to the average of the true state, but we also want an estimator that results in the smallest possible variation of the state estimate. Mathematically, the error variance of the estimate should be less. The Kalman filter is the estimator that satisfies these two criteria. But the Kalman filter solution does not apply unless we can satisfy certain assumptions about the noise that affects our system. In the system model, w is the process noise and z is the measurement noise. We assume that the average value of w and z is zero and also assume that there is no correlation between w and z . i.e., at any time k , w_k and z_k are independent random variables. Then the noise covariance matrices S_w and S_z are defined as:

Process noise covariance:

$$S_w = E(w_k w_k^T) \quad (3)$$

Measurement noise covariance:

$$S_z = E(z_k z_k^T) \quad (4)$$

where w^T and z^T indicate the transpose of the w and z random noise vectors, and $E(\cdot)$ means the expected value. So the Kalman Filter equations can be depicted in general as:

$$K_k = A P_k C^T (C P_k C^T + S_z)^{-1} \quad (5)$$

which gives the Kalman gain at k^{th} instant.

$$\hat{x}_{k+1} = A \hat{x}_k + B u_k + K_k (y_{k+1} - C \hat{x}_k) \quad (6)$$

is the estimate of x at $(k+1)^{\text{st}}$ instant.

$$P_{k+1} = A P_k A^T + S_w - A P_k C^T S_z^{-1} C P_k A^T \quad (7)$$

gives the estimation error covariance.

Equation (5) shows that if the measurement noise is large, S_z will be large, so gain K will be small and much credibility is given to the measurement y when computing the next. But if the measurement noise is small, S_z will be small, so K will be large and a lot of credibility is given to the measurement when computing the next [6].

IV. EXISTING SYSTEM DESCRIPTION AND PROBLEM FORMULATION

A. Existing System Description

The existing system presents a recursive finite-horizon filter design with modification of Riccati equations to incorporate measurement missing owing to temporary sensor failures, network congestions etc. Data missing is modeled by introducing a series of mutually independent random variables satisfying the Bernoulli binary distribution taking values on either 1 or 0, 1 for perfect signal delivery and 0 for a miss. The finite-horizon filter is designed such that an upper bound on the filtering error covariance is guaranteed and such an upper bound is subsequently minimized by the designed filter at each sampling instant. The proposed filter scheme is given in terms of the solutions to two Riccati-like difference equations, and therefore the algorithm is suitable for recursive computations.

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 9, September 2015

Consider the following class of time-varying nonlinear systems given by the Riccati equations [1]:

$$x_{k+1} = f(x_k) + w_k \quad (8)$$

$$y_k = \bar{E}_k C_k x_k + v_k \quad (9)$$

where x_k is the system state to be estimated, w_k is the process noise, y_k is the measurement sample, C_k is the transformation matrix, \bar{E}_k stands for missing measurement and v_k is the measurement noise. In the model (9), $C_k x_k$ represents the measurement output subject to probabilistic information loss characterized by the matrix \bar{E}_k , and v_k is the random exogenous noise acting on the measurement output. In other words, the model (9) is inclusive of all the practical cases of probabilistic missing measurements and external additive disturbances, thereby reflecting the engineering practice in a more realistic way. Simulation results of the existing system have shown that the MSE for the states stay below their upper bounds and the existing scheme can perform well to estimate the system states.

B. Problem Formulation

In the existing system, the missing measurements are modeled by Bernoulli binary distribution. There is no mention about the extent to which missing of measurements can be continuous. The variation of the upper bound of estimation error covariance when consecutive samples miss is not picturised. The effectiveness of the filtering method for such a scenario need to be studied since there arise practical situations where a sensor temporarily fails for a short duration without a redundant measurement provision. It does not discuss the effect of rate of update of state on the upper error bound guaranteed. The Kalman filter is a recursive estimator. This means that only the estimated state from the previous time step and the current measurement are needed to compute the estimate for the current state. The rate of change of its state variable matters, when the filter is put for estimating the parameters of a moving object. If the state dynamics of the object is more, the filter errors build up. This is a general case for filters designed to estimate the state of moving objects in space with definite velocity and acceleration. In such cases, even a single measurement miss can be fatal. The existing system does not refer to this problem. The finite-horizon filter in the existing system is designed such that an upper bound on the filtering error covariance is guaranteed and such an upper bound is subsequently minimized by the designed filter at each sampling instant. But it does not disclose any idea about the sampling rate chosen for each state update. The variation of filtering error with the increase in sample rate has to be noted.

V. PROPOSED EKF MODELLING FOR TRACKING RADAR WITH MISSING MEASUREMENTS

The radar measures range, azimuth and elevation in which range is a linear parameter and the other two are bearings. In case of a linear Kalman Filter these three measurements cannot be combined to form a single state vector. It may require running three separate filters. In such a case, an erroneous reading in any one parameter result in divergence of the corresponding filter alone and a combined inference is difficult. Hence an EKF can prove effective in such a situation where state vector is formed in Cartesian system and the residuals are computed in polar domain. Here it requires converting the state variables between Cartesian and polar system. Target motion is described by:

$$X(t_{k+1}) = \varphi(t_k) X(t_k) + A(t_k) + A_p(t_k) \quad (10)$$

where $X(t_k)$ is the target state at time t_k consisting of position and velocity components and $\varphi(t_k)$ is a transition matrix that moves the target linearly over an elapsed time, $T_k = t_{k+1} - t_k$, from time t_k to time t_{k+1} ; $A(t_k)$ is the target state change due to an unknown acceleration caused by a maneuver or atmospheric drag; and $A_p(t_k)$ is the target state change due to a known acceleration that can be corrected, such as gravity for a falling object or Coriolis acceleration.

A. EKF modelling without measurement miss

The Kalman filter update procedure continues as follows [5]:

First, predict a new target state estimate $\hat{X}(t_{k+1}|t_k)$ of the state $X(t_{k+1})$ at time t_{k+1} given all measurements upto time t_k .

$$\hat{X}(t_{k+1}|t_k) = \varphi(t_k) X(t_k) + A_p(t_k) \quad (11)$$

along with its covariance

$$P(k+1|k) = \varphi(t_k) P(k|k) \varphi(t_k)^T + Q(t_k) \quad (12)$$

Then update the target state using $(k+1)^{st}$ radar measurement

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 9, September 2015

$$\hat{X}(t_{k+1} | t_{k+1}) = \hat{X}(t_{k+1} | t_k) + K_k [Y_{k+1} - H(t_{k+1})\hat{X}(t_{k+1} | t_k)] \quad (13)$$

and its covariance

$$P(k+1 | k+1) = [I - K_{k+1} H(t_{k+1})]P(k+1 | k) \quad (14)$$

using the Kalman gains

$$K_{k+1} = P(k+1 | k)H^T(t_{k+1}) [H(t_{k+1})P(k+1 | k)H^T(t_{k+1}) + \mathfrak{R}_k]^{-1} \quad (15)$$

B.EKF modeling with measurement miss

The Riccati difference equations (13)-(15) are modified to incorporate measurement miss. It is done by using the Bernoulli distribution model with 0 indicating a miss and 1 indicating a perfect signal delivery. The modified Riccati equations are given as:

1. Target updation :

$$\hat{X}(t_{k+1} | t_{k+1}) = \hat{X}(t_{k+1} | t_k) + K_k [Y_{k+1} - (BD)H(t_{k+1})\hat{X}(t_{k+1} | t_k)] \quad (16)$$

2. Covariance calculation:

$$P(k+1 | k+1) = ([I - K_{k+1}(BD)H(t_{k+1})]P(k+1 | k)[I - K_{k+1}(BD)H(t_{k+1})]^T + K_{k+1}(BD)(J + \mathfrak{R})K_{k+1}^T) \quad (17)$$

where

$$J = BD (H(t_{k+1}) I H^T(t_{k+1}))$$

3. Kalman gain computation

$$K_{k+1} = P(k+1 | k)H^T(t_{k+1})(BD)[K_a + K_b + \mathfrak{R}_k]^{-1} \quad (18)$$

$$K_a = (BD)H(t_{k+1})P(k+1 | k)H^T(t_{k+1})(BD)$$

$$K_b = (BD)H(t_{k+1})P(k+1 | k)H^T(t_{k+1})$$

In (16) – (18), BD stands for Bernoulli distribution matrix which is a 3X3 matrix in this case.

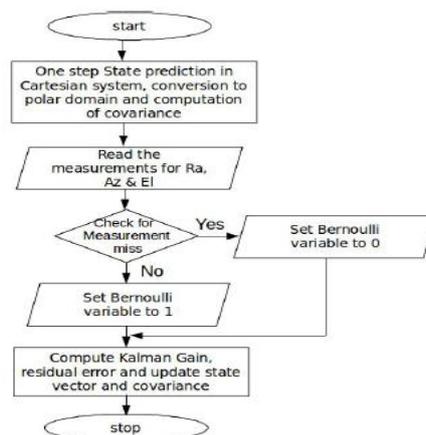


Fig. 1. Flow diagram of EKF model

VI. SIMULATION RESULTS

The EKF model has been simulated using MATLAB and the filter performance has been validated using three different target trajectories namely that of a slow moving balloon, a satellite and an accelerating launch vehicle. The filter has been run for normal trajectories without measurement miss and the first difference or noise plots show that the filter is converging for all the three targets. The measured and filtered noise corresponds to range, azimuth and elevation parameters. The sampling rate used for simulation is 10Hz. As we move on from a balloon to a launch vehicle, the target rate increases.

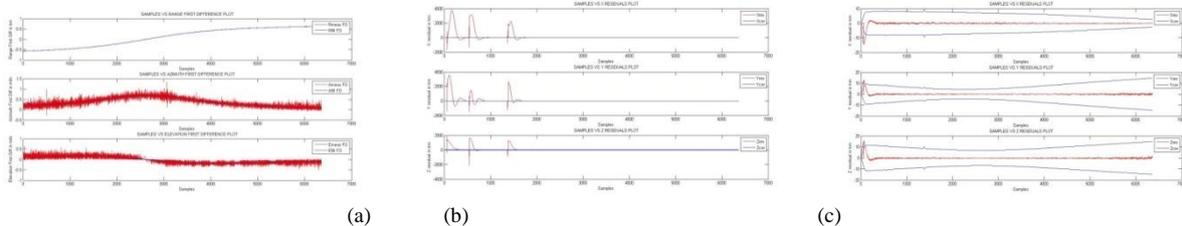


Fig.2.Noise plot for satellite trajectory (a) without measurement miss (b) with measurement miss (c) with measurement miss modeled by Bernoulli distribution .

The figure above shows residuals shooting up during sample miss and goes beyond the acceptable covariance limits. The residual errors are brought to safe limits by using Bernoulli modeling for missing measurements. The case of missing measurement is simulated by replacing a random number of samples from the trajectory data file with zeroes. A measurement miss is characterized by the value 0 and a good, reliable measurement is represented with value 1 in Bernoulli distribution. Hence for perfect measurements, the residual remains a nonzero value which is then used along with the Kalman Gain to smoothen the predicted state vector. For a missing measurement, the residual becomes zero and hence the filter goes in a prediction mode. This prevents sudden rising of filter residuals thereby checking the filter from deviation.

It could be noted that irrespective of the target dynamics, the filter is stable and the filter model holds good for measurement miss. For a satellite, initially, the residuals are high due to its higher dynamics. Also, it can be understood that for an accelerating launch vehicle, the residuals as well as the covariance limits increase as time proceeds.

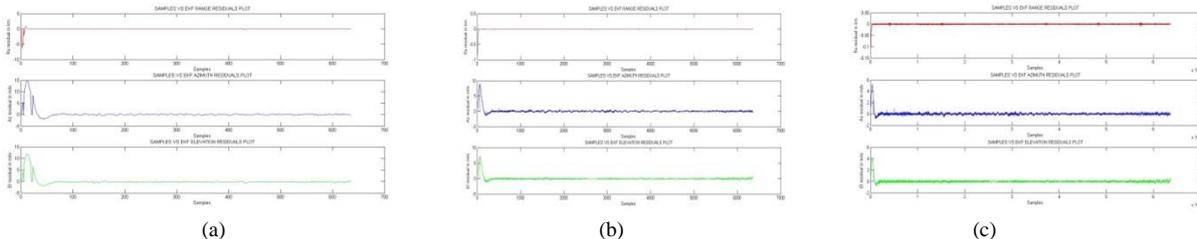


Fig. 5. Range, Azimuth and Elevation residuals of satellite data for (a) 1Hz (b) 10 Hz (c) 100 Hz .

The performance of the filter for different sampling rates is studied by using 1Hz, 10 Hz and 100 Hz data for satellite trajectory. From Fig.5, it is evident that as the sampling rate increases, the filter residuals at the initial stage come down. This is because of the higher dynamics of satellite trajectory. Once the filter is run, it takes some samples depending on the target dynamics to get stabilized. Till then the residual errors will be more which can be minimized by using higher sample rates. Otherwise, the Kalman filter should be initialized with some other batch filters like GHK. For the satellite data used in simulation, the initial errors build up to 5 mils in Azimuth (Az) and 4 mils in Elevation (El) for 100Hz, 8 mils in Az and 6 mils in El for 10 Hz and 15 mils in Az and 12 mils in El for 1 Hz data. For radar with 1 degree (18 mils) beamwidth, if the beam is positioned using the predicted data from filter, the target moves out of the beam once the angular errors exceed 9 mils.

VII. CONCLUSION AND FUTURE SCOPE

Missing measurements are quite common in modern communication systems employing multiple sensors and overcrowded networks due to network congestions, single point failures, dropouts, etc. An EKF model that takes care of missing measurements is discussed and simulated by projecting its application for a tracking radar. The missing measurements are modeled by a series of mutually independent random variables obeying Bernoulli distributions and the filter performance has been validated for consecutive sample misses for balloon, satellite and launch vehicle. The filter residuals are well within the error covariance limits during sample miss. The initial residuals are more as the filter being a recursive one, requires time to settle down. The simulation results for satellite data at three sampling rates show that irrespective of the target dynamics and sampling rate, the filter errors are within acceptable levels. The

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 4, Issue 9, September 2015

modifications incorporated in the filter design for modelling the measurement miss eliminate the need for keeping separate routines for state prediction and state updation. Also, the need for reinitializing the filter during sample miss could be avoided to some extent.

The design of Kalman filter is limited to first order because of the higher order of matrices involved which leads to computational overhead. Multiplication and inverse operations of matrices with higher order consumes processor time. The realization of a second order filter in order to get better filtering can be taken up as a future improvement of the work. Also, the filter model can be used for taking care of incomplete or corrupted measurements like wild samples. When there is something other than a measurement miss like a sample which is slightly different from what is expected, the filter can be used to reduce the residuals by using a value between 0 and 1 for the Bernoulli distribution. Similarly, there arises the case of getting more than one measurement samples for a tracking radar when an object other than the desirable one comes in the beamwidth. The filter prediction can be used to select and proceed with the correct measurement.

REFERENCES

1. Jun Hu; Zidong Wang; Bo Shen; Chenxiao Cai; Lam, J., "Recursive filtering for a class of nonlinear systems with missing measurements", UKACC International Conference on Control (CONTROL), 2012, vol 40., no. 7, pp.929-934, 3-5 Sept. 2012.
2. Moayed, M.; Foo, Y.K.; Soh, Y.C., "Adaptive Kalman Filtering in Networked Systems With Random Sensor Delays, Multiple Packet Dropouts and Missing Measurements", IEEE Transactions on Signal Processing, vol.58, no.3, pp.1577-1588, March 2010.
3. Xiong Kai; Chunling Wei; Liangdong Liu, "Robust Extended Kalman Filtering for Nonlinear Systems With Stochastic Uncertainties", IEEE Transactions on Systems, Man and Cybernetics, Part A: Systems and Humans, , vol.40, no.2, pp.399-405, March 2010.
4. Kluge, S.; Reif, K.; Brokate, M., "Stochastic Stability of the Extended Kalman Filter With Intermittent Observations", IEEE Transactions on Automatic Control, vol.55, no.2, pp.514-518, Feb. 2010
5. Merrill I. Skolnik, Radar Handbook, 3rd edition, 1 Jan 2008
6. Maria Isabel Ribeiro, "Kalman and Extended Kalman Filters: Concept, Derivation and Properties," Institute for Systems and Robotics, Feb. 2004
7. Simon, Dan. "Kalman filtering." Embedded systems programming pp. 72-79, 14 June 2001.
8. Peng Shi; Boukas, E.-K.; Agarwal, R.K., "Kalman filtering for continuous-time uncertain systems with Markovian jumping parameters", IEEE Transactions on Automatic Control, vol.44, no.8, pp.1592-1597, Aug 1999.
9. Samuel Blackman, Robert Popoli, Design and Analysis of Modern Tracking Systems, 30 Jun 1999
10. Barton, D. K., and H. R. Ward, Handbook of Radar Measurements, Dedham, MA: Artech House, 1984.