

On T_c Sets in Fuzzy Topological Spaces

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ABSTRACT: This paper deals with a new class of sets namely, T_c - fuzzy sets in fuzzy topological spaces (via) θ - sets. Some characterizations and properties of these sets are investigated. Further, we introduce the concept of T_c - fuzzy continuous and T_c - fuzzy irresolute mappings.

KEYWORDS: T_c - fuzzy set, T_c - fuzzy continuous mappings, T_c - fuzzy irresolute mappings.

I. INTRODUCTION

In 1970, Levine [10] introduced the notion of generalized closed sets in topological spaces as a generalization of closed sets. Since then, many concepts related to generalized closed sets were defined and investigated. The concepts θ - closure and θ - interior were first introduced by Velicko [16] for the purpose of studying the important class of H -closed spaces. I. Arockiarani & D. Sasikala [2] introduced the concept of T_c -set in topological spaces. Balasubramanian and Sundaram [2] introduced the concept of generalized closed sets in fuzzy setting. Also, they studied various generalizations fuzzy continuous mappings. El-Shafei and Zakari [12] introduced new types of generalized closed fuzzy continuous mappings. In this present paper, we introduce the concept of T_c - fuzzy set and study some of their properties. Moreover, we use the new class of fuzzy set to construct new types of fuzzy mappings.

II. PRELIMINARIES

Let X be any set and I be the closed unit interval $[0,1]$. A fuzzy set in X is an element of the set of all functions from X into I . The family of all fuzzy sets in X is denoted by I^X . A fuzzy singleton x_t is a fuzzy set in X defined by $x_t(x) = t$, $x_t(y) = 0$ for all $y \neq x$, $t \in (0, 1]$. The set of all fuzzy singletons in X is denoted by $S(X)$. For every $x_t \in S(X)$ and $\mu \in I^X$, $x_t \in \mu$ iff $t \leq \mu(x)$. For two fuzzy sets μ and ν , we shall write $\mu q \nu$ to mean that μ is quasi - coincident with ν . If μ is not quasi - coincident with ν , then we write $\mu \not q \nu$. For $\mu \in I^X$, the closure, interior, and complement of μ are denoted by $cl(\mu)$, $int(\mu)$, and μ^c , respectively. The θ - closure and θ - interior of a fuzzy set μ , denoted by $cl_\theta(\mu)$ and $int_\theta(\mu)$ respectively, are defined by: $x_t q cl_\theta(\mu) \Leftrightarrow cl(\eta) q \mu$ for each $\eta \in N(x_t)$ and $x_t \in int_\theta(\mu) \Leftrightarrow cl(\eta) \leq \mu$ for some $\eta \in N(x_t)$, where $N(x_t)$ is the family of all open neighborhoods of x_t .

The fuzzy subset μ is called θ - open (θ - closed) if $\mu = int_\theta(\mu)$ ($\mu = cl_\theta(\mu)$). The complement of a θ - open fuzzy set is closed.

Lemma 2.01. [6] Let (X, τ) be a fuzzy topological space (fts for short) and $\mu \in I^X$. Then

- (i) $cl(\mu) \leq cl_\theta(\mu)$.
- (ii) If $\mu \leq \nu$, then $cl_\theta(\mu) \leq cl_\theta(\nu)$.
- (iii) $cl_\theta(\mu) = \bigwedge \{ cl(\eta) : \mu \leq \eta, \eta \in \tau \}$.

Definition 2.02 [4][11][13] A fuzzy subset μ of a fts (X, τ) is called:

- (i) α - open iff $\mu \leq int(cl(int(\mu)))$.
- (ii) α - closed iff $cl(int(cl(\mu))) \leq \mu$.
- (iii) Preopen iff $\mu \leq int(cl(\mu))$.
- (iv) Preclosed iff $cl(int(\mu)) \leq \mu$.

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(v) β - open iff $\mu \leq \text{cl}(\text{int}(\text{cl}(\mu)))$.

The α - closure of $\mu \in I^X$, denoted by $\text{cl}_\alpha(\mu)$, is the smallest α - closed fuzzy set containing μ .

Definition 2.03 [2]

- (1) A fuzzy subset μ of a fts (X, τ) is called a generalized closed (g - closed, for short) fuzzy set if $\mu \leq \eta$ and $\eta \in \tau$ implies that $\text{cl}(\mu) \leq \eta$.
- (2) A fts (X, τ) is called $\text{FT}_{1/2}$ - space iff every g -closed fuzzy set is closed.
- (3) A fuzzy subset μ of a fts (X, τ) is called a θ - generalized closed (θ -g - closed, for short) fuzzy set if $\mu \leq \eta$ and $\eta \in \tau$ implies that $\text{cl}_\theta(\mu) \leq \eta$.

Definition 2.04 [7]. A fts (X, τ) is called

- (1) FR_0 - space iff $x_t q \text{cl}(y_t)$ implies that $y_t q \text{cl}(x_t)$.
- (2) FR_1 - space iff $x_t q \text{cl}(y_t)$ implies there exist $\eta \in N(x_t)$ and $v \in N(y_t)$ such that $\eta q v$.
- (3) FR_2 - space (Fuzzy Regular Space) iff $x_t q \lambda$, λ is closed fuzzy set, implies that $\eta \in N(x_t)$ and $v \in \tau$, $\lambda \leq v$ such that $\eta q v$.
- (4) FT_1 - space iff $x_t q (y_t)$ implies that $x_t q \text{cl}(y_t)$ and $y_t q \text{cl}(x_t)$.
- (5) FT_2 - space (Fuzzy Hausdroff space) iff $x_t q (y_t)$ implies that there exist $\eta \in N(x_t)$, $v \in N(y_t)$ such that $\eta q v$.

Proposition 2.05 [6,8]. Let (X, τ) be a fts. Then:

- (i) (X, τ) is FT_1 - space iff $\text{cl}(x_t) = \text{cl}_\theta(x_t)$ for every $x_t \in S(X)$.
- (ii) (X, τ) is FT_2 - space iff $x_t = \text{cl}_\theta(x_t)$, for each $x_t \in S(X)$.

Definition 2.06 [6]. Let (X, τ) be a fts and $\mu \in I^X$. Then:

- (1) The family $\gamma = \{\eta_j : j \in J\} \subset \tau$ is called an open P - cover of μ iff for every $x_t \in \mu$ there exist $j_0 \in J$ such that $x_t \in \eta_{j_0}$.
- (2) μ is called a C - set iff every open P - cover of μ has a finite sub cover.

It is obvious that every finite fuzzy set is a C set.

Proposition 2.07 [6,8]. Let (X, τ) be a FT_2 - space, Then:

- (i) $x_t = \bigwedge \{ \text{cl}(\eta) : \eta \in N(x_t) \}$.
- (ii) Every C - set in X is θ - closed.

Definition 2.08 [2]. A mapping $f: (X, \tau) \rightarrow (Y, \delta)$ is called f-g-continuous if $f^{-1}(\lambda)$ is g - closed in (X, τ) for every closed fuzzy set in (Y, δ) .

Definition 2.09 [9]. A mapping $f: (X, \tau) \rightarrow (Y, \delta)$ is called f- θ -continuous iff for each $x_t \in S(X)$ and for each $v \in N(f(x_t))$ there exist $\eta \in N(x_t)$ such that $f(\text{cl}(\eta)) \leq \text{cl}(v)$.

Definition 2.09 [6]. A mapping $f: (X, \tau) \rightarrow (Y, \delta)$ is called f-strongly θ -continuous iff for each $x_t \in S(X)$ and for each $v \in N(f(x_t))$ there exist $\eta \in N(x_t)$ such that $f(\text{cl}(\eta)) \leq v$.

III. Tc SETS IN FUZZY TOPOLOGICAL SPACES

Definition 3.1. A fuzzy subset μ of a fuzzy topological space (X, τ) is called a Tc fuzzy set if $\text{Cl}_\theta(\mu) \leq \eta$, whenever $\mu \leq \eta$ and η is β - open in (X, τ) . The complement of a Tc - fuzzy set is called Tc - fuzzy open.

Proposition 3.2. Every fuzzy θ - closed set is a Tc - set.

Proof: Obvious

Proposition 3.3. Every Tc - fuzzy set is θ - g- closed fuzzy set.

Proof: Let η be an open set such that $\mu \leq \eta$. It implies $\text{cl}_\theta(\mu) \leq \eta$. Therefore μ is θ - g-closed fuzzy set.

Proposition 3.4. Every Tc - fuzzy set is g-closed

Proof: Let η be an open set such that $\mu \leq \eta$. It implies $\text{cl}(\mu) \leq \text{cl}_\theta(\mu) \leq \eta$ (i.e.) $\text{cl}(\mu) \leq \eta$. Therefore μ is g-closed set.

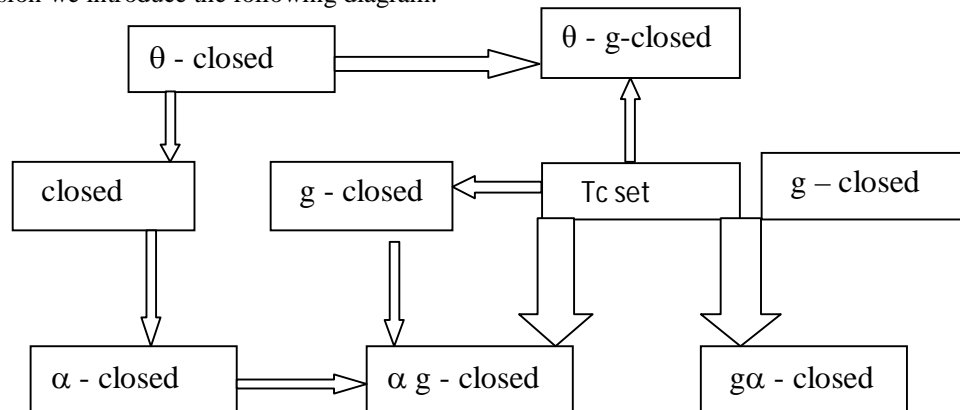
Proposition 3.5. Every Tc - fuzzy set is α g - closed and $g\alpha$ - closed.

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From the above discussion we introduce the following diagram:



Proposition 3.6. A fuzzy subset μ of an FR_2 - fts (X, τ) is a Tc - set iff μ is g - closed.

Proof: Suppose that (X, τ) is FR_2 - fts, $\mu \in I^X$. It suffices to prove that $cl(\mu) = cl_\theta(\mu)$. Let $x_t \in cl(\mu)$. Since (X, τ) is FR_2 and $cl(\mu)$ is closed, then there exist two open fuzzy sets η, ν such that $\eta \in N(x_t)$, $cl(\mu) \leq \nu$ and $\eta \cap \nu = \emptyset$. Then $\eta \leq \nu^c$ and hence $cl(\eta) \leq \nu^c$. Thus there exists $\eta \in N(x_t)$ such that $cl(\eta) \not\leq \mu$ and so $x_t \notin cl_\theta(\mu)$. Therefore $cl_\theta(\mu) \leq cl(\mu)$. From lemma 2.1 (i), we have $cl(\mu) = cl_\theta(\mu)$.

Lemma 3.7. If μ is a preopen fuzzy subset of a fts (X, τ) , then

- (i) $cl_\alpha(\mu) = cl(\mu)$
- (ii) $cl_\theta(\mu) = cl(\mu)$

Proof: (i) If μ is a preopen fuzzy set, then μ is closed iff μ is α - closed and so $cl_\alpha(\mu) = cl(\mu)$.

(ii) By Lemma 2.1 (i), it suffices to show that $cl_\theta(\mu) = cl(\mu)$. Let $x_t \in cl(\mu)$, then there exist $\nu \in N(x_t)$ such that $\nu \cap \mu = \emptyset$, i. e, there exists $\nu \in N(x_t)$ such that $\nu \leq \mu^c$. But μ is preopen, so $cl(\nu) = cl(int(\nu)) \leq cl(int(\mu^c)) \leq \mu^c$. This implies that there exists $\nu \in N(x_t)$ such that $cl(\nu) \cap \mu = \emptyset$. Then $x_t \notin cl_\theta(\mu)$. Therefore $cl_\theta(\mu) = cl(\mu)$.

Proposition 3.8. Let μ be a preopen fuzzy subset of a fts (X, τ) , then the following are equivalent:

- (i) μ is a Tc set
- (ii) μ is g - closed
- (iii) μ is αg - closed

Lemma 3.9 : If μ and η are fuzzy subsets of a fts (X, τ) , then

- (1) $cl_\theta(\mu \vee \eta) = cl(\mu) \vee cl_\theta(\eta)$.
- (2) $cl_\theta(\mu \wedge \eta) = cl(\mu) \wedge cl_\theta(\eta)$.

Proof: Obvious

Proposition 3.10. A finite union of is always a Tc fuzzy set.

Proof: Suppose that $\mu, \eta \in I^X$ are Tc - fuzzy sets and let ν be a β - open set such that $\mu \vee \eta \leq \nu$.

Since μ and η are Tc- fuzzy sets, then we have $cl(\mu) \vee cl_\theta(\eta) \leq \nu$ and by lemma 3.8, $cl_\theta(\mu \vee \eta) \leq \nu$. Hence $\mu \vee \eta$ is a Tc - fuzzy set.

The following example shows that the finite intersection of Tc - fuzzy sets may fail to be Tc - fuzzy set.

Example 3.11. Let $X = \{x, y\}$ and $\tau = \{0_x, x_{1/4}, y_{3/4}, x_{1/4} \vee y_{3/4}, 1_x\}$. If $\mu = x_{1/2} \vee y_{1/4}$, $\eta = x_{1/4} \vee y_1$, then μ and η are Tc - fuzzy sets, since the only open superset of μ and η is 1_x . But $\mu \wedge \eta = x_{1/4} \vee y_{1/4}$ is not a Tc-set.

Proposition 3.12. If a fts (X, τ) is $FT_{1/2}$, then every Tc - fuzzy set is closed.

Proof: Since every Tc - fuzzy set is g - closed, then by $FT_{1/2}$ - ness the result is given.

IV. Tc – CONTINUOUS IN FUZZY TOPOLOGICAL SPACES

Definition 4.01. A mapping $f: (X, \tau) \rightarrow (Y, \delta)$ is called Tc - fuzzy continuous if the inverse image of a closed fuzzy set in (Y, δ) is Tc - fuzzy set in (X, τ) .

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Proposition 4.02. Every T_c – fuzzy continuous mapping is fuzzy g – continuous.

Proof: Obvious.

Proposition 4.03. Every θ - g - closed fuzzy mapping is T_c – fuzzy continuous mapping

The following example shows that the converse of the above theorem is not true.

Example 4.04. Let $X = \{x\}$, $\tau = \{0_x, x_{2/3}, x_{3/4}, 1_x\}$ and $\delta = \{0_x, x_{3/4}, 1_x\}$. Let $f: (X, \tau) \rightarrow (Y, \delta)$ be the identity fuzzy mapping, it is easy to show that f is f – continuous and hence f - g - continuous but not T_c – fuzzy continuous.

Proposition 4.05. Every f – strongly θ - continuous mapping is T_c – fuzzy continuous.

Proof: Let $f: (X, \tau) \rightarrow (Y, \delta)$ be f – strongly θ - continuous mapping. Let λ be a closed fuzzy set in Y and let $f^{-1}(\lambda) \leq \eta$, where η is an β - open fuzzy set in X . Now, let $x_t \in \eta$. Then $x_t \in f^{-1}(\lambda)$ and so $f(x_t) \in \lambda^c$, λ^c is open fuzzy set in Y .

Proposition 4.06. The composition of two T_c – fuzzy continuous mappings does not give a T_c – fuzzy continuous mapping.

Definition 4.07. A mapping $f: (X, \tau) \rightarrow (Y, \delta)$ is called T_c – fuzzy- irresolute if $f^{-1}(\lambda)$ is a T_c – fuzzy set of (X, τ) for every T_c –fuzzy set λ of (Y, δ) .

Clearly, every T_c – fuzzy irresolute map is T_c fuzzy - continuous.

Theorem 4.08: Let $f: (X, \tau) \rightarrow (Y, \delta)$ be bijective, f – open and T_c fuzzy - continuous. Then f is T_c – fuzzy irresolute.

Proof: Let λ be a T_c – fuzzy set in Y and let $f^{-1}(\lambda) \leq \eta$. Clearly $\lambda \leq f(\eta)$. Since f is f – open, then $f(\eta) \in \delta$. Since λ is a T_c – fuzzy set in Y , then $cl_0(\lambda) \leq f(\eta)$ and thus $f^{-1}(cl_0(\lambda)) \leq \eta$. Since f is T_c fuzzy – continuous and since $cl_0(\lambda)$ is closed in Y , then $cl_0(f^{-1}(cl_0(\lambda))) \leq \eta$ and hence $cl_0(\lambda) \leq \eta$. Therefore $f^{-1}(\lambda)$ is T_c – fuzzy set in X . Hence f is T_c – fuzzy irresolute.

V. CONCLUSION

We conclude this paper by looking briefly on the basic concept of T_c fuzzy set, T_c – fuzzy continuous, T_c – fuzzy irresolute in fuzzy topological spaces. We have illustrated some examples and proved some interesting results. These concepts may have extensive applicational values.

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