

# On Neighborly Pseudo and Strongly Pseudo Irregular Bipolar Fuzzy graphs

Dr. N.R. Santhi Maheswari<sup>1</sup>, Dr. C. Sekar<sup>2</sup>

Department of Mathematics, G. Venkataswamy Naidu College, Kovilpatti, Tamil Nadu, India<sup>1</sup>

P.G. Extension Centre, Manonmaniam Sundranar University, Nagercoil, Tamil Nadu, India<sup>2</sup>

**ABSTRACT:** In this paper, neighbourly pseudo irregular bipolar fuzzy graphs, strongly pseudo irregular bipolar fuzzy graphs, neighbourly pseudo totally irregular bipolar fuzzy graphs and strongly pseudo totally irregular bipolar fuzzy graphs are introduced and some of their properties are discussed.

**KEYWORDS:** pseudo degree in bipolar fuzzy graph, total pseudo degree in bipolar fuzzy graph, pseudo regular bipolar fuzzy graph, pseudo irregular bipolar fuzzy graph, neighbourly pseudo irregular fuzzy graphs, strongly pseudo irregular fuzzy graphs.

**AMS Mathematics Subject Classification.** 05C12, 03E72, 05C72.

## I. INTRODUCTION

In this paper, we consider only finite, simple, connected graphs. We denote the vertex set and the edge set of a graph  $G$  by  $V(G)$  and  $E(G)$  respectively. The degree of a vertex  $v$  is the number of edges incident at  $v$ , and it is denoted by  $d(v)$ . A graph  $G$  is regular if all its vertices have the same degree. The 2-degree of  $v$  [4] is the sum of the degrees of the vertices adjacent to  $v$  and it is denoted by  $t(v)$ . We call  $\frac{t(v)}{d(v)}$ , the average degree of  $v$ . A graph is called pseudo-regular if every vertex of  $G$  has equal average degree [3].

Fuzzy set theory was first introduced by Zadeh in 1965 [17]. The first definition of fuzzy graph was introduced by Haufmann in 1973 based on Zadeh's fuzzy relations in 1971. In 1975, A. Rosenfeld introduced the concept of fuzzy graphs [8]. Now, fuzzy graphs have many applications in branches of engineering and technology.

Zhang initiated the concept of bipolar fuzzy sets as a generalization of fuzzy sets in 1994. Bipolar fuzzy sets whose range of membership degree is  $[-1,1]$ . In bipolar fuzzy sets, membership degree 0 of an element means that the element is irrelevant to the corresponding property, the membership degree within  $(0, 1]$  of an element indicates that the element somewhat satisfies the property, and the membership degree within  $[-1,0)$  of an element indicates the element somewhat satisfies the implicit counter property. It is noted that positive information represents what is granted to be possible, while negative information represents what is considered to be impossible [2].

## II. REVIEW OF LITERATURE

Nagoorgani and Radha introduced the concept of degree, total degree, regular fuzzy graphs in 2009 [5]. Nagoorgani and Latha introduced the concept of irregular fuzzy graphs, neighbourly irregular fuzzy graphs and highly irregular fuzzy graphs in 2012 [4]. Mathew, Sunitha and Anjali introduced some connectivity concepts in bipolar fuzzy graphs [19]. Akram and Dudek introduced the notions of regular bipolar fuzzy graphs and also introduced intuitionistic fuzzy graphs [1]. Samanta and Pal introduced the concept of irregular bipolar fuzzy graphs [18]. Madhumangal Pal and Hossein Rashmanlou introduced irregular interval-valued fuzzy graphs [21]. K. Radha and N. Kumaravel introduced the concept of an edge degree, total edge degree in bipolar fuzzy graphs and edge regular bipolar fuzzy graphs and discussed about the degree of an edge in some bipolar fuzzy graphs [7]. N.R.Santhi Maheswari and C.Sekar introduced strongly edge irregular bipolar fuzzy graphs and discussed its properties [10]. N.R.Santhi Maheswari and C.Sekar introduced neighbourly edge irregular bipolar fuzzy graphs and discussed some of its properties [9]. N. R. Santhi

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Maheswari and C. Sekar introduced an m-Neighbourly Irregular bipolar fuzzy graphs [12]. N.R.Santhi Maheswari and C.Sekar introduced pseudo regular fuzzy graphs and discussed its properties [14]. N.R.Santhi Maheswari and M.Sudha introduced pseudo irregular fuzzy graphs, highly pseudo irregular fuzzy graphs and discussed its properties [15]. N.R.Santhi Maheswari and M. Rajeswari introduced strongly pseudo irregular fuzzy graphs and discussed its properties [16]. N.R.Santhi Maheswari and C.Sekar introduced pseudo regular bipolar fuzzy graphs and pseudo irregular bipolar fuzzy graphs and discussed its properties[17].

These motivate us to introduce neighbourly pseudo irregular bipolar fuzzy graphs, strongly pseudo irregular bipolar fuzzy graphs and discussed some of their properties. Throughout this paper, the vertices take the membership value  $A = (m_1^+, m_1^-)$  and edges take the membership value  $B = (m_2^+, m_2^-)$ , where  $(m_1^+, m_2^+)$  in  $[0,1]$  and  $(m_1^-, m_2^-)$  in  $[-1,0]$ . These motivate us to introduce neighbourly pseudo irregular bipolar fuzzy graphs, strongly pseudo irregular bipolar fuzzy graphs and discussed some of their properties. Throughout this paper, the vertices take the membership value  $A = (m_1^+, m_1^-)$  and edges take the membership value  $B = (m_2^+, m_2^-)$ , where  $(m_1^+, m_2^+)$  in  $[0,1]$  and  $(m_1^-, m_2^-)$  in  $[-1,0]$ .

### III. PRELIMINARIES

We present some known definitions and results for ready reference to go through the work presented in this paper.

**Definition 3.1.** A Fuzzy graph denoted by  $G : (\sigma, \mu)$  on the graph  $G^* : (V,E)$ : is a pair of functions  $(\sigma, \mu)$  where  $\sigma : V \rightarrow [0; 1]$  is a fuzzy subset of a set  $V$  and  $\mu : V \times V \rightarrow [0; 1]$  is a symmetric fuzzy relation on  $\sigma$  such that for all  $u, v$  in  $V$  the relation  $\mu(u, v) = \mu(uv) \leq \sigma(u) \wedge \sigma(v)$  is satisfied[8].

**Definition 3.2.** Let  $G : (\sigma, \mu)$  be a fuzzy graph on  $G^* : (V, E)$ . The 2-degree of a vertex  $v$  in  $G$  is defined as the sum of degrees of the vertices adjacent to  $v$  and is denoted by  $t_G(v)$ . That is,  $t_G(v) = \sum d_G(u)$ , where  $d_G(u)$  is the degree of the vertex  $u$  which is adjacent with the vertex  $v$ [14]

**Definition 3.3.** Let  $G : (\sigma, \mu)$  be a fuzzy graph on  $G^* : (V, E)$ . A pseudo (average) degree of a vertex  $v$  in fuzzy graph  $G$  is denoted by  $d_a(v)$  and is defined by  $d_a(v) = \frac{t_G(v)}{d_G^*(v)}$ , where  $d_G^*(v)$  is the number of edges incident at  $v$ . The total pseudo degree of a vertex  $v$  in  $G$  is denoted by  $t d_a(v)$  and is defined as  $t d_a(v) = d_a(v) + \sigma(v)$ , for all  $v$  in  $G$ [14].

**Definition 3.4.** Let  $G : (\sigma, \mu)$  be a fuzzy graph on  $G^* : (V,E)$ . Then,  $G$  is said to be a neighbourly pseudo irregular fuzzy graph if every pair of adjacent vertices have distinct pseudo degrees.

**Definition 3.5.** Let  $G : (\sigma, \mu)$  be a fuzzy graph on  $G^* : (V,E)$ . Then,  $G$  is said to be a strongly pseudo irregular fuzzy graph if every pair of vertices have distinct pseudo degrees.

**Definition 3.6.** A bipolar fuzzy graph with an underlying set  $V$  is defined to be a pair  $(A, B)$ , where  $A = (m_1^+, m_1^-)$  is a bipolar fuzzy set on  $V$  and  $B = (m_2^+, m_2^-)$  is a bipolar fuzzy set on  $E$  such that  $m_2^+(x, y) \leq \min\{(m_1^+(x), m_1^+(y))\}$  and  $m_2^-(x, y) \geq \max\{(m_1^-(x), m_1^-(y))\}$  for all  $(x,y)$  in  $E$ . Here,  $A$  is called bipolar fuzzy vertex set on  $V$  and  $B$  is called bipolar fuzzy edge set on  $E$  [ ].

**Definition 3.7** Let  $G : (A, B)$  be a bipolar fuzzy graph on  $G^*(V, E)$ . The positive degree of a vertex  $u$  in  $G$  is defined as  $d^+(u) = \sum m_2^+(u,v)$ , for  $uv$  in  $E$ . The negative degree of a vertex  $u$  in  $G$  is defined as  $d^-(u) = \sum m_2^-(u,v)$ , for  $uv$  in  $E$  and  $\sum m_2^+(u,v) = \sum m_2^-(u,v) = 0$  if  $uv$  not in  $E$ . The degree of a vertex  $u$  is defined as  $d(u) = (d^+(u), d^-(u))$  [ ].

**Definition 3.8.** Let  $G : (A, B)$  be a bipolar fuzzy graph on  $G^*(V, E)$ . The positive total degree of a vertex  $u$  in  $G$  is defined as  $td^+(u) = \sum m_2^+(u,v) + m_1^+(u)$ , for  $uv$  in  $E$ . The negative degree of a vertex  $u$  in  $G$  is defined as  $td^-(u) = \sum m_2^-(u,v) + m_1^-(u)$ , for  $uv$  in  $E$  [ ].

**Definition 3.9.** Let  $G : (\sigma, \mu)$  be a fuzzy graph on  $G^* : (V, E)$ . The 2-degree of a vertex  $v$  in  $G$  is defined as the sum of the degrees of the vertices adjacent to  $v$  and is denoted by  $t_G(v) = (t_G^+(v), t_G^-(v))$ . That is, the positive 2-degree of  $v$  is  $t_G^+(v) = \sum d_G^+(u)$ , where  $d_G^+(u)$  is the positive degree of the vertex  $u$  which is adjacent with the vertex  $v$  and the negative 2-degree of  $v$  is  $t_G^-(v) = \sum d_G^-(u)$ , where  $d_G^-(u)$  is the negative degree of the vertex  $u$  which is adjacent with the vertex  $v$ .

**Definition 3.10** Let  $G : (A, B)$  be a bipolar fuzzy graph on  $G^*(V, E)$ . The positive pseudo (average) degree of a vertex  $v$  in bipolar fuzzy graph  $G$  is denoted by  $d_a^+(v)$  and is defined by  $d_a^+(v) = \frac{t_G^+(v)}{d_G^*(v)}$ , where  $d_G^*(v)$  is the number of edges incident at  $v$ . The negative pseudo (average) degree of a vertex  $v$  in bipolar fuzzy graph  $G$  is denoted by  $d_a^-(v)$  and is defined by  $d_a^-(v) = \frac{t_G^-(v)}{d_G^*(v)}$ , where  $d_G^*(v)$  is the number of edges incident at  $v$ . The pseudo degree of a vertex  $u$  in bipolar fuzzy graph  $G$  is defined as  $d_a(v) = (d_a^+(v), d_a^-(v))$ .

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**Definition 3.11** The positive total pseudo degree of a vertex  $v$  in  $G$  is denoted by  $t d_a^+(v)$  and is defined as  $t d_a^+(v) = d_a^+(v) + m_1^+(v)$ , for all  $v$  in  $G$ . The negative total pseudo degree of a vertex  $v$  in  $G$  is denoted by  $t d_a^-(v)$  and is defined as  $t d_a^-(v) = d_a^-(v) + m_1^-(v)$ , for all  $v$  in  $G$ . The total pseudo degree of a vertex  $v$  in bipolar fuzzy graph  $G$  is defined by  $t d_a(v) = (t d_a^+(v), t d_a^-(v))$ .

**Definition 3.12** Let  $G : (A, B)$  be a bipolar fuzzy graph on  $G^*(V, E)$ . If  $d_a(v) = (f_1, f_2)$ , for all  $v$  in  $V$ , then  $G$  is called  $(f_1, f_2)$  - pseudo regular bipolar fuzzy graph.

**Definition 3.13** Let  $G : (A, B)$  be a bipolar fuzzy graph on  $G^*(V, E)$ . If  $td_a(v) = d_a(v) + A(v) = (k_1, k_2)$ , for all  $v$  in  $V$ , then  $G$  is called  $(k_1, k_2)$  - totally pseudo regular bipolar fuzzy graph.

**Definition 3.14** Let  $G : (A, B)$  be a bipolar fuzzy graph on  $G^*(V, E)$ , where  $A = (m_1^+, m_1^-)$  and  $B = (m_2^+, m_2^-)$  be two bipolar fuzzy sets on a non empty set  $V$  and  $E \subseteq V \times V$  respectively. Then  $G$  is said to be a pseudo irregular bipolar fuzzy graph if there exists a vertex which is adjacent to the vertices with distinct pseudo degrees.

**Definition 3.15** Let  $G : (A, B)$  be a bipolar fuzzy graph on  $G^*(V, E)$ , where  $A = (m_1^+, m_1^-)$  and  $B = (m_2^+, m_2^-)$  be two bipolar fuzzy sets on a non empty set  $V$  and  $E \subseteq V \times V$  respectively. Then  $G$  is said to be a pseudo totally irregular bipolar fuzzy graph if there exists a vertex which is adjacent to the vertices with distinct total pseudo degrees.

**Definition 3.16** Let  $G : (A, B)$  be a bipolar fuzzy graph on  $G^*(V, E)$ , where  $A = (m_1^+, m_1^-)$  and  $B = (m_2^+, m_2^-)$  be two bipolar fuzzy sets on a non empty set  $V$  and  $E \subseteq V \times V$  respectively. Then  $G$  is said to be a highly pseudo irregular bipolar fuzzy graph if each vertex of  $G$  is adjacent to the vertices with distinct pseudo degrees.

**Definition 3.17** Let  $G : (A, B)$  be a bipolar fuzzy graph on  $G^*(V, E)$ , where  $A = (m_1^+, m_1^-)$  and  $B = (m_2^+, m_2^-)$  be two bipolar fuzzy sets on a non empty set  $V$  and  $E \subseteq V \times V$  respectively. Then  $G$  is said to be a highly pseudo totally irregular bipolar fuzzy graph if each vertex of  $G$  is adjacent to the vertices with distinct total pseudo degrees.

## IV. NEIGHBOURLY PSEUDO IRREGULAR BIPOLAR FUZZY GRAPHS

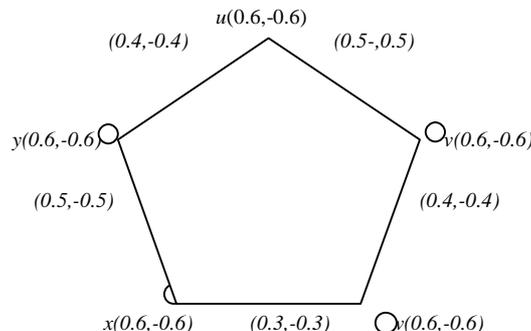
In this section, neighbourly pseudo irregular bipolar fuzzy graphs and neighbourly pseudo totally irregular bipolar fuzzy graphs are defined and discussed some of their properties.

**Definition 4.1.** Let  $G : (A, B)$  be a bipolar fuzzy graph on  $G^*(V, E)$ , where  $A = (m_1^+, m_1^-)$  and  $B = (m_2^+, m_2^-)$  be two bipolar fuzzy sets on a non empty set  $V$  and  $E \subseteq V \times V$  respectively. Then  $G$  is said to be a neighbourly pseudo irregular bipolar fuzzy graph if every pair of adjacent vertices have distinct pseudo degrees.

**Definition 4.2.** Let  $G : (A, B)$  be a bipolar fuzzy graph on  $G^*(V, E)$ , where  $A = (m_1^+, m_1^-)$  and  $B = (m_2^+, m_2^-)$  be two bipolar fuzzy sets on a non empty set  $V$  and  $E \subseteq V \times V$  respectively. Then  $G$  is said to be a neighbourly pseudo totally irregular bipolar fuzzy graph if every pair of adjacent vertices have distinct total pseudo degrees.

**Example 4.3.** A bipolar fuzzy graph which is both neighbourly pseudo irregular bipolar fuzzy graph and neighbourly pseudo totally irregular bipolar fuzzy graph.

Consider  $G^* : (V, E)$  where  $V = \{u, v, w, x\}$  and  $E = \{uv, vw, wx, xu\}$ .



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From Figure 1,  $d_G(u) = (0.9, -0.9)$   $d_G(v) = (0.9, -0.9)$   $d_G(w) = (0.7, -0.7)$   $d_G(x) = (0.8, -0.8)$ ,  $d_G(y) = (0.9, -0.9)$ . Here,  $d_G^*(u) = 2$ , for all  $u$  in  $G$ . Also,  $d_a(u) = (0.9, -0.9)$ ,  $d_a(v) = (0.8, -0.8)$ ,  $d_a(w) = (0.85, -0.85)$ ,  $d_a(x) = (0.8, -0.8)$ ,  $d_a(y) = (0.85, -0.85)$ . So, pseudo degree of all the adjacent vertices are distinct. Hence  $G$  is neighbourly pseudo irregular bipolar fuzzy graph. Also, total pseudo degrees of all the adjacent vertices are distinct. Hence  $G$  is neighbourly pseudo totally irregular bipolar fuzzy graph.

**Remark 4.4.** A neighbourly pseudo irregular bipolar fuzzy graph need not be a neighbourly pseudo totally irregular bipolar fuzzy graph.

**Remark 4.5.** A neighbourly pseudo totally irregular bipolar fuzzy graph need not be a neighbourly pseudo irregular bipolar fuzzy graph.

**Theorem 4.6.** Let  $G : (A, B)$  be a bipolar fuzzy graph on  $G^* : (V, E)$  and  $A$  is a constant function. Then the following are equivalent.

- (i)  $G$  is a neighbourly pseudo irregular bipolar fuzzy graph.
- (ii)  $G$  is a neighbourly pseudo totally irregular bipolar fuzzy graph.

Proof. Let  $G : (A, B)$  be a connected bipolar fuzzy graph on  $G^* : (V, E)$  and  $A$  is a constant function.

Then  $A(u) = (c_1, c_2)$ , for all  $u$  in  $V$ , where  $c_1$  and  $c_2$  are constant. Assume that  $G$  is neighbourly pseudo irregular bipolar fuzzy graph. Then every pair of adjacent vertices in  $G$  have distinct pseudo degrees. Let  $u_1$  and  $u_2$  be any two adjacent vertices with distinct pseudo degrees  $(k_1, k_2)$  and  $(f_1, f_2)$  respectively. Then  $(k_1, k_2) \neq (f_1, f_2)$ .

Suppose  $G$  is not a neighbourly pseudo totally irregular bipolar fuzzy graph. Then atleast one pair of adjacent vertices in  $G$  with same total pseudo degree  $\Rightarrow tda(u_1) = tda(u_2) \Rightarrow da(u_1) + A(u_1) = da(u_2) + A(u_2) \Rightarrow (k_1, k_2) + (c, c) = (f_1, f_2) + (c, c) \Rightarrow (k_1, k_2) = (f_1, f_2)$ , which is a contradiction to  $(k_1, k_2) \neq (f_1, f_2)$ . Hence  $G$  is neighbourly pseudo totally irregular bipolar fuzzy graph. Thus (i)  $\Rightarrow$  (ii) is proved.

Now, suppose  $G$  is a neighbourly pseudo totally irregular bipolar fuzzy graph. Then every pair adjacent vertices in  $G$  have distinct total pseudo degree. Let  $u_1$  and  $u_2$  be any pair of adjacent vertices in  $G$  with distinct total pseudo degrees. Now,  $tda(u_1) \neq tda(u_2) \Rightarrow da(u_1) + A(u_1) \neq da(u_2) + A(u_2) \Rightarrow da(u_1) + (c_1, c_2) \neq da(u_2) + (c_1, c_2) \Rightarrow da(u_1) \neq da(u_2)$ . Hence  $G$  is neighbourly pseudo irregular bipolar fuzzy graph. Thus (ii)  $\Rightarrow$  (i) is proved.

**Remark 4.7.** If every pseudo regular bipolar fuzzy graph  $G$  with membership values of all the adjacent vertices are distinct, then  $G$  is neighbourly pseudo totally irregular bipolar fuzzy graph.

## V. STRONGLY PSEUDO IRREGULAR BIPOLAR FUZZY GRAPHS

In this section, Strongly pseudo irregular bipolar fuzzy graphs and Strongly pseudo totally irregular bipolar fuzzy graphs are defined and discussed some of their properties.

**Definition 5.1.** Let  $G : (A, B)$  be a bipolar fuzzy graph on  $G^*(V, E)$ , where  $A = (m_1^+, m_1^-)$  and  $B = (m_2^+, m_2^-)$  be two bipolar fuzzy sets on a non empty set  $V$  and  $E \subseteq V \times V$  respectively. Then  $G$  is said to be a strongly pseudo irregular bipolar fuzzy graph if every pair of vertices have distinct pseudo degrees.

**Definition 5.2.** Let  $G : (A, B)$  be a bipolar fuzzy graph on  $G^*(V, E)$ , where  $A = (m_1^+, m_1^-)$  and  $B = (m_2^+, m_2^-)$  be two bipolar fuzzy sets on a non empty set  $V$  and  $E \subseteq V \times V$  respectively. Then  $G$  is said to be a strongly pseudo totally irregular bipolar fuzzy graph if every pair of vertices have distinct total pseudo degrees.

**Remark 5.3** Strongly pseudo irregular bipolar fuzzy graph need not be a Strongly pseudo totally irregular bipolar fuzzy graph.

**Remark 5.4** Strongly pseudo totally irregular bipolar fuzzy graph need not be a Strongly pseudo irregular bipolar fuzzy graph.

**Remark 5.5** If every pseudo regular bipolar fuzzy graph  $G$  with membership values of all the vertices are distinct, then  $G$  is strongly pseudo totally irregular bipolar fuzzy graph.

**Theorem 5.6.** Let  $G : (A, B)$  be a bipolar fuzzy graph on  $G^* : (V, E)$  and  $A$  is a constant function. Then the following are equivalent.

- (i)  $G$  is a strongly pseudo irregular bipolar fuzzy graph.
- (ii)  $G$  is a strongly pseudo totally irregular bipolar fuzzy graph.

Proof. Let  $G : (A, B)$  be a connected bipolar fuzzy graph on  $G^* : (V, E)$  and  $A$  is a constant function.

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Then  $A(u) = (c_1, c_2)$ , for all  $u$  in  $V$ , where  $c_1$  and  $c_2$  are constant. Assume that  $G$  is strongly pseudo irregular bipolar fuzzy graph. Then every pair of vertices in  $G$  have distinct pseudo degrees. Let  $u_1$  and  $u_2$  be any two vertices with distinct pseudo degrees  $(k_1, k_2)$  and  $(f_1, f_2)$  respectively. Then  $(k_1, k_2) \neq (f_1, f_2)$ . Suppose  $G$  is not a strongly pseudo totally irregular bipolar fuzzy graph. Then at least one pair of vertices in  $G$  with same total pseudo degree  $\Rightarrow tda(u_1) = tda(u_2) \Rightarrow da(u_1) + A(u_1) = da(u_2) + A(u_2) \Rightarrow (k_1, k_2) + (c_1, c_2) = (f_1, f_2) + (c_1, c_2) \Rightarrow (k_1, k_2) = (f_1, f_2)$ , which is a contradiction to  $(k_1, k_2) \neq (f_1, f_2)$ . Hence  $G$  is strongly pseudo totally irregular bipolar fuzzy graph. Thus (i)  $\Rightarrow$  (ii) is proved.

Now, suppose  $G$  is a strongly pseudo totally irregular bipolar fuzzy graph. Then every pair vertices in  $G$  have distinct total pseudo degree. Let  $u_1$  and  $u_2$  be any pair of vertices in  $G$  with distinct total pseudo degrees. Now,  $tda(u_1) \neq tda(u_2) \Rightarrow da(u_1) + A(u_1) \neq da(u_2) + A(u_2) \Rightarrow da(u_1) + (c_1, c_2) \neq da(u_2) + (c_1, c_2) \Rightarrow da(u_1) \neq da(u_2)$ . Hence  $G$  is strongly pseudo irregular bipolar fuzzy graph. Thus (ii)  $\Rightarrow$  (i) is proved.

**Theorem 5.7.** Every strongly pseudo irregular bipolar fuzzy graph is neighbourly pseudo irregular bipolar fuzzy graph.

*Proof.* Let  $G$  be a strongly pseudo irregular bipolar fuzzy graph. Then every pair of vertices in  $G$  having distinct pseudo degrees  $\Rightarrow$  every pair of adjacent vertices in  $G$  having distinct pseudo degrees. Hence the graph  $G$  is neighbourly pseudo irregular bipolar fuzzy graph.

**Theorem 5.8.** Every strongly pseudo totally irregular bipolar fuzzy graph is neighbourly pseudo totally irregular bipolar fuzzy graph.

*Proof.* Proof is similar to the above theorem.

**Remark 5.9.** Example 4.3 shows that neighbourly pseudo irregular bipolar fuzzy graph need not be a strongly pseudo irregular bipolar fuzzy graph.

**Theorem 5.10.** Let  $G : (A, B)$  be a bipolar fuzzy graph on  $G^* : (V, E)$ . Then  $G$  is neighbourly pseudo irregular bipolar fuzzy graph and highly pseudo irregular bipolar fuzzy graph if and only if  $G$  is strongly pseudo irregular bipolar fuzzy graphs.

*Proof.* Assume that  $G$  is strongly irregular bipolar fuzzy graph. Then pseudo degrees of all the vertices of  $G$  are distinct. Then clearly  $G$  is neighbourly pseudo irregular bipolar fuzzy graph and highly pseudo irregular bipolar fuzzy graph. Conversely, assume that  $G$  is neighbourly pseudo irregular bipolar fuzzy graph and highly pseudo irregular bipolar fuzzy graph. Let  $G$  be a bipolar fuzzy graph with  $n$  vertices  $v_1, v_2, v_3, \dots, v_n$ . Let  $v_1$  is adjacent with the vertices  $v_2, v_3, \dots, v_n$  with pseudo degrees  $(k_2^+ k_2^-), (k_3^+ k_3^-) \dots, ((k_n^+ k_n^-))$  respectively. Then degrees  $(k_2^+ k_2^-) \neq (k_3^+ k_3^-) \neq \dots, ((k_n^+ k_n^-))$ , and the pseudo degree of  $v_1$  cannot be any one of  $(k_2^+ k_2^-), (k_3^+ k_3^-) \dots, ((k_n^+ k_n^-))$ . So, pseudo degrees of all vertices of  $G$  are distinct. Hence  $G$  is strongly pseudo irregular bipolar fuzzy graph

**Remark 5.11.** If every pseudo regular bipolar fuzzy graph  $G$  with membership values of all the vertices are distinct, then  $G$  is strongly pseudo totally irregular bipolar fuzzy graph.

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### BIOGRAPHY



Dr.N.R.Santhi Maheswari is Head and Associate professor in Department of Mathematics, G.Venkataswamy Naidu College, Kovilpatti, Tamil Nadu, India. Her area of research is regularity and irregularity concept in graph theory. She is doing one minor research project in fuzzy graph theory. She acted as a PG board of study member in Manonmaniam Sundranar University, Tirunelveli and Sadhakathullah appa college, Tirunelveli. She is a member in professional body ADMA. She is senate member in Manonmaniam Sundranar University, Tirunelveli. She published more than 35 papers in various esteemed peer reviewed international journals.