

Prediction of Natural Frequency of Wind Turbine off Mangalore Coast

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ABSTRACT: Offshore wind turbines have minimal environmental effects and visual impacts as compared to onshore wind turbine. It is necessary to consider the safety factors for efficient performance and stability of the offshore wind turbine. These structures are long slender column with rotating mass at the top. In the present study, dynamic behaviour of the structure and estimation of the natural frequency of a wind turbine system subjected to environmental forces and forces from rotor vibration will be discussed. The offshore wind turbine is modelled as Euler-Bernoulli beam column with elastic end supports. The elastic end-supports resemble flexible nature of soil-structure interaction. Natural frequencies of structure will be obtained from analytical solution. Analytical approach is successfully tried for example wind turbine for Indian soil conditions.

KEYWORDS: Dynamic analysis, Natural frequency, Euler-Bernoulli beam.

I. INTRODUCTION

Wind turbines are one of the renewable energy sources for power generation. Being high slender and low stiffened dynamical structure, Offshore Wind Turbine (OWT) subjected to wind, and wave loadings. Hence, Dynamic analysis of wind turbine is required to assure its stability, performance, operation and safety. OWT towers are long slender columns with a rotor and blade assembly placed on the top. Cyclic loads are generated in the foundations due to the variety of dynamic loadings (arising from the rotor dynamics, out-of-balance masses, varying wind loading intensity, wave loading), which are three dimensional in nature. Different types of foundations for offshore wind turbines in practice are, suction caisson, monopile, multipod, gravity based etc., [Adhikari *et al.*, 2011]. In addition, the tower may consist of jacket type structure or tubular column type structure. Figure 1a represents the tubular type of offshore wind turbine with monopile foundation which is assumed in the present study and foundation was modelled by two types of springs having rotational and lateral spring constants k_r and k_l , respectively. The top mass of the turbine i.e. weight of the rotor-hub and the blades are assumed to be an axial load N (Refer Figure 1c).

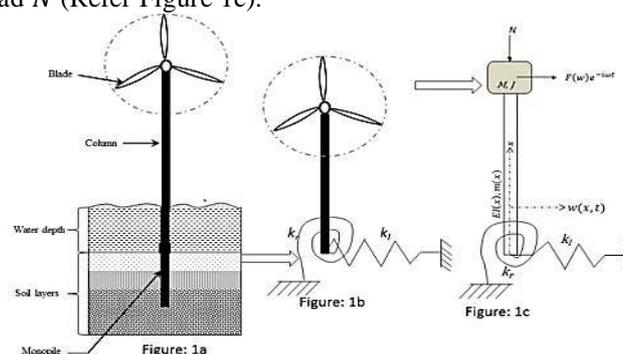


Figure 1: Idealization of a wind turbine using Euler Bernoulli beam with a top mass. Flexible springs are assumed to model the soil-structure interaction. The weight of the rotor-hub and blades are assumed to be N .

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Overall, the whole structure was idealised as Euler Bernoulli beam having a rotating top mass with flexible springs. In this paper, analytical method is used for better understanding of soil-structure interaction in terms of frequency with respect to axial load, rotational k_r and lateral k_l stiffness parameters and procedure is adopted from Adhikari *et al.*, (2012). Molenaar and Tempel (2002) described the safer zones of frequencies to avoid resonance. To avoid resonance type of failure, the wind turbine tower should be designed such that the first natural frequency should not be close to either 0.37 Hz or 1.11 Hz or close to the frequency of the wind and the wave at the location i.e. more than 0.1 Hz [Molenaar *et al.*, 2002].

II. LITERATURE

Molenaar *et al.*, (2002) proposed a method to carry out dynamic analysis of a flexible wind turbine system. The wind turbine is modelled as a flagpole of length L , mass per unit length m having bending stiffness EI with a top mass M . The flagpole is assumed to be fixed at the bottom. The first natural frequency Eq.(1) which is simple to use, which does not consider the flexibility of the foundation and stiffness softening effect due to the axial load arising from the mass of the rotor and blade system

$$f_1 \approx \sqrt{\frac{3.04EI}{(M+0.227mL)4\pi^2L^3}} \quad (1)$$

Adhikari *et al.*, (2012) carried out dynamic analysis considering the flexibility of the foundation and also the effect of the axial load. The foundation is modelled by two springs denoted by k_r (rotational spring) and k_l (lateral spring). The method is based on an Euler-Bernoulli beam-column with elastic end supports. A closed-form expression of the characteristic equation governing all the natural frequencies of the system has been derived.

Bhattacharya *et al.*, (2011) experimentally investigated the response of a wind turbine model considering foundation as a monopile for 17 test cases. Experimental results show that the natural frequencies and the damping factors of the wind turbine tower varies significantly with the type of soil/foundation. Analytical results for the natural frequencies matches reasonably well with the experimental results and finite element results.

Arany *et al.*, (2015) conducted a study on offshore wind turbines (OWTs) supported on flexible foundation. In their study an analytical model suitable for preliminary design was presented. They idealized the foundation as three coupled springs- lateral, rotational and cross-coupling. The vertical spring is neglected as the structure is very stiff vertically. The tower was modeled using both Euler-Bernoulli and Timoshenko beam models and compared the obtained results. From this study, it was concluded that the Timoshenko beam model does not improve the results significantly.

Equation of Motion and Estimation of First Natural Frequency

Consider a Wind turbine (Refer Figure 1c) which is modelled as a flagpole of length L , mass per unit length m having bending stiffness EI with a top mass M and rotary inertia J with its bottom fixed. $m(x)$ and $r(x)$ are the mass per unit length and radius of gyration of the beam respectively. The foundation is modelled by two springs rotational and lateral spring constants k_r and k_l respectively. The beam is subjected to a constant compressive axial load $N = Mg$ where M is the lumped mass at the top of the turbine. The force N in general is a function of time due to the rotational motion of the hub. Eqn 1 represents the fourth-order partial differential equation for free vibration of column subjected to axial compressive load N .

$$EI \frac{\partial^4 w(x,t)}{\partial x^4} + N(x) \frac{\partial^2 w(x,t)}{\partial x^2} - mr^2 \frac{\partial^2 \ddot{w}(x,t)}{\partial x^2} + m\ddot{w}(x,t) = 0 \quad (2)$$

Here $w(x, t)$ is the transverse deflection of the beam, L is the height of the structure, t is time, $(\dot{\cdot})$ denotes derivative with respect to time and four boundary conditions associated with this equation can be expressed as in Table 1. Assuming harmonic solution (the separation of variable) we have

$$w(x, t) = W(\xi)e^{-i\omega t}, \quad \text{where } \xi = x/L \quad (3)$$

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Special Issue 9, May 2016

Table 1: Four boundary conditions associated with Equation 1

$x = 0$	Bending moment	$EIw''(0, t) - k_r w'(0, t) = 0$
	Shear force	$EIw'''(0, t) + Nw'(0, t) + k_l w(0, t) + mr^2 \frac{\partial^2 \ddot{w}(0, t)}{\partial x^2} = 0$
$x = L$	Bending moment	$EIw''(L, t) + J\ddot{w}(L, t) = 0$
	Shear force	$EIw'''(L, t) + Nw'(L, t) - M\ddot{w}(L, t) - mr^2 \frac{\partial^2 \ddot{w}(L, t)}{\partial x^2} = 0$

It is convenient to express the boundary conditions given in Table 1 in terms of non-dimensional parameters as following equations and these are mentioned in Table 2. η_r and η_l will be called as the soil-structure interaction parameters.

$$\frac{\partial^4 W(\xi)}{\partial \xi^4} + \frac{\partial^2 W(\xi)}{\partial \xi^2} - \Omega^2 W(\xi) = 0 \tag{4}$$

$$W''(0) - \eta_r W'(0) = 0 \tag{5}$$

$$W'''(0) + \check{\nu} W'(0) + \eta_l W(0) = 0 \tag{6}$$

$$W''(1) - \beta \Omega^2 W'(1) = 0 \tag{7}$$

$$W'''(1) + \check{\nu} W'(1) + \alpha \Omega^2 W(1) = 0 \tag{8}$$

Using the equation for Frequency parameter from Table 2, the natural frequencies is estimated by

$$\omega_j = \Omega_j c_0; \quad j = 1, 2, 3, \dots \tag{9}$$

Determination of Natural Frequency:

Turbine system frequencies can be estimated from the ‘free vibration problem’ by considering no force on the system. Assuming a solution of the form $W(\xi) = e^{\lambda \xi}$ substituting in Eqn. (4)

$$\lambda^4 + \check{\nu} \lambda^2 - \Omega^2 = 0 \tag{10}$$

This equation is often known as the dispersion relationship. Thus, Eqn (10) governing for the natural frequencies of the beam. Solving, we obtained

$$\lambda = \pm i\lambda_1, \quad \pm i\lambda_2 \tag{11}$$

where

$$\lambda_1 = \left(\sqrt{\left(\frac{\check{\nu}}{2}\right)^2 + \Omega^2 + \frac{\check{\nu}}{2}} \right)^{1/2}, \quad \lambda_2 = \left(\sqrt{\left(\frac{\check{\nu}}{2}\right)^2 + \Omega^2 - \frac{\check{\nu}}{2}} \right)^{1/2} \tag{12}$$

The solution $W(\xi)$ can be expressed as

$$W(\xi) = a_1 \sin \lambda_1 \xi + a_2 \cos \lambda_1 \xi + a_3 \sinh \lambda_2 \xi + a_4 \cosh \lambda_2 \xi \text{ or } W(\xi) = s^T(\xi) a \tag{13}$$

$$\text{where, } s(\xi) = \{\sin \lambda_1 \xi, \cos \lambda_1 \xi, \sinh \lambda_2 \xi, \cosh \lambda_2 \xi\}^T, \quad a = \{a_1, a_2, a_3, a_4\}^T \tag{14}$$

Applying the boundary conditions in Eqn. (4)– (7) and in the expression of $W(\xi)$ [Eqn. 14] we have

$$Ra = 0 \tag{15}$$

International Journal of Innovative Research in Science, Engineering and Technology

(An ISO 3297: 2007 Certified Organization)

Vol. 5, Special Issue 9, May 2016

$$R = \begin{bmatrix} s_1''(0) - \eta_r s_1'(0) & s_2''(0) - \eta_r s_2'(0) \\ s_1'''(0) + \check{v} s_1'(0) + \eta_l s_1(0) & s_2'''(0) + \check{v} s_2'(0) + \eta_l s_2(0) \\ s_1''(1) - \beta \Omega^2 s_1'(1) & s_2''(1) - \beta \Omega^2 s_2'(1) \\ s_1'''(1) + \check{v} s_1'(L) + \alpha \Omega^2 S(1) & s_2'''(1) + \check{v} s_2'(1) + \alpha \Omega^2 S(1) \\ s_3''(0) - \eta_r s_3'(0) & s_4''(0) - \eta_r s_4'(0) \\ s_3'''(0) + \check{v} s_3'(0) + \eta_l s_3(0) & s_4'''(0) + \check{v} s_4'(0) + \eta_l s_4(0) \\ s_3''(1) - \beta \Omega^2 s_3'(1) & s_4''(1) - \beta \Omega^2 s_4'(1) \\ s_3'''(1) + \check{v} s_3'(L) + \alpha \Omega^2 S(1) & s_4'''(1) + \check{v} s_4'(1) + \alpha \Omega^2 S(1) \end{bmatrix} \quad (16)$$

Substituting functions $s_j(\xi)$, $j = 1, 2, 3, 4$ in Eqn. (14) and simplifying we obtain, the constant vector \mathbf{a} which cannot be zero. Therefore, the natural frequencies is given by

$$|R| = 0 \quad (17)$$

Eqn. (17) is a transcendental equation. Due to the nonlinearity of this equation, it can be solved numerically. Matlab software has been used to solve the equation for Ω . The actual natural frequencies of the wind turbine system can be calculated using Eqn. (9). Eqn. (17) can be used for general case i.e. axial load, elastic end restraints, mass and rotary inertia of the rotor. Adhikari *et al.* (2012) modified the Eqn. (17) which is simple and better understanding to find first natural frequency of the system for different support condition.

$$f_1 = \frac{\omega_1}{2\pi} = \frac{\Omega_1 c_0}{2\pi} \quad (18)$$

Determination of the soil-structure interaction parameters

Subgrade reaction approach is an analytical method where soil is treated as set of non-linear independent springs often known as $p - y$ springs down the length of the pile so called Winkler idealization of soil. Closed-form expressions developed by Fleming *et al.*, (1992) based on Matlock and Rees (1961) derived the relation between spring stiffness k and bending rigidity of pile EI_p

$$\frac{k}{G} \approx 10 \left(\frac{E_p}{G} \right)^{-0.14} \quad (19)$$

Where $G = aN^b$ (Ohsaki and Twasaki 1973) (20)

predicted the values of $a = 124$ and $b = 0.78$ for all type of soils. Based on Fleming *et al.*, (1992) the expression for L_c is given by,

$$L_c = 4 \left(\frac{EI_p}{4k} \right)^{1/4} \quad (21)$$

where L_c = critical length of the pile defined as the length beyond which the pile behaves as if it were infinitely long. Below expressions [Eqn. (22)] are the translational and rotational stiffness respectively, where G is the Dynamic shear Modulus of the soil and E_p is the equivalent modulus of elasticity of the pile. Eqn. 19 is based on the interpretation of

$$k_l = \frac{k}{\sqrt{2}} \left(\frac{4}{L_c} \right)^{-1} \quad \& \quad k_r = \frac{k}{\sqrt{2}} \left(\frac{4}{L_c} \right)^{-3} \quad (22)$$

Table 3: Stiffness of soil calculation

Soil type	G (MPa)	Soil stiffness k (MPa)	k_l (GN/m)	k_r (GN/m)	η_l	η_r
Very loose	42.57	129.426	0.65	32.95	2609.44	20.13
Very Stiff	191.56	718.995	2.36	50.58	9442.27	30.90
Stiff	157.416	574.812	1.99	47.83	7983.19	29.22

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Vol. 5, Special Issue 9, May 2016

the work carried out by Baguelinet *al.*, (1977) and presented by Randolph and Steward (1999). The theoretical concept is are applied to Indian soil condition and the results are summarized in Table 3

III. RESULTS AND DISCUSSION

The first natural frequency of the wind turbine system is function of non-dimensional foundation stiffness η_r, η_l and also non-dimensional axial force, ν . Figure 2a, 2b, and 2c describes the variation of the natural frequency with respect to non-dimensional axial force for fixed values of non-dimensional rotational stiffness and lateral stiffness as estimated by Eqn. (8). The natural frequency shows that sudden fall for small values of η_r and increases as η_r increases. However, for $\eta_r \geq 100$, frequency of the turbine system remains constant and further increase in rotational and lateral stiffness parameter values do not change the natural frequency. This is because stiffness properties stabilize the system. Thus soil-structure interaction can be essentially considered as fixed. It has been observed from the Figures 2a, 2b & 2c that natural frequency is very sensitive to the lower range (say $\nu = 0 - 0.5$) of axial force. Frequency of the turbine system decreases with the increase in values of ν and for smaller values of η_r (say $\eta_r = 1, 2, 5, 10$) an average increase of 40% is observed. For low soil stiffness parameter with high axial force values, system starts buckling due to instability (Refer figure 2a 2b & 2c) and the results have been verified with Adhikari *et al.*, (2012).

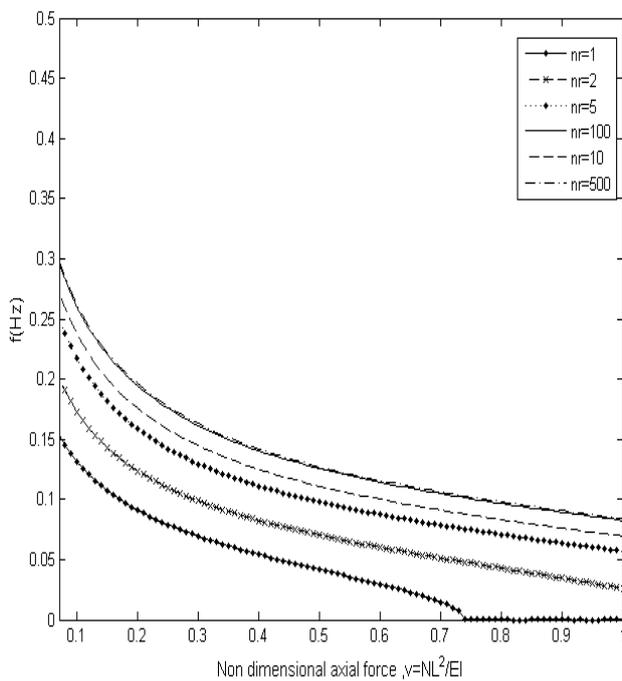


Figure 2a: $\eta_l = 5$

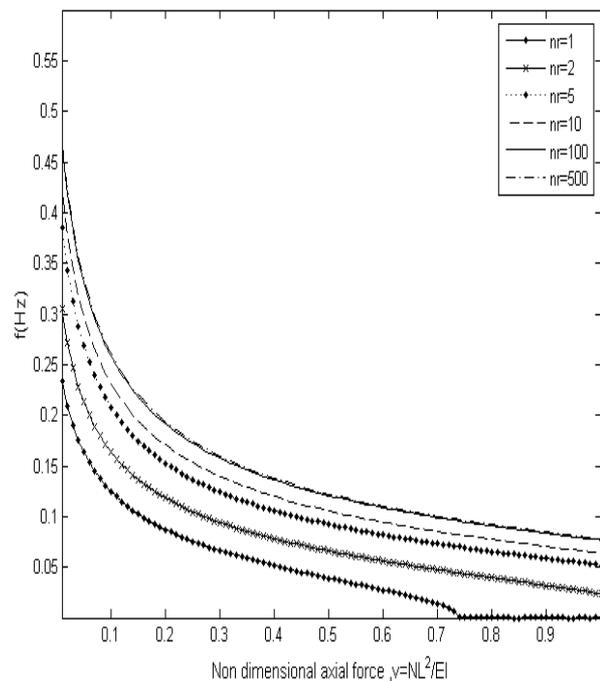


Figure 2b: $\eta_l = 10$

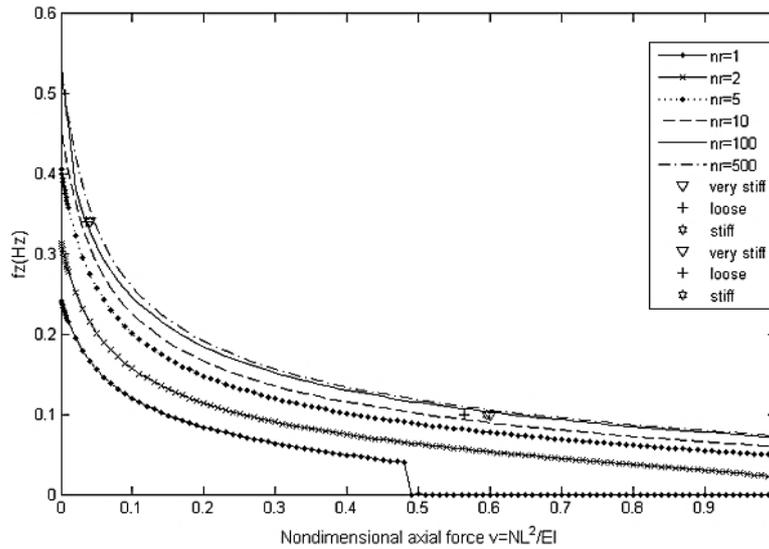


Figure 2c: $\eta_l = 100$ [α , + and \diamond represents the critical axial force for resonance for different soil layers in the Mangalore coast]

Table 4 gives the details of soil properties used in the present study. The data was collected from New Mangalore Port Trust (NMPT) which was a field investigation carried by WAPCOS Ltd. It is assumed that same soil profile is extended to the offshore region as well. The structural parameters of the wind turbine is adopted from Molenaaret *al.*, (2002). The dynamic shear modulus $\gamma'G'$ for different layers of soil is calculated by using theoretical concept (Eqn. 20) suggested by Ohsakiet *al.*, (1973) and the values are comparatively equal with Bowles (1997).

Table 4: SPT and soil characteristics of Mangalore coast (Soils reports: WAPCOS Limited)

Depth (m)	Soil type	Corrected $N(1)_{60}$	$\gamma' (kN/m^3)$
-3.00	Very loose	5	4.19
-12.00	Very Stiff	35	12.19
-15.00	Stiff	27	12.19

Molenaaret *al.*, (2002) discussed different types of frequencies, wind turbine systems are subjected to, which can lead to resonance type of failure. Here, resonance frequency is kept constant and axial force is back calculated for different values of η_r and η_l as described in Table 3. Obtained v values are plotted against to two type frequencies namely rotor frequency (0.34 Hz) and frequency spectrum of wind gust (below 0.1 Hz) shown in Figure 2c. It is observed that critical axial load will be less for loose soil compared to other type of soils (Refer Table 5). To avoid the resonance type of failure designers are advised to check the limits of critical load for this region.

Table 5: shows the calculated values crucial axial force for resonance frequencies

Soil type	v (Non-dimensional axial force)		N kN(Axial load)	
	$f = 0.34 \text{ Hz}$	$f = 0.1 \text{ Hz}$	$f = 0.34 \text{ Hz}$	$f = 0.1 \text{ Hz}$
Very loose	0.0360	0.5683	730.11	11530
Very Stiff	0.0392	0.5916	795.25	12000
Stiff	0.0396	0.5945	803.47	12060

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Vol. 5, Special Issue 9, May 2016

IV. CONCLUSION

Dynamic Analysis for Offshore Wind turbine is important for safety and its efficient performance. As a case study, the soil profile in the Mangalore Coast is considered and having thought of Wind Turbine in the offshore region Wind turbine foundation is modelled by spring analogy. The assumption of elastic end supports to account the soil-foundation interaction is crucial for OWT's has obtained less importance in literature. Analytical approach is used to calculate the natural frequency and critical axial loads are estimated by keeping excitation frequency constant for different layers of soil.

REFERENCES

- [1]. Adhikari S and Bhattacharaya S, "Dynamic analysis of wind turbine towers on flexible foundations", Shock and Vibration, Vol.19pp.37-56,2012.
- [2]. Adhikari S and Bhattacharaya S, "Vibrations of wind-structures considering soil-structure interaction", Wind and Structures, Vol.14, No 2 pp.85-112, 2011.
- [3]. Arany L., Bhattacharaya S., Adhikari S., Hogan S J and Macdonald J H G, "An analytical model to predict the natural frequency of offshore wind turbines on three spring flexible foundations using two different beam models", Soil Dynamics and Earthquake Engineering, Vol. 74 pp.40-45, 2015.
- [4]. Baguein SY, and FrankR, "Theoretical study of lateral reaction mechanism of piles", Geotechnique, Vol.27 pp.405-434, 1977.
- [5]. Bhattacharaya S and Adhikari S, "Experimental validation of soil-structure interaction of offshore wind turbines", Soil Dynamics and Earthquake Engineering, Vol. 31 pp.805-816, 2011.
- [6]. Bowles J, "Foundation Analysis and Design", 5th edition, Mc Graw-Hill International editions, 1997.
- [7]. Davenport A G, "Transcendental eigenvalue problem and its applications", Quarterly Journal of Royal Meteorological Society, Vol.87 pp.194-211, 1961.
- [8]. David W sykora, "A report on Examination Of Existing Shear, Wave Velocity And Shear Modulus Correlations in Soils", US Army Corps of Engineers, 1987.
- [9]. Fleming W., Weltman A., Randolph MF and Elson W, "Piling Engineering", John Wiley and Sons, New York, USA, 1992.
- [10]. Geradin M and Rixen D. "Mechanical Vibrations", John Wiley and sons, New York, NY, 2nd edition, 1991.
- [11]. Kreyszig E, "Advanced engineering mathematics", John Wiley & Sons, New York, 9th edition, 2006.
- [12]. Matlock H. and Reese LC, "Generalized solutions for laterally loaded piles", Journal of the Soil Mechanics Division, ASCE, Vol.86 pp.673-694.
- [13]. Molenaar DP and Tempel DP, "Wind turbine structural dynamics- A review of the principles for modern power generation, onshore and offshore", Wind Engineering, Vol.26 pp.211-220, 2002.
- [14]. Moskowitz L, "Estimates of the power spectrums for fully developed seas for wind speeds of 20 to 40 knots", Journal of Geophysical Research, Vol. 69 pp.5161-5179, 1964.
- [15]. Randolph M.F and Steward D.P, "Manual for the program PYGM", University of Western Australia, Australia, 1999.