

Some Separation Properties involving Zero set in Product Closure Spaces

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ABSTRACT: In this paper we discuss the separation properties in product closure spaces

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I. INTRODUCTION

In [1] E. Čech defined closure spaces, product of Closure spaces and their associated topological spaces. In [3] T.A Sunitha discussed morphisms between closure spaces and some separation properties in closure spaces. These motivated the present study.

0.0 Preliminaries

We give below the necessary preliminaries taken from [1], [2], [3] and [4].

0.1 Definition

A function c from the power set of X to itself is called a closure operation on X provided that the following conditions are satisfied.

- (i) $c(\phi) = \phi$
- (ii) $A \subseteq c(A)$ for all $A \subseteq X$
- (iii) $c(A \cup B) = c(A) \cup c(B)$ for all $A, B \subseteq X$

X with a closure operator c is called a closure space.

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0.2 Remark

The associated topological space is the one whose closed sets are precisely those subsets of X for which $cA=A$. The (Kuratowski) closure operation 'cl' in this associated topological space associates to each set A , the smallest closed set $cl(A)$ containing A .

0.3 Definition

Let $(X,c),(Y,c')$ be closure spaces and cl,cl' be the Kuratowski closure operations in the respective associated topologies. A map $f : X \rightarrow Y$ is said to be a,

- (i) c - c' morphism if $f(cA) \subseteq c'f(A)$ for all $A \subseteq X$
- (ii) c - cl' morphism if $f(cA) \subseteq cl'f(A)$ for all $A \subseteq X$
- (iii) cl - cl' morphism or a continuous map if $f(clA) \subseteq cl'f(A)$ for all $A \subseteq X$.

0.4 Definition

Let $\{(X_a, c_a) : a \in A\}$ be a family of closure spaces, X be the product of the family $\{X_a\}$ that is, $X = \prod X_a$ of underlying sets, π_a be the projection of X onto X_a for each a , then the product closure c is the coarsest closure on the product of underlying sets such that all the projections are morphisms.

0.5 Definition

A closure space (X,c) is said to be Hausdorff if for any two distinct points, there exist neighborhoods U of x and V of y such that $U \cap V = \emptyset$.

0.6 Definition

A closure space (X,c) is said to be completely regular if for every point x and a closed set A not containing x , there exist a c - cl morphism $f: X \rightarrow [0,1]$ such that $f(x)=0$ and $f(y)=1$ for all $y \in A$

0.7 Definition

- (i) $c(X) = \{f: X \rightarrow \mathbb{R} \mid f \text{ is continuous from } (X,t) \text{ to } \mathbb{R} \text{ with usual topology, here 't' is any topology on } X\}$
- (ii) $c^*(X) = \{f \in c(X) \mid f \text{ is bounded}\}$
- (iii) For $f \in c(X)$, $Z(f) = \{x \in X \mid f(x) = 0\}$ zero set of f and $Z(X) = \{Z(f) \mid f \in c(X)\}$

1. Some separation properties in closure spaces

In this section we discuss some separation properties involving zero set.

1.1 Notation

We denote $C_1(X) = \{f: X \rightarrow \mathbb{R} \mid f \text{ is } c\text{-}cl \text{ morphism where } c \text{ is the closure operator on } X \text{ and } cl \text{ is the Kuratowski closure operator on } \mathbb{R}\}$ and the zero set of X is denoted by $Z_1(X) = \{Z_1(f) \mid f \in C_1(X)\}$ and $Z_1(f) = \{x \in X \mid f(x) = 0\}$.

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1.2 Theorem

A c -Hausdorff space X is c -completely regular if and only if the family $Z_1(X) = \{ Z_1(f) : f \in C_1(X) \}$ is a base for the closed sets in the associated topology of X .

Proof

Suppose X is c -completely regular. Then whenever F is closed set and $x \in F^c$ there exist $f \in C_1(X)$ such that $f(x) = 1$ and $f(F) = \{0\}$. Then $F \subseteq Z_1(f)$ and x is not an element of $Z_1(f)$. Thus $Z_1(X)$ is a base.

Conversely, suppose $Z_1(X)$ is a base. So if F is closed set and $x \in F^c$ then there exist $g \in C_1(X)$ such that $F \subseteq Z_1(g)$ and x is not an element of $Z_1(g)$. Let $r = g(x)$ then $r \neq 0$ and let $f = gr^{-1} \in C_1(X)$ and $f(x) = 1$, $f(F) = \{0\}$ so that Hausdorff space X is completely regular. But by [3] completely regular implies c -completely regular. Hence the theorem.

1.3 Remark

It is known that [2] a Hausdorff space X is completely regular if and only if $\{ Z(f) \mid f \in C^*(X) \}$ forms a base for the closed set. Also continuity implies c -cl' morphism by [1.3].

1.4 Remark

Using result 2.2 and remark 2.3 we have the result 'A Hausdorff space is completely regular then $Z_1(X) = \{ Z_1(f) : f \in C_1(X) \}$ is a base for the closed sets in the associated topology'.

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