

Fluid Flow through PAKKA-Model Structures in Multiphase Flow

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ABSTRACT: In Fluid dynamics, the lattice-Boltzmann method is used to solve 3D Newtonian fluid flow problems in porous media given by tomographic imaging and by numerical modelling. In the Lattice Boltzmann method particles are allowed to move and collide on a lattice. The rules governing the collisions are designed such that the time-average motion of the particles is consistent with the Navier-Stokes equations. The Lattice Boltzmann method has multiple advantages as its time and space efficient computations are straightforward to parallelize, it handles complex boundaries without difficulty and it directly links microscopic and macroscopic phenomena.

By the lattice-Boltzmann simulations there are examination of the quality of numerical models of porous media and tomographic imaging techniques. The different models were taken to give a qualitative dependence of permeability on porosity, but also effects of other structural properties were demonstrated. These include specific surface area, tortuosity, and shape and orientation of particles in the medium. The transverse and in-plane flow simulations for PAKKA-model samples augmented the characteristics of the samples and revealed e.g. the effect of fibre flexibility. The simulations show that results on paper can be sensitive to imaging techniques, since the permeability differences between low and high-resolution images can be seen though the results within each technique were consistent. The table and graphs show the variation and dependency of porosity on the Darcy permeability and tortuosity.

KEYWORDS: PAKKA-model structure, Lattice-Boltzmann method, Multiphase flow.

I. INTRODUCTION

In the three-dimensional images, fluids are depicted with iso-surfaces and volume visualization techniques using color, intensity, and transparency for indicating the fluid density. The fluid movement is evaluated by assembling visualizations at a series of time steps into animations. The three-dimensional images also use iso-surfaces to delineate the structure of medium or the surface of the structure in which the fluid flow is being modeled.

The paper consists of fibres, fines, fillers and some other additives that are configured in a complex, spatial solid materials formed in the paper making process. We know that the porous structure is given explicitly when the fibre network is given and the term paper structure can be taken as a common phenomenon. The paper structure gives the actual paper properties directly or indirectly together with the material properties of the solid materials. The three-dimensional paper structure is an important phenomenon for the paper material; it is important to know the detailed knowledge of it. But it is a difficult task as the three-dimensional structure is in its nature hard to access. The indirect methods have therefore been applied for characterizing the features of the paper structure. But, only such indirect methods have been used for characterizing the paper structure. The microscopy and x-ray techniques and increased computer capacity allow the procedures to develop the complete three-dimensional paper structure. The methods allow the development of knowledge on the paper structure and how it affects the macroscopic paper properties. So that the methods to explore how the three-dimensional paper structure is related to the macroscopic paper properties are now emerging, alternative indirect methods have been applied for a lot of years. The well experienced papermakers know how to vary the production settings to improve certain paper characteristics and thus change the structure of the paper [4], [5].

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II. METHOD AND EQUATIONS

The motion of fluid is given by the basic fluid mechanics equation, i.e., the equation of continuity given by

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho u) = 0 \quad (1.1)$$

The Lattice-Boltzman method was well analyzed and the equations were derived by Aaltosalmi, U., et al. [1], [2], [3]. The Lattice-Boltzman hydrodynamics is a mesoscopic approach for the computational fluid mechanics. The macroscopic dynamical system can be shown for satisfying the Navier-Stokes equation. In the present analysis, the fluid is modeled by the particle distributions that moves on the lattice divided to different fluid particles. In the one lattice time period, the particles propagate to their adjacent lattice points and distribute the momentum in the collision. The velocity and density of the entering particle at site r and time t in the arbitrary directions c_i is given by the equation (1.2).

By taking $f_i(r, t)$ the particle density entering at site r at time t with a velocity pointing in direction c_i , where i labels the lattice directions. For a lattice time step η and lattice spacing λ , the particle velocities are

$$v_i = \frac{\lambda}{\eta} c_i \quad (1.2)$$

The evolution equation for f_i is given as

$$f_i(r + \lambda c_i, t + \eta) = f_i(r, t) + \omega_i(r, t) \quad (1.3)$$

Where $\omega_i(r, t)$ is a model dependent collision term that decreases or increases the density of type i particles. Just like the kinetic theory, the quantities i.e., density ρ , velocity v and momentum tensor M are given by the equations

$$\rho(r, t) = \sum_{i=1}^n f_i(r, t) \quad (1.4)$$

$$\rho(r, t) v(r, t) = \sum_{i=1}^n v_i f_i(r, t) \quad (1.5)$$

$$M_{jk}(r, t) = \sum_{i=1}^n v_{i\alpha} v_{i\beta} f_i(r, t) \quad (1.6)$$

Where j and k are spatial components of the velocity vectors and n shows the number of possible directions c_i . In the lattice-Boltzman dynamics, we have

$$M_i = \frac{1}{\xi} (f_i^{(0)}(r, t) - f_i(r, t)) \quad (1.7)$$

This is based on the single collision relaxation. So the evolution equation for the system can be written as

$$f_i(r + \lambda c_i, t + \eta) = f_i(r, t) + \frac{1}{\xi} (f_i^{(0)}(r, t) - f_i(r, t)) \quad (1.8)$$

Here ξ is the relaxation parameter. The term shown in the equation $f_i^{(0)}$ is the equilibrium distribution as chosen to give the required behavior of the fluid.

The expansion of equation (1.8) by the Taylor's method on retaining the term up to second order and neglecting the higher order terms, we have

$$\eta \frac{\partial}{\partial t} f_i + \lambda (c_i \cdot \nabla) f_i + \frac{\eta^2}{2} \frac{\partial^2}{\partial t^2} f_i + \frac{\lambda^2}{2} (c_i \cdot \nabla)^2 f_i + \lambda \eta (c_i \cdot \nabla) \frac{\partial}{\partial t} f_i = \omega_i \quad (1.9)$$

$$f_i = f_i^{(0)} + \varepsilon f_i^{(1)} + \varepsilon^2 f_i^{(2)} + \dots + \varepsilon^n f_i^{(n)} + \dots \quad (1.10)$$

Here ε is a small parameter. For finding the term $f_i^{(n)}$, $n = 0, 1, 2, \dots$, we have

$$\rho = \sum_{i=1}^n f_i^{(0)}, \rho v = \sum_{i=1}^n v_i f_i^{(0)} \quad (1.11)$$

$$\sum_{i=1}^n f_i^{(n)} = 0, \quad \sum_{i=1}^n v_i f_i^{(n)} = 0; n \geq 1 \quad (1.12)$$

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Taking

$$\frac{\eta}{t_1} = o(\varepsilon), \frac{\eta}{t_2} = o(\varepsilon^2) \quad (1.13)$$

Where t_1 and t_2 are macroscopic time scales.

Also,

$$\frac{\lambda}{L_1} = o(\varepsilon) \quad (1.14)$$

Also by taking,

$$\frac{t_1}{\varepsilon} + \frac{t_2}{\varepsilon^2} = t, r = \frac{r_1}{\varepsilon} \quad (1.15)$$

Where r_1 is the macroscopic scale variable. We have

$$\frac{\partial}{\partial t} = \varepsilon \frac{\partial}{\partial t_1} + \varepsilon^2 \frac{\partial}{\partial t_2} \quad (1.16)$$

$$\frac{\partial}{\partial j} = \varepsilon \frac{\partial}{\partial (1j)} \quad (1.17)$$

Now all the velocities do not have the same mass m_i , so we take

$$\sum_{i=1}^n m_i c_{ij} = 0 \quad (1.18)$$

Let us take the function $f_i^{(0)}$ as the function of quantities ρ and ρu , we have

$$f_i^{(0)} \cong m_i \left(a_1 \rho + \frac{a_2}{v^2} \rho v_i \cdot u + \frac{a_3}{v^2} \rho \frac{u^2}{v^2} + \frac{a_4}{v^4} \rho (v_i \cdot u)^2 \right) \quad (1.19)$$

Here a_1, a_2, a_3 and a_4 are arbitrary constants. The collision operator ω_i will conserve the mass and momentum, so that

$$\sum_i \omega_i = 0, \sum_i \omega_i v_i = 0 \quad (1.20)$$

The derivation of Navier-Stokes equation by expanding the Boltzman equations (1.13), (1.14) and (1.20), [3] we get

$$\frac{\partial \rho u_j}{\partial t} + \frac{\partial}{\partial k} \left[M_{jk} + \frac{\eta}{2} \left(\varepsilon \frac{\partial}{\partial t_1} M_{jk}^{(0)} + \frac{\partial}{\partial l} \left(\sum_i m_i v_{ij} v_{ik} v_{il} F_i^{(0)} \right) \right) \right] = 0 \quad (1.21)$$

The momentum tensor M_{jk} [3] can be determined by

$$M_{jk} = M_{jk}^{(0)} + M_{jk}^{(1)} + M_{jk}^{(2)} + \dots \quad (1.22)$$

$$F_i^{(1)} = -\eta \varepsilon \frac{a_2}{v^2} \left(v_{ij} v_{ik} - \frac{a_1 v^2}{a_2} \delta_{jk} \right) \frac{\partial}{\partial (1j)} (\rho u_k) \quad (1.23)$$

III. NUMERICAL SIMULATIONS AND FIGURES

In PAKKA-model Structure, the fibre webs are grown randomly depositing flexible fibres having rectangular cross section on the left substrate. These are periodic in xy-plane. For making the structures homogeneous, the surface layers perpendiculars to z-axis are removed. The porosity of the sample is organized by varying the flexibility of fibres. In this model the fibre width to length ratio are 1/20 and the sample size is four fibre lengths in xy-plane and ten fibre width in z-direction. For the determination of anisotropy of permeability, the imposed boundary conditions are selected so as to use the structural symmetry inherent in the model. The different sample models [1], [2], [3] are shown in the figures- 3.1, 3.2, 3.3 and 3.4.

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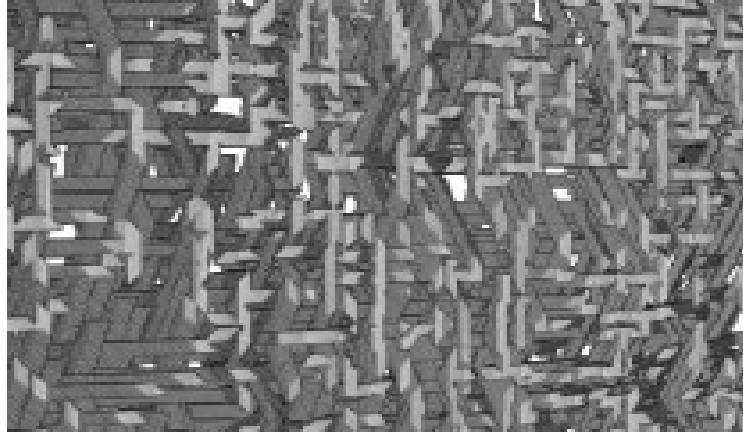


Figure-3.1

(PAKKA-model sample: A sample with fibres oriented in two perpendicular directions [3])

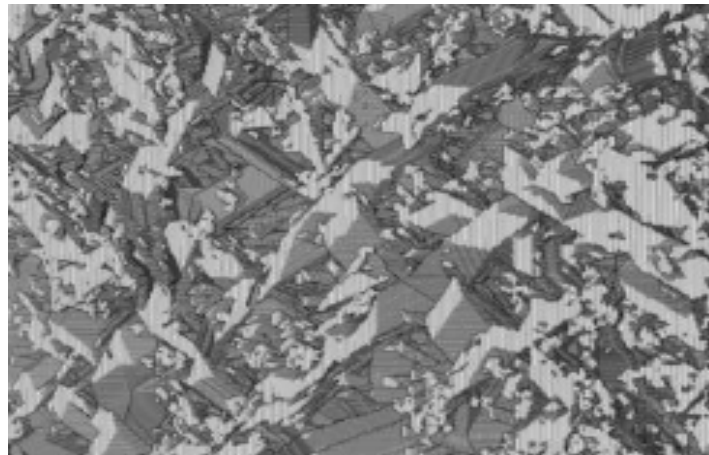


Figure-3.2

(PAKKA-model sample: A sample with fines and randomly oriented fibres [3])



Figure-3.3

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(The figure represents the 2D horizontal sections in the middle of the simulated flow fields in PAKKA samples of 0% of fines content. Black color indicates solid area, and white and dark colors in the pore area indicate low and high velocities respectively [3])



Figure-3.4

(The figure represents the 2D horizontal sections in the middle of the simulated flow fields in PAKKA samples of 50 % of fines content. Black color indicates solid area, and white and dark colors in the pore area indicate low and high velocities respectively [3])

For the transverse flow the fluid layers of not less than 1/10 of the thickness of sample are added up and down side of the sample. The periodic boundary conditions are used in z-direction and also in xy-plane due to periodicity of PAKKA-model in these directions. For in plane flow the frictionless slip conditions are imposed at the boundaries perpendicular to z-direction. The values calculated by the experiments are shown in the table-2.1 [3], [6], [7], [8].

Table-2.1. (The structured parameters and simulated quantities the four different samples)

Sample	T (mm) Thickness	ϕ Porosity	k (10^{-10} m^2) Darcy permeability	S ($10^3 / \text{m}$) Pore surface area per volume of sample	τ Tortuosity
A	0.58	0.479	0.510	20.2	1.190
B	0.87	0.526	1.47	16.6	1.171
C	0.66	0.530	0.937	20.5	1.174
D	0.87	0.560	0.61	15.4	1.166

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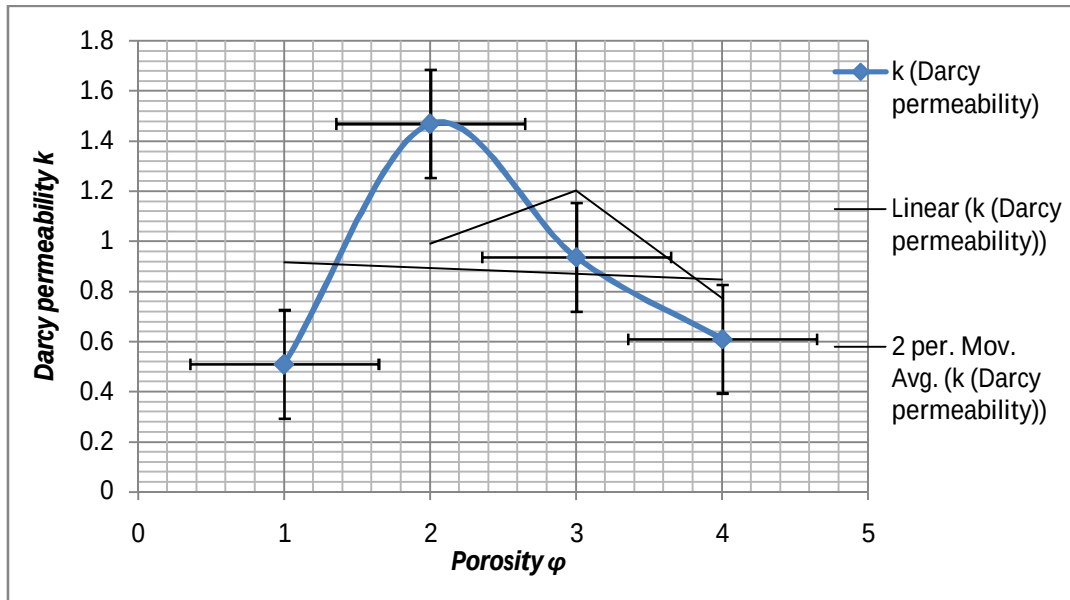


Figure-5 (Variation of Darcy permeability k with porosity ϕ with error bars with standard errors and two period moving averages)

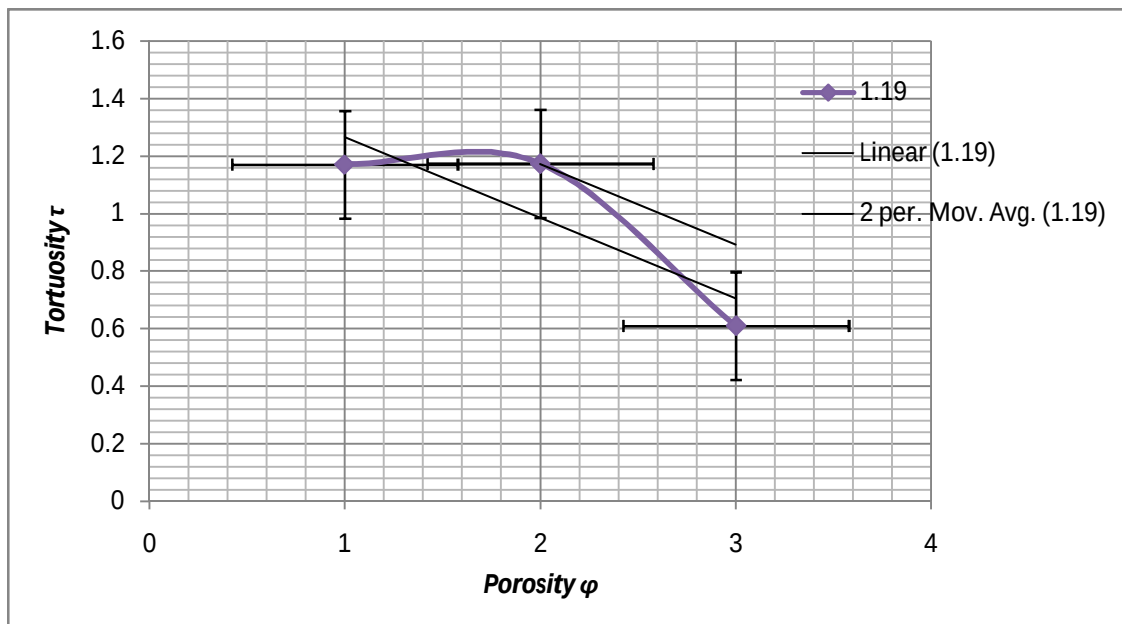


Figure-6 (Variation of Tortuosity τ with porosity ϕ with error bars with standard errors and two period moving averages)

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IV. CONCLUSION

The lattice-Boltzmann method is used to solve 3D Newtonian fluid-flow problems in porous media given by tomographic imaging and by numerical modelling. By the lattice-Boltzmann simulations there are examination of the quality of numerical models of porous media and tomographic imaging techniques. The different models were taken to give a qualitatively dependence of permeability on porosity, but also effects of other structural properties were demonstrated. These include specific surface area, tortuosity, and shape and orientation of particles in the medium. The transverse and in-plane flow simulations for PAKKA-model samples augmented the characteristics of the samples and revealed e.g. the effect of fibre flexibility. The simulations show that results on paper can be sensitive to imaging techniques, since the permeability differences between low- and high-resolution images can be seen though the results within each technique were consistent. The table and graphs show the variation and dependency of porosity on the Darcy permeability and tortuosity.

The results obtained show that fluid-flow simulations for PAKKA-model samples and tomographic images of some paper grades, can produce results that are similar to those found in experiments. There are obviously also experimental factors, such as e.g. swelling of fibres in water, which are not included in the present simulations. The fluid flow simulation results of various paper grades could increase the reliability of modelling the paper-making processes, and those of wires the possibilities to examine and develop the paper-machine clothing.

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