

On $b\hat{g}$ -Quotient Mappings in Topological Spaces

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ABSTRACT: In this paper, we define new type of quotient mappings namely $b\hat{g}$ -quotient mappings and we prove some of their basic properties. Also, we introduce strongly $b\hat{g}$ -open map, strongly $b\hat{g}$ -quotient map, $b\hat{g}^*$ -quotient map and we discuss some of their relationship among other mappings.

KEYWORDS: $b\hat{g}$ – closed set, $b\hat{g}$ -continuous function, $b\hat{g}$ -irresolute function, $b\hat{g}$ -quotient mapping, strongly $b\hat{g}$ -open map, strongly $b\hat{g}$ -quotient map, $b\hat{g}^*$ -quotient map.

I. INTRODUCTION

In 1963, N.Levine[5] introduced semi-open sets in topology and studied their properties. Andrijevic[1] introduced one such new version called b -open sets in 1996. N.Levine[6] introduced the concept of generalized closed sets and studied their properties in 1970. By considering the concept of g -closed sets many concepts of topology have been generalized and interesting results have been obtained by several mathematician. M.K.R.S.Veerakumar[9] defined \hat{g} -closed sets in 2003. R.Subasree and M.Mariasingam[7] introduced $b\hat{g}$ -closed sets in 2013. Also, we introduced $b\hat{g}$ -continuous maps [8] and studied their properties.

In this paper, we define new class of mappings namely $b\hat{g}$ -quotient mappings and we prove some of their basic properties. Also, we introduce some new type of mappings namely strongly $b\hat{g}$ -open map, strongly $b\hat{g}$ -quotient map, $b\hat{g}^*$ -quotient map and we discuss some of their relationship among other mappings.

II. PRELIMINARIES

Throughout this paper (X, τ) (or simply X) represents topological spaces in which no separation axioms are assumed unless otherwise mentioned. For a subset A of (X, τ) , $Cl(A)$, $Int(A)$ and A^c denote the closure of A , interior of A and the complement of A respectively. We are giving some definitions.

Definition 2.1: A subset A of a topological space (X, τ) is called

- (i) semi-open set[5] if $A \subseteq Cl(Int(A))$.
- (ii) α -open set[7] if $A \subseteq Int(Cl(Int(A)))$.
- (iii) b -open set[1] if $A \subseteq Cl(Int(A)) \cup Int(Cl(A))$.

Definition 2.2: A subset A of a topological space (X, τ) is called $ab\hat{g}$ -closed set[7], if $bCl(A) \subseteq U$ whenever $A \subseteq U$ and U is \hat{g} -open in X .

The complements of the above mentioned closed sets are their respective open sets.

Remark 2.3: The family of all $b\hat{g}$ -closed sets and $b\hat{g}$ -open sets are denoted by $b\hat{g}-C(X)$ and $b\hat{g}-O(X)$.

Definition 2.4: A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is called a

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- (i) continuous[11] if $f^{-1}(V)$ is closed in X for every closed set V in Y .
- (ii) α -continuous[14] if $f^{-1}(V)$ is α -closed in X for every closed set V in Y .
- (iii) $b\hat{g}$ -continuous[8] if $f^{-1}(V)$ is $b\hat{g}$ -closed in X for every closed set V in Y .
- (iv) $b\hat{g}$ – irresolute map[8] if $f^{-1}(V)$ is $b\hat{g}$ -closed in (X,τ) for every $b\hat{g}$ -closed set V in (Y,σ) .
- (v) $b\hat{g}$ - open map[8] if $f(V)$ is $b\hat{g}$ – open in (Y,σ) for every open set V in (X,τ) .
- (vi) quotient[11] if f is continuous and $f^{-1}(V)$ is open in X implies V is open in Y .
- (vii) α -quotient[10] if f is α -continuous and $f^{-1}(V)$ is open in X implies V is α -open in Y .
- (viii) \hat{g} -quotient[12] if f is \hat{g} -continuous and $f^{-1}(V)$ is open in X implies V is \hat{g} -open in Y .
- (ix) $\alpha\hat{g}$ -quotient[13] if f is $\alpha\hat{g}$ -continuous and $f^{-1}(V)$ is open in X implies V is $\alpha\hat{g}$ -open in Y .
- (x) strongly $\alpha\hat{g}$ -quotient [13] if provided a set V of (Y, σ) is closed if and only if $f^{-1}(V)$ is $\alpha\hat{g}$ -closed in (X, τ) .

Definition 2.5: A space X is called a $T_{b\hat{g}^*}$ -space[2] if every $sb\hat{g}$ -closed set in it is closed.

III. $b\hat{g}$ -QUOTIENT MAP

In this section, we introduce two new types of mappings namely $b\hat{g}$ -quotient map and strongly $b\hat{g}$ -open map and discuss some of its basic properties.

Definition 3.1: A surjective map $f:(X,\tau) \longrightarrow (Y,\sigma)$ is said to be a $b\hat{g}$ -quotient map, if f is $b\hat{g}$ -continuous and $f^{-1}(V)$ is open in $X \Rightarrow V$ is $b\hat{g}$ -open in Y .

Example 3.2: Let $X=\{a,b,c,d\}$ and $Y = \{p,q,r\}$ with topologies
 $\tau = \{X, \Phi, \{a\}, \{a,c\}, \{a,b,d\}\}$ and $\sigma = \{Y, \{p\}, \{p,q\}, \{p,r\}\}$
 Define a function $f: (X,\tau) \longrightarrow (Y, \sigma)$ by $f(a) = p, f(b) = r, f(c) = q, f(d) = r$.
 $b\hat{g}\text{-}O(X)=\{X,\Phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$
 $b\hat{g}\text{-}O(Y)=\{Y,\Phi, \{p\}, \{p,q\}, \{p,r\}\}$
 Clearly f is a $b\hat{g}$ -quotient map.

Definition 3.3: A function $f: (X,\tau) \longrightarrow (Y, \sigma)$ is said to be a strongly $b\hat{g}$ -open map, if $f(U)$ is $b\hat{g}$ -open in Y for each $b\hat{g}$ -open set U in X .

Example 3.4: Let $X=Y=\{a,b,c,d\}$ with topologies
 $\tau = \{X, \Phi, \{b\}, \{a,b\}, \{b,c,d\}\}$ and $\sigma = \{Y, \Phi, \{a\}, \{a,c\}, \{a,b,d\}\}$
 Define a function $f: (X,\tau) \longrightarrow (Y, \sigma)$ by $f(a) = d, f(b) = a, f(c) = c, f(d) = b$.
 $b\hat{g}\text{-}O(X)=\{X,\Phi, \{b\}, \{c\}, \{d\}, \{a,b\}, \{b,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{b,c,d\}\}$
 $b\hat{g}\text{-}O(Y)=\{X,\Phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,c,d\}, \{a,b,d\}\}$
 Clearly f is strongly $b\hat{g}$ -open map.

Proposition 3.5: If a map $f: (X,\tau) \longrightarrow (Y, \sigma)$ is surjective, $b\hat{g}$ -continuous and $b\hat{g}$ -open map, then f is a $b\hat{g}$ -quotient map.

Proof: We need only to prove that $f^{-1}(V)$ is open in X implies V is a $b\hat{g}$ -open set in Y . Let $f^{-1}(V)$ is open in X . Since f is $b\hat{g}$ -open map, then $f(f^{-1}(V))$ is a $b\hat{g}$ -open set in Y . Hence V is a $b\hat{g}$ -open set in Y , as f is surjective $f(f^{-1}(V)) = V$. Thus f is a $b\hat{g}$ -quotient map.

Proposition 3.6:

- (i) Every quotient map is $b\hat{g}$ -quotient map.
- (ii) Every \hat{g} -quotient map is $b\hat{g}$ -quotient map.

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(iii) Every $\alpha\hat{g}$ -quotient map is $b\hat{g}$ -quotient map.

Proof:

- (i) The proof is straight forward, since from [8] proposition 3.3, "Every continuous map is $b\hat{g}$ -continuous" and from [7] remark 3.23, "Every open set is $b\hat{g}$ -open".
- (ii) The proof is straight forward, since from [8] proposition 3.16, "Every \hat{g} -continuous map is $b\hat{g}$ -continuous" and from [7] proposition 3.9, "Every \hat{g} -open set is $b\hat{g}$ -open".
- (iii) The proof is straight forward, since from [8] proposition 3.28, "Every $\alpha\hat{g}$ -continuous map is $b\hat{g}$ -continuous" and from [7] proposition 3.20, "Every $\alpha\hat{g}$ -open set is $b\hat{g}$ -open".

The converse of the above proposition is not true in general as seen in the following proposition.

Proposition 3.7:

- (i) Every $b\hat{g}$ -quotient map need not be a quotient map.
- (ii) Every $b\hat{g}$ -quotient map need not be a \hat{g} -quotient map.
- (iii) Every $b\hat{g}$ -quotient map need not be an $\alpha\hat{g}$ -quotient map.

Proof:

- (i) Let $X=Y=\{a,b,c,d\}$ with topologies
 $\tau = \{X, \Phi, \{a\}, \{a,c\}, \{a,b,d\}\}$ and $\sigma = \{Y, \Phi, \{b\}, \{a,b\}, \{b,c,d\}\}$
 Define a function $f: (X, \tau) \longrightarrow (Y, \sigma)$ by $f(a) = b, f(b) = a, f(c) = c, f(d) = d$.
 $b\hat{g}\text{-}O(X) = \{X, \Phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,c,d\}, \{a,b,d\}\}$
 $b\hat{g}\text{-}O(Y) = \{Y, \Phi, \{b\}, \{c\}, \{d\}, \{a,b\}, \{b,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{b,c,d\}\}$
 Clearly f is $b\hat{g}$ -quotient map, but not a quotient map. Since f is not continuous and also $f^{-1}(\{b,c\}) = \{a,c\}$ which is open in X but $\{b,c\}$ is not open in Y .
- (ii) Let $X=Y=\{a,b,c\}$ with topologies
 $\tau = \{X, \Phi, \{a,c\}\}$ and $\sigma = \{Y, \Phi, \{a\}, \{b\}, \{a,b\}, \{a,c\}\}$
 Define a function $f: (X, \tau) \longrightarrow (Y, \sigma)$ by $f(a) = a, f(b) = c, f(c) = b$.
 $\hat{g}\text{-}C(X) = \{X, \Phi, \{b\}, \{a,b\}, \{b,c\}\}$
 $b\hat{g}\text{-}C(X) = \{X, \Phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{b,c\}\}$
 $\hat{g}\text{-}O(Y) = \{Y, \Phi, \{a\}, \{b\}, \{a,b\}, \{a,c\}\}$
 $b\hat{g}\text{-}O(Y) = \{Y, \Phi, \{a\}, \{b\}, \{a,b\}, \{a,c\}\}$
 Clearly f is $b\hat{g}$ -quotient map, but not \hat{g} -quotient map, since f is not \hat{g} -continuous. However $f^{-1}(\{a,b\}) = \{a,c\}$ which is open in X implies $\{a,b\}$ is \hat{g} -open in Y .
- (iii) Let $X=Y=\{a,b,c\}$ with topologies
 $\tau = \{X, \Phi, \{a\}, \{b,c\}\}$ and $\sigma = \{Y, \Phi, \{a\}, \{b\}, \{a,b\}\}$
 Define a function $f: (X, \tau) \longrightarrow (Y, \sigma)$ by $f(a) = a, f(b) = b, f(c) = c$.
 $\alpha\hat{g}\text{-}C(X) = \{X, \Phi, \{a\}, \{b,c\}\}$
 $b\hat{g}\text{-}C(X) = \{X, \Phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$
 $\alpha\hat{g}\text{-}O(Y) = \{Y, \Phi, \{a\}, \{b\}, \{a,b\}\}$
 $b\hat{g}\text{-}O(Y) = \{Y, \Phi, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{b,c\}\}$
 Clearly f is $b\hat{g}$ -quotient map, but not $\alpha\hat{g}$ -quotient map, since f is not $\alpha\hat{g}$ -continuous. Also, for an open set $\{b,c\}$ in X , $f^{-1}(\{b,c\}) = \{b,c\}$ which is open in X implies $\{a\}$ is $\alpha\hat{g}$ -open in Y .

IV. STRONG FORM OF $b\hat{g}$ -QUOTIENT MAP

In this section we introduce strong form of $b\hat{g}$ -quotient map and $b\hat{g}^*$ -quotient map and obtained their basic properties.

Definition 4.1: Let $f: (X, \tau) \longrightarrow (Y, \sigma)$ be a surjective map. Then f is called strongly $b\hat{g}$ -quotient map, provided a set V of (Y, σ) is open in Y if and only if $f^{-1}(V)$ is $b\hat{g}$ -open in X .

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Example 4.2: Let $X=\{a,b,c,d\}$ and $Y=\{p,q,r\}$ with topologies

$\tau = \{X, \Phi, \{a\}, \{b,c\}, \{a,b,c\}\}$ and $\sigma = \{Y, \Phi, \{p\}, \{q\}, \{p,q\}, \{p,r\}\}$

Define a function $f: (X, \tau) \longrightarrow (Y, \sigma)$ by $f(a) = f(b) = p, f(c) = q, f(d) = r.$

$b\hat{g}\text{-}O(X) = \{X, \Phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}\}$

Clearly f is a strongly $b\hat{g}$ -quotient map.

Proposition 4.3:

(i) Every strongly $b\hat{g}$ -quotient map is $b\hat{g}$ -open map.

(ii) Every strongly $b\hat{g}$ -quotient map is strongly $b\hat{g}$ -open.

Proof:

(i) Let f be a strongly $b\hat{g}$ -quotient map and V be any open set in X . Since, from [7] remark 3.23, "Every open set is $b\hat{g}$ -open", V is also a $b\hat{g}$ -open set in (X, τ) . That is $f^{-1}(f(V))$ is $b\hat{g}$ -open in (X, τ) . Since f is strongly $b\hat{g}$ -quotient, $f(V)$ is open in Y and hence $f(V)$ is $b\hat{g}$ -open in Y . Hence f is a $b\hat{g}$ -open map.

(ii) Let f be a strongly $b\hat{g}$ -quotient map and V be any $b\hat{g}$ -open set in X . That is $f^{-1}(f(V))$ is $b\hat{g}$ -open in (X, τ) . Since f is strongly $b\hat{g}$ -quotient, $f(V)$ is open in Y and hence $b\hat{g}$ -open in Y . Hence f is a strongly $b\hat{g}$ -open map.

The converse of the above Proposition need not be true as seen in the following Proposition.

Proposition 4.4:

(i) Every $b\hat{g}$ -open map need not be strongly $b\hat{g}$ -quotient map.

(ii) Every strongly $b\hat{g}$ -open map need not be strongly $b\hat{g}$ -quotient map.

Proof:

(i) Let $X=Y=\{a,b,c,d\}$ with topologies

$\tau = \{X, \Phi, \{a\}, \{a,c\}, \{a,b,d\}\}$ and $\sigma = \{Y, \Phi, \{a\}, \{b,c\}, \{a,b,c\}\}$

Define a function $f: (X, \tau) \longrightarrow (Y, \sigma)$ by $f(a) = a, f(b) = b, f(c) = c, f(d) = d.$

$b\hat{g}\text{-}O(X) = \{X, \Phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,c,d\}, \{a,b,d\}\}$

$b\hat{g}\text{-}O(Y) = \{Y, \Phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}\}$

Clearly f is a $b\hat{g}$ -open map but not strongly $b\hat{g}$ -quotient map. Since $f^{-1}(\{b\}) = \{b\}$ which is $b\hat{g}$ -open in X , but $\{b\}$ is not open in Y .

(ii) Let $X=\{a,b,c\}$ and $Y=\{p,q,r\}$ with topologies

$\tau = \{X, \Phi, \{a\}, \{b\}, \{a,b\}\}$ and $\sigma = \{Y, \Phi, \{p,r\}\}$

Define a function $f: (X, \tau) \longrightarrow (Y, \sigma)$ by $f(a) = p, f(b) = r, f(c) = q.$

$b\hat{g}\text{-}O(X) = \{X, \Phi, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{b,c\}\}$

$b\hat{g}\text{-}O(Y) = \{Y, \Phi, \{p\}, \{r\}, \{p,q\}, \{p,r\}, \{q,r\}\}$

Clearly f is a strongly $b\hat{g}$ -open map, but not strongly $b\hat{g}$ -quotient map. Since $f^{-1}(\{p\}) = \{a\}$ which is $b\hat{g}$ -open in X , but $\{p\}$ is not open in Y .

Definition 4.5: A surjective map $f: (X, \tau) \longrightarrow (Y, \sigma)$ is said to be a $b\hat{g}^*$ -quotient map, if f is $b\hat{g}$ -irresolute and $f^{-1}(V)$ is $b\hat{g}$ -open in X implies V is open in Y .

Example 4.6: Let $X = \{a,b,c,d\}$ and $Y = \{p,q,r\}$ with topologies

$\tau = \{X, \Phi, \{b\}, \{a,b\}, \{b,c,d\}\}$ and $\sigma = \{Y, \Phi, \{p\}, \{q\}, \{p,q\}, \{p,r\}\}$

Define a function $f: (X, \tau) \longrightarrow (Y, \sigma)$ by $f(a) = r, f(b) = p, f(c) = q, f(d) = p.$

$b\hat{g}\text{-}O(X) = \{X, \Phi, \{b\}, \{c\}, \{d\}, \{a,b\}, \{b,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{b,c,d\}\}$

$b\hat{g}\text{-}O(Y) = \{Y, \Phi, \{p\}, \{q\}, \{p,q\}, \{p,r\}\}$

Clearly f is a $b\hat{g}^*$ -quotient map.

Proposition 4.7:

(i) Every $b\hat{g}^*$ -quotient map is $b\hat{g}$ -irresolute.

(ii) Every $b\hat{g}^*$ -quotient map is strongly $b\hat{g}$ -open map.

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Proof:

- (i) Proof follows from definitions 4.5 and 2.4 (iv).
- (ii) Let f be a $b\hat{g}^*$ -quotient map. We need to prove that for each $b\hat{g}$ -open set V in X , $f(V)$ is $b\hat{g}$ -open in Y . Let V be any $b\hat{g}$ -open set in X . As f is surjective, $f^{-1}(f(V))$ is $b\hat{g}$ -open in (X, τ) . Since f is $b\hat{g}^*$ -quotient, $f(V)$ is open in Y and hence from [7] remark 3.23, $f(V)$ is also $b\hat{g}$ -open in Y . Hence f is a strongly $b\hat{g}$ -open map.

The converse of the above Proposition need not be true as seen in the following Proposition.

Proposition 4.8:

- (i) Every $b\hat{g}$ -irresolute map need not be $b\hat{g}^*$ -quotient map.
- (ii) Every strongly $b\hat{g}$ -open map need not be $b\hat{g}^*$ -quotient map.

Proof:

- (i) Let $X=\{a,b,c,d\}$ and $Y=\{p,q,r\}$ with topologies $\tau = \{X, \Phi, \{a\}, \{b,c\}, \{a,b,c\}\}$ and $\sigma = \{Y, \Phi, \{p\}, \{q,r\}\}$
Define a function $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = r, f(b) = q, f(c) = p, f(d) = r$.
 $b\hat{g}\text{-}O(X) = \{X, \Phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}\}$
 $b\hat{g}\text{-}O(Y) = \{Y, \Phi, \{p\}, \{q\}, \{r\}, \{p,q\}, \{p,r\}, \{q,r\}\}$
Clearly f is $b\hat{g}$ -irresolute but not $b\hat{g}^*$ -quotient map. Since $f^{-1}(\{q\}) = \{b\}$ which is $b\hat{g}$ -open in X but $\{q\}$ is not open in Y .
- (ii) Let $X=Y=\{a,b,c,d\}$ with topologies $\tau = \{X, \Phi, \{b\}, \{a,b\}, \{b,c,d\}\}$ and $\sigma = \{Y, \Phi, \{a\}, \{a,c\}, \{a,b,d\}\}$
Define a function $f: (X, \tau) \rightarrow (Y, \sigma)$ by $f(a) = d, f(b) = a, f(c) = c, f(d) = b$.
 $b\hat{g}\text{-}O(X) = \{X, \Phi, \{b\}, \{c\}, \{d\}, \{a,b\}, \{b,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{b,c,d\}\}$
 $b\hat{g}\text{-}O(Y) = \{Y, \Phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,c,d\}, \{a,b,d\}\}$
Clearly f is a strongly $b\hat{g}$ -open map but not $b\hat{g}^*$ -quotient map. Since f is not $b\hat{g}$ -irresolute (i.e.,) the inverse image of a $b\hat{g}$ -closed set $\{a,b,c\}$ in Y is $\{b,c,d\}$ which is not $b\hat{g}$ -closed in X .

V. COMPARISONS

In this section, we compared the quotient maps such as $b\hat{g}$ -quotient map, strongly $\alpha\hat{g}$ -quotient map, strongly $b\hat{g}$ -quotient map and $b\hat{g}^*$ -quotient map and found the result through counter examples.

Proposition 5.1: Every $b\hat{g}^*$ -quotient map is strongly $b\hat{g}$ -quotient.

Proof: Let V be any open set in Y , Then from [7] remark 3.23, V is also $b\hat{g}$ -open in Y . Since f is $b\hat{g}$ -irresolute, $f^{-1}(V)$ is $b\hat{g}$ -open in X . This means that V is open in Y implies $f^{-1}(V)$ is $b\hat{g}$ -open in X . Now if $f^{-1}(V)$ is $b\hat{g}$ -open in X and since f is $b\hat{g}^*$ -quotient map, V is an open set in Y . This means that $f^{-1}(V)$ is $b\hat{g}$ -open in X implies V is open in Y . Hence f is a strongly $b\hat{g}$ -quotient map.

The converse of the above Proposition is also true as seen in the following proposition.

Proposition 5.2: Every strongly $b\hat{g}$ -quotient map is $b\hat{g}^*$ -quotient.

Proof: Let V be any $b\hat{g}$ -open set in Y , As f is surjective, $f(f^{-1}(V))$ is $b\hat{g}$ -open in Y . By Proposition 4.3(ii), "Every strongly $b\hat{g}$ -quotient map is strongly $b\hat{g}$ -open" then, f is also strongly $b\hat{g}$ -open map. Hence, $f(f^{-1}(V))$ is $b\hat{g}$ -open in Y implies $f^{-1}(V)$ is $b\hat{g}$ -open in X . Thus, f is a $b\hat{g}$ -irresolute map. Now if, $f^{-1}(V)$ is $b\hat{g}$ -open in X and since f is strongly $b\hat{g}$ -quotient map, V is an open set in Y . This means that $f^{-1}(V)$ is $b\hat{g}$ -open in X implies V is open in Y . Hence f is a $b\hat{g}^*$ -quotient map.

Proposition 5.3:

- (i) Every strongly $\alpha\hat{g}$ -quotient map is strongly $b\hat{g}$ -quotient map.
- (ii) Every strongly $b\hat{g}$ -quotient map is $b\hat{g}$ -quotient map.

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(iii) Every $b\hat{g}^*$ -quotient map is $b\hat{g}$ -quotient map.

Proof:

- (i) The proof is straightforward, since from [7] Proposition 3.20, "Every $\alpha\hat{g}$ -closed set is $b\hat{g}$ -closed set".
- (ii) Let V be any open set in Y . Since f is strongly $b\hat{g}$ -quotient map, $f^{-1}(V)$ is $b\hat{g}$ -open in X . This means that f is $b\hat{g}$ -continuous. Now let $f^{-1}(V)$ be any open set in X . Since from Proposition [7] remark 3.23, $f^{-1}(V)$ is also $b\hat{g}$ -open in X and since f is strongly $b\hat{g}$ -quotient map, V is an open set in Y . Therefore V is also $b\hat{g}$ -open in Y . This means that $f^{-1}(V)$ is open in X implies V is $b\hat{g}$ -open in Y . Hence f is a $b\hat{g}$ -quotient map.
- (iii) Let V be any open set in Y , then from [7] remark 3.23, V is also $b\hat{g}$ -open in Y . Since f is $b\hat{g}^*$ -quotient map, $f^{-1}(V)$ is $b\hat{g}$ -open in X . This means that for every open set V in Y , $f^{-1}(V)$ is $b\hat{g}$ -open in X . Hence f is $b\hat{g}$ -continuous. Now, let $f^{-1}(V)$ be any open set in X . Since, "Every open set is $b\hat{g}$ -open", $f^{-1}(V)$ is also $b\hat{g}$ -open in X and since f is $b\hat{g}^*$ -quotient map, V is an open set in Y . Therefore V is also $b\hat{g}$ -open in Y . This means that $f^{-1}(V)$ is open in X implies V is $b\hat{g}$ -open in Y . Hence f is a $b\hat{g}$ -quotient map.

The converse of the above proposition is not true as seen in the following proposition.

Proposition 5.4:

- (i) Every strongly $b\hat{g}$ -quotient map need not be strongly $\alpha\hat{g}$ -quotient map.
- (ii) Every $b\hat{g}$ -quotient map need not be strongly $b\hat{g}$ -quotient map.
- (iii) Every $b\hat{g}$ -quotient map need not be $b\hat{g}^*$ -quotient map.

Proof:

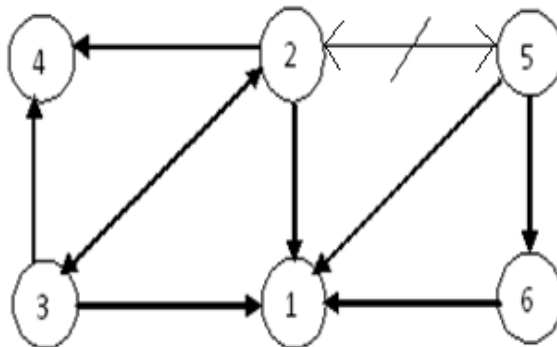
- (i) Let $X=\{a,b,c,d\}$ and $Y = \{p,q,r\}$ with topologies
 $\tau = \{X, \Phi, \{a\}, \{b,c\}, \{a,b,c\}\}$ and $\sigma = \{Y, \Phi, \{p\}, \{q\}, \{p,q\}, \{p,r\}\}$
 Define a function $f: (X, \tau) \longrightarrow (Y, \sigma)$ by $f(a) = p, f(b) = p, f(c) = q, f(d) = r$.
 $b\hat{g}-O(X) = \{X, \Phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{a,d\}, \{b,c\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}, \{b,c,d\}\}$
 $\alpha\hat{g}-O(X) = \{X, \Phi, \{a\}, \{b\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,c\}\}$
 Clearly f is strongly $b\hat{g}$ -quotient, but not strongly $\alpha\hat{g}$ -quotient map. Since for an open set $\{p,r\}$ in Y , $f^{-1}(\{p,r\}) = \{a,b,d\}$ which is $b\hat{g}$ -open in X , but not $\alpha\hat{g}$ -open in X .
- (ii) Let $X=\{a,b,c,d\}$ and $Y = \{p,q,r\}$ with topologies
 $\tau = \{X, \Phi, \{b\}, \{a,b\}, \{b,c,d\}\}$ and $\sigma = \{Y, \Phi, \{p\}, \{r\}, \{p,r\}, \{q,r\}\}$
 Define a function $f: (X, \tau) \longrightarrow (Y, \sigma)$ by $f(a) = r, f(b) = p, f(c) = q, f(d) = r$.
 $b\hat{g}-O(X) = \{X, \Phi, \{b\}, \{c\}, \{d\}, \{a,b\}, \{b,c\}, \{b,d\}, \{a,b,c\}, \{a,b,d\}, \{b,c,d\}\}$
 $b\hat{g}-O(Y) = \{Y, \Phi, \{p\}, \{r\}, \{p,r\}, \{q,r\}\}$
 Clearly f is a $b\hat{g}$ -quotient map but not strongly $b\hat{g}$ -quotient map. Since $f^{-1}(\{q\}) = \{c\}$ which is $b\hat{g}$ -open in X , but $\{q\}$ is not open in Y .
- (iii) Let $X=\{a,b,c,d\}$ and $Y = \{p,q,r\}$ with topologies
 $\tau = \{X, \Phi, \{a\}, \{a,c\}, \{a,b,d\}\}$ and $\sigma = \{Y, \Phi, \{p\}, \{p,q\}, \{p,r\}\}$
 Define a function $f: (X, \tau) \longrightarrow (Y, \sigma)$ by $f(a) = p, f(b) = r, f(c) = q, f(d) = r$.
 $b\hat{g}-O(X) = \{X, \Phi, \{a\}, \{b\}, \{c\}, \{d\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$
 $b\hat{g}-O(Y) = \{Y, \Phi, \{p\}, \{p,q\}, \{p,r\}\}$
 Clearly f is a $b\hat{g}$ -quotient map but not $b\hat{g}^*$ -quotient map. Since $f^{-1}(\{q\}) = \{c\}$ which is $b\hat{g}$ -open in X , but $\{q\}$ is not open in Y .

Remark 5.5: From the above discussions and known results, we have the following diagram. In this diagram, $A \longrightarrow B$ means A implies B but not conversely. $A \not\longleftrightarrow B$ means A and B are independent of each other.

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- | | | |
|----------------------------------|--------------------------------------|-----------------------------------|
| 1. $b\hat{g}$ -quotient map | 2. strongly $b\hat{g}$ -quotient map | 3. $b\hat{g}^*$ -quotient map |
| 4. strongly $b\hat{g}$ -open map | 5. quotient map | 6. $\alpha\hat{g}$ -quotient map. |

VI. APPLICATIONS OF $b\hat{g}$ -QUOTIENT MAPS

In this section, we proved several theorems on composite functions based on $b\hat{g}$ -quotient map.

Proposition 6.1: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an open surjective, $b\hat{g}$ -irresolute map and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be a $b\hat{g}$ -quotient map. Then their composition $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is a $b\hat{g}$ -quotient map.

Proof: Let V be any open set in (Z, η) . Since g is a $b\hat{g}$ -quotient map, $g^{-1}(V)$ is $b\hat{g}$ -open in Y and since f is $b\hat{g}$ -irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is a $b\hat{g}$ -open set in (X, τ) . This means that for every open set V in (Z, η) , $(g \circ f)^{-1}(V)$ is $b\hat{g}$ -open in (X, τ) . Hence $g \circ f$ is a $b\hat{g}$ -continuous map.

Now, assume that $(g \circ f)^{-1}(V)$ is open in (X, τ) for $V \subseteq Z$, that is $f^{-1}(g^{-1}(V))$ is open in X . Since f is an open map, $f[f^{-1}(g^{-1}(V))]$ is open in Y . It follows that $g^{-1}(V)$ is open in Y , because f is surjective. Since g is a $b\hat{g}$ -quotient map, V is $b\hat{g}$ -open in Z . This means that, for every $(g \circ f)^{-1}(V)$ open in (X, τ) implies V is $b\hat{g}$ -open in Z . Hence $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is a $b\hat{g}$ -quotient map.

Proposition 6.2: Let $h: (X, \tau) \rightarrow (Y, \sigma)$ is a $b\hat{g}$ -quotient map and $g: (X, \tau) \rightarrow (Z, \eta)$ is a continuous map that is constant on each set $h^{-1}(y)$ for $y \in Y$, then g induces a $b\hat{g}$ -continuous map $f: (Y, \sigma) \rightarrow (Z, \eta)$ such that $f \circ h = g$.

Proof: We have to prove, f is $b\hat{g}$ -continuous. Since g is constant on $h^{-1}(y)$ for each $y \in Y$, the set $g[h^{-1}(y)]$ is a one point set in (Z, η) . If $f(y)$ denote this point, then it is clear that f is well-defined and for each $x \in X$, $f(h(x)) = g(x)$. For if, we let V be any open set in (Z, η) then $g^{-1}(V)$ is an open set as g is continuous. But $g^{-1}(V) = h^{-1}(f^{-1}(V))$ is open in (X, τ) . Since h is a $b\hat{g}$ -quotient map, $f^{-1}(V)$ is a $b\hat{g}$ -open set. Hence f is $b\hat{g}$ -continuous.

Proposition 6.3: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a strongly $b\hat{g}$ -open, surjective and $b\hat{g}$ -irresolute map and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be a strongly $b\hat{g}$ -quotient map. Then $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is a strongly $b\hat{g}$ -quotient map.

Proof: We have to prove, $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is a strongly $b\hat{g}$ -quotient map. Let V be any open set in (Z, η) . Since g is a strongly $b\hat{g}$ -quotient map, $g^{-1}(V)$ is $b\hat{g}$ -open set in Y and since f is $b\hat{g}$ -irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is a $b\hat{g}$ -open set in (X, τ) . This means that for every open set V in (Z, η) , $(g \circ f)^{-1}(V)$ is $b\hat{g}$ -open in (X, τ) . Hence $g \circ f$ is a $b\hat{g}$ -continuous map.

Now, assume that $(g \circ f)^{-1}(V)$ is $b\hat{g}$ -open in (X, τ) for $V \subseteq Z$, that is $f^{-1}(g^{-1}(V))$ is $b\hat{g}$ -open in X . Since f is strongly $b\hat{g}$ -open, $f[f^{-1}(g^{-1}(V))]$ is $b\hat{g}$ -open in Y . Since f is surjective, it follows that, $g^{-1}(V)$ is $b\hat{g}$ -open in Y . This gives that V is an open set in Y , since g is strongly $b\hat{g}$ -quotient. This implies that, if $(g \circ f)^{-1}(V)$ is $b\hat{g}$ -open in X implies V is open in Y . Hence $g \circ f$ is strongly $b\hat{g}$ -quotient map.

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Theorem 6.4: Let $h: (X, \tau) \rightarrow (Y, \sigma)$ be a $b\hat{g}$ -quotient map where (X, τ) and (Y, σ) are $Tb\hat{g}^*$ -spaces. Then $f: (Y, \sigma) \rightarrow (Z, \eta)$ is strongly $b\hat{g}$ -continuous if and only if the composite map $f \circ h: (X, \tau) \rightarrow (Z, \eta)$ is strongly $b\hat{g}$ -continuous.

Proof: Let V be any open set in (Z, η) . Then $f^{-1}(V)$ is open in Y , since f is strongly $b\hat{g}$ -continuous. Now $h^{-1}(f^{-1}(V)) = (f \circ h)^{-1}(V)$ is $b\hat{g}$ -open in X , since h is a $b\hat{g}$ -quotient map. Since X is a $Tb\hat{g}^*$ -space, $h^{-1}(f^{-1}(V))$ is open in X . This implies that for every $b\hat{g}$ -open set V in Z , $h^{-1}(f^{-1}(V)) = (f \circ h)^{-1}(V)$ is open in X . Thus $f \circ h$ is strongly $b\hat{g}$ -continuous. Conversely, let $f \circ h$ is strongly $b\hat{g}$ -continuous, then for every $b\hat{g}$ -open set V in Z , $h^{-1}(f^{-1}(V)) = (f \circ h)^{-1}(V)$ is open in X . Since h is a $b\hat{g}$ -quotient map, $f^{-1}(V)$ is $b\hat{g}$ -open in Y . Since Y is a $Tb\hat{g}^*$ -space, $f^{-1}(V)$ is open in Y . Hence f is a strongly $b\hat{g}$ -continuous map.

Theorem 6.5: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a surjective, strongly $b\hat{g}$ -open and $b\hat{g}$ -irresolute map and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be a $b\hat{g}^*$ -quotient map. Then $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is a $b\hat{g}^*$ -quotient map.

Proof: Let V be a $b\hat{g}$ -open set in (Z, η) . Since g is a $b\hat{g}^*$ -quotient map, $g^{-1}(V)$ is $b\hat{g}$ -open in Y . Since f is $b\hat{g}$ -irresolute, $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$ is a $b\hat{g}$ -open set in (X, τ) . This means that for every $b\hat{g}$ -open set V in (Z, η) , $(g \circ f)^{-1}(V)$ is $b\hat{g}$ -open in (X, τ) . Hence $g \circ f$ is a $b\hat{g}$ -irresolute map.

Now, suppose that $(g \circ f)^{-1}(V)$ is $b\hat{g}$ -open in (X, τ) for $V \subseteq Z$, that is $f^{-1}(g^{-1}(V))$ is $b\hat{g}$ -open in X . Since f is strongly $b\hat{g}$ -open, $f[f^{-1}(g^{-1}(V))]$ is $b\hat{g}$ -open in Y . Since f is surjective, it follows that, $g^{-1}(V)$ is $b\hat{g}$ -open in Y . This gives that V is an open set in Z , since g is $b\hat{g}^*$ -quotient. This implies that, if $(g \circ f)^{-1}(V)$ is $b\hat{g}$ -open in X implies V is open in Z . Hence $g \circ f$ is a $b\hat{g}^*$ -quotient map.

Theorem 6.6: The composition of two $b\hat{g}^*$ -quotient maps is again $b\hat{g}^*$ -quotient.

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \eta)$ be two $b\hat{g}^*$ -quotient maps. We have to prove, their composition $g \circ f: (X, \tau) \rightarrow (Z, \eta)$ is a $b\hat{g}^*$ -quotient map.

Let V be a $b\hat{g}$ -open set in (Z, η) . Since g is $b\hat{g}^*$ -quotient, $g^{-1}(V)$ is $b\hat{g}$ -open in Y . Since f is $b\hat{g}^*$ -quotient, $f^{-1}(g^{-1}(V))$ is $b\hat{g}$ -open in X . That is $(g \circ f)^{-1}(V)$ is $b\hat{g}$ -open in X . Hence $g \circ f$ is $b\hat{g}$ -irresolute. Let $(g \circ f)^{-1}(V)$ is $b\hat{g}$ -open in X . That is $f^{-1}(g^{-1}(V))$ is $b\hat{g}$ -open in X . Since f is $b\hat{g}^*$ -quotient, $g^{-1}(V)$ is open in Y , then $g^{-1}(V)$ is $b\hat{g}$ -open in Y . Since g is $b\hat{g}^*$ -quotient, V is open in Z . This implies that $(g \circ f)^{-1}(V)$ is $b\hat{g}$ -open in X implies V is open in Z . Hence $g \circ f$ is $b\hat{g}^*$ -quotient map.

VII. CONCLUSION

The properties of $b\hat{g}$ -quotient map, $b\hat{g}^*$ -quotient map and strong form of $b\hat{g}$ -quotient map have been studied. Further the relation between strong and weak form of $b\hat{g}$ -quotient map have been investigated. Based on composite function, several theorems have been proved by using $b\hat{g}$ -quotient map.

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