

On Pseudo Regular and Irregular Intuitionistic Fuzzy Graphs

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ABSTRACT: In this paper, pseudo regular intuitionistic fuzzy graphs, pseudo irregular intuitionistic fuzzy graphs, highly pseudo irregular intuitionistic fuzzy graphs, neighbourly pseudo irregular intuitionistic fuzzy graphs, strongly pseudo irregular intuitionistic fuzzy graphs are introduced and some of their properties are discussed.

KEYWORDS: Pseudo degree in a fuzzy graph, Pseudo regular fuzzy graph, pseudo irregular fuzzy graph, highly pseudo irregular fuzzy graph, neighbourly pseudo irregular fuzzy graphs, strongly pseudo irregular fuzzy graph, intuitionistic fuzzy graph.

AMS Subject Classification. 05C12, 03E72, 05C72.

I. INTRODUCTION

In this paper, we consider only finite, simple, connected graphs. We denote the vertex set and the edge set of a graph G by $V(G)$ and $E(G)$ respectively. The degree of a vertex v is the number of edges incident at v , and it is denoted by $d(v)$. A graph G is regular if all its vertices have the same degree. The 2-degree of v [4] is the sum of the degrees of the vertices adjacent to v and it is denoted by $t(v)$. We call $\frac{t(v)}{d(v)}$, the average degree of v . A graph is called pseudo-regular if every vertex of G has equal average degree [3].

In 1965, Lofti A. Zadeh [16] introduced the concept of fuzzy subset of a set as method of representing the phenomena of uncertainty in real life situation. K.T. Atanassov [1] introduced the concept of intuitionistic fuzzy sets as a generalization of fuzzy sets. K.T. Atanassov added a new component (which determines the degree of non-membership) in the definition of fuzzy set. The fuzzy sets give the degree of membership of an element in a given set (and the non- membership degree equals one minus the degree of membership), while intuitionistic fuzzy sets give both a degree of membership and a degree of non-membership which are more-or-less independent from each other, the only requirement is that the sum of these two degrees is not greater than one. Intuitionistic fuzzy sets have been applied in a wide variety of fields including computer science, engineering, mathematics, medicine, chemistry and economics [1, 2].

Azriel Rosenfeld introduced the concept of fuzzy graphs in 1975 [10]. It has been growing fast and has numerous application in various fields. Bhattacharya [5] gave some remarks on fuzzy graphs, and some operations on fuzzy graphs were introduced by Morderson and Peng [9]. Krassimir T Atanassov [2] introduced the intuitionistic fuzzy graph theory. R.Parvathi and M.G.Karunambigai [8] introduced intuitionistic fuzzy graphs as a special case of Atanassov's IFG and discussed some properties of regular intuitionistic fuzzy graphs [6]. M.G. Karunambigai and R.Parvathi and R.Buvanewari introduced constant intuitionistic fuzzy graphs [7]. M. Akram, W. Dudek [3] introduced the regular intuitionistic fuzzy graphs. M.Akram and Bijan Davvaz [4] introduced the notion of strong intuitionistic fuzzy graphs and discussed some of their properties.

N.R.Santhi Maheswari and C.Sekar introduced $(m, (c_1, c_2))$ -regular intuitionistic fuzzy graphs. N.R.Santhi Maheswari and C.Sekar introduced pseudo regular fuzzy graphs and discussed its properties [9]. N.R.Santhi Maheswari and M.Sudha introduced pseudo irregular fuzzy graphs and highly pseudo irregular fuzzy graphs and discussed their properties [9]. N.R.Santhi Maheswari and M. Rajeswari introduced strongly pseudo irregular fuzzy graphs and

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discussed its properties [9]. N.R.Santhi Maheswari and S.Ravinarayanan introduced neighbourly pseudo irregular fuzzy graphs and discussed its properties [9].

These motivate us to introduce pseudo degree, total pseudo degree of a vertex in intuitionistic fuzzy graph and also we introduce pseudo regular intuitionistic fuzzy graphs, pseudo irregular intuitionistic fuzzy graphs, neighbourly pseudo irregular intuitionistic fuzzy graphs, highly pseudo irregular intuitionistic fuzzy graphs, strongly pseudo irregular intuitionistic fuzzy graphs and discussed some of their properties.

II. PRELIMINARIES

We present some known definitions and results for ready reference to go through the work presented in this paper.

Definition 2.1. A Fuzzy graph denoted by $G : (\sigma, \mu)$ on the graph $G^* : (V, E)$: is a pair of functions (σ, μ) where $\sigma : V \rightarrow [0; 1]$ is a fuzzy subset of a set V and $\mu : V \times V \rightarrow [0; 1]$ is a symmetric fuzzy relation on σ such that for all u, v in V the relation $\mu(u, v) = \mu(v, u) \leq \sigma(u) \wedge \sigma(v)$ is satisfied [7].

Definition 2.2. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. The 2-degree of a vertex v in G is defined as the sum of degrees of the vertices adjacent to v and is denoted by $t_G(v)$. That is, $t_G(v) = \sum d_G(u)$, where $d_G(u)$ is the degree of the vertex u which is adjacent with the vertex v [13].

Definition 2.3. A pseudo (average) degree of a vertex v in fuzzy graph G is denoted by $d_a(v)$ and is defined by $d_a(v) = \frac{t_G(v)}{d_G^*(v)}$, where $d_G^*(v)$ is the number of edges incident at v . The total pseudo degree of a vertex v in G is denoted by $t d_a(v)$ and is defined as $t d_a(v) = d_a(v) + \sigma(v)$, for all v in G [13].

Definition 2.4. Let $G : (\sigma, \mu)$ be a fuzzy graph on $G^* : (V, E)$. If $d_a(v) = k$, for all v in V ; then G is called k - pseudo regular fuzzy graph [13].

Definition 2.5. A fuzzy graph G is said to be pseudo irregular fuzzy graph if there exists a vertex which is adjacent to the vertices with distinct pseudo degrees [14].

Definition 2.6. A fuzzy graph G is said to be highly pseudo irregular fuzzy graph if each vertex is adjacent to the vertices having distinct pseudo degrees [14].

Definition 2.7. A fuzzy graph G is said to be a neighbourly pseudo irregular fuzzy graph if every pair of adjacent vertices of G have distinct pseudo degrees.

Definition 2.8. A fuzzy graph G is said to be a strongly pseudo irregular fuzzy graph if every pair of vertices of G have distinct pseudo degrees.

Definition 2.9 An intuitionistic fuzzy graph with underlying set V is defined to be a pair $G = (V, E)$ where (i) $V = \{v_1, v_2, v_3, \dots, v_n\}$ such that $\mu_1 : V \rightarrow [0, 1]$ and $\gamma_1 : V \rightarrow [0, 1]$ denote the degree of membership and non membership of the element $v_i \in V$, ($i=1,2,3,\dots,n$), such that $0 \leq \mu_1(v_i) + \gamma_1(v_i) \leq 1$

(ii) $E \subseteq V \times V$, where $\mu_2 : V \times V \rightarrow [0, 1]$ and $\gamma_2 : V \times V \rightarrow [0, 1]$ are such that $\mu_2(v_i, v_j) \leq \min \{\mu_1(v_i), \mu_1(v_j)\}$ and $\gamma_2(v_i, v_j) \leq \max \{\mu_1(v_i), \mu_1(v_j)\}$ and $0 \leq \mu_2(v_i, v_j) + \gamma_2(v_i, v_j) \leq 1$, for every $(v_i, v_j) \in E$, ($i, j = 1, 2, \dots, n$) [7].

Definition 2.10 Let $G = (V, E)$ be an intuitionistic fuzzy graph on $G^*(V, E)$. Then the degree of a vertex $v_i \in G$ is defined by $d(v_i) = (d_{\mu_1}(v_i), d_{\gamma_1}(v_i))$, where $d_{\mu_1}(v_i) = \sum \mu_2(v_i, v_j)$ and $d_{\gamma_1}(v_i) = \sum \gamma_2(v_i, v_j)$, for $v_i, v_j \in E$ and $\mu_2(v_i, v_j) = 0$ and $\gamma_2(v_i, v_j) = 0$ for $v_i, v_j \notin E$ [7].

Definition 2.11 Let $G = (V, E)$ be an intuitionistic fuzzy graph on $G^*(V, E)$. Then the total degree of a vertex $v_i \in G$ is defined by $td(v_i) = (td_{\mu_1}(v_i), td_{\gamma_1}(v_i))$, where $td_{\mu_1}(v_i) = d_{\mu_1}(v_i) + \mu_1(v_i)$ and $td_{\gamma_1}(v_i) = d_{\gamma_1}(v_i) + \gamma_1(v_i)$ [7].

III. PSEUDO DEGREE OF A VERTEX IN INTUITIONISTIC FUZZY GRAPH

Definition 3.1. Let $G = (V, E)$ be an intuitionistic fuzzy graph on $G^*(V, E)$. Then 2 - degree of a vertex $v \in G$ is defined by $t(v) = (t_{\mu_1}(v), t_{\gamma_1}(v))$, where $t_{\mu_1}(v) = \sum d_{\mu_1}(u)$ and $t_{\gamma_1}(v) = \sum d_{\gamma_1}(u)$, the vertex u is the adjacent to the vertex v .

Definition 3.2 Let $G = (V, E)$ be an intuitionistic fuzzy graph on $G^*(V, E)$. Then pseudo (average) degree of a vertex v in intuitionistic fuzzy graph G is defined by $d_a(v) = (d_{a_{\mu_1}}(v), d_{a_{\gamma_1}}(v))$, where $d_{a_{\mu_1}}(v) = \frac{t_{\mu_1}(v)}{d_G^*(v)}$, where $d_G^*(v)$ is the number of edges incident at v and $d_{a_{\gamma_1}}(v)$ and is defined by $d_{a_{\gamma_1}}(v) = \frac{t_{\gamma_1}(v)}{d_G^*(v)}$, where $d_G^*(v)$ is the number of edges incident at v .

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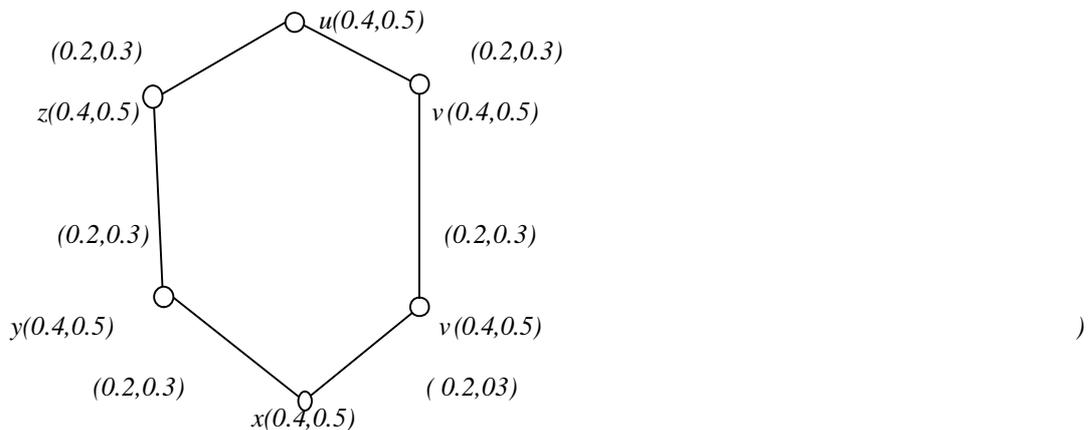
Definition 3.3 Let $G = (V, E)$ be an intuitionistic fuzzy graph on $G^*(V, E)$. The total pseudo degree of a vertex v in intuitionistic fuzzy graph G is defined by $td_a(v) = (td_{a\mu}(v), td_{a\gamma}(v))$, where $td_{a\mu}(v) = d_{a\mu}(v) + \mu_1(v)$, for all v in G and $td_{a\gamma}(v) = d_{a\gamma}(v) + \gamma_1(v)$, for all v in G .

IV PSEUDO REGULAR INTUITIONISTIC FUZZY GRAPHS

Definition 4.1. Let $G = (V, E)$ be an intuitionistic fuzzy graph on $G^*(V, E)$. If $d_a(v) = (f_1, f_2)$, for all v in V , then G is called (f_1, f_2) - pseudo regular intuitionistic fuzzy graph.

Definition 4.2. Let $G = (V, E)$ be an intuitionistic fuzzy graph on $G^*(V, E)$. If $td_a(v) = (k_1, k_2)$, for all v in V , then G is called (k_1, k_2) - totally pseudo regular intuitionistic fuzzy graph.

Example 4.3. Consider an intuitionistic fuzzy graph on $G^*(V, E)$.



$$d_{a\mu}(u) = 0.4; d_{a\gamma}(u) = 0.6$$

$d_a(u) = (0.4, 0.6)$, $d_a(v) = (0.4, 0.6)$, $d_a(w) = (0.4, 0.6)$, $d_a(x) = (0.4, 0.6)$, $d_a(y) = (0.4, 0.6)$, $d_a(z) = (0.4, 0.6)$. So, all vertices have same pseudo degree. Hence G is $(0.4, 0.6)$ -regular intuitionistic fuzzy graph. Also, $td_a(u) = (0.8, 1.1)$, $td_a(v) = (0.8, 1.1)$, $td_a(w) = (0.8, 1.1)$, $td_a(x) = (0.8, 1.1)$, $td_a(y) = (0.8, 1.1)$, $td_a(z) = (0.8, 1.1)$. So, all vertices have same total pseudo degree. Hence G is totally $(0.8, 1.1)$ -regular intuitionistic fuzzy graph.

Remark 4.4. A pseudo regular intuitionistic fuzzy graph need not be a totally pseudo regular intuitionistic fuzzy graph.

Remark 4.5. A totally pseudo regular intuitionistic fuzzy graph need not be a pseudo regular intuitionistic fuzzy graph.

Theorem 4.6. Let $G : (V, E)$ be an intuitionistic fuzzy graph on $G^*(V, E)$. Then $A(v) = (\mu_1(v), \gamma_1(v))$, for all $v \in V$ is a constant function if and only if the following are equivalent.

- (i) G is a pseudo regular intuitionistic fuzzy graph.
- (ii) G is a totally pseudo regular intuitionistic fuzzy graph.

Proof. Assume that $A(v) = (\mu_1(v), \gamma_1(v))$ is a constant function. Let $A(v) = (\mu_1(v), \gamma_1(v)) = (c_1, c_2)$, for all $v \in V$. Suppose G is a pseudo regular intuitionistic fuzzy graph. Then $d_a(u) = (f_1, f_2)$, for all $u \in V$. Now, $td_a(u) = d_a(u) + A(u) = (f_1, f_2) + (\mu_1(u), \gamma_1(v)) = (f_1, f_2) + (c_1, c_2) = (f_1 + c_1, f_2 + c_2)$, for all $u \in V$. Hence G is a totally pseudo regular intuitionistic fuzzy graph. Thus (i) \Rightarrow (ii) is proved.

Suppose G is a totally pseudo regular intuitionistic fuzzy graph. Then $td_a(u) = (k_1, k_2)$, for all $u \in V \Rightarrow d_a(u) + A(u) = (k_1, k_2)$, for all $u \in V \Rightarrow d_a(u) + (c_1, c_2) = (k_1, k_2)$, for all $u \in V \Rightarrow d_a(u) = (k_1, k_2) - (c_1, c_2) = (k_1 - c_1, k_2 - c_2)$, all $u \in V$. Hence G is a pseudo regular intuitionistic fuzzy graph. Thus (ii) \Rightarrow (i) is proved. Hence (i) and (ii) are equivalent.

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Conversely, suppose (i) and (ii) are equivalent. Let G be a pseudo regular intuitionistic fuzzy graph and a totally pseudo regular intuitionistic fuzzy graph. Then $d_a(u) = (f_1, f_2)$ and $td_a(u) = (k_1, k_2)$, for all $u \in V$. Now $td_a(u) = (k_1, k_2)$, for all $u \in V \Rightarrow d_a(u) + A(u) = (k_1, k_2)$, for all $u \in V \Rightarrow (f_1, f_2) + A(u) = (k_1, k_2)$, for all $u \in V \Rightarrow A(u) = (k_1 - f_1, k_2 - f_2)$, for all $u \in V$. Hence A is a constant function.

Theorem 4.7. Let $G : (A, B)$ be a intuitionistic fuzzy graph on $G^*(V, E)$. If G is both pseudo regular and totally pseudo regular intuitionistic fuzzy graph then A is a constant function.

Proof. Assume that G is both pseudo regular and totally pseudo regular intuitionistic fuzzy graph. Then $d_a(u) = (f_1, f_2)$ and $td_a(u) = (k_1, k_2)$, for all $u \in V$. Now, $td_a(u) = (k_1, k_2) \Rightarrow d_a(u) + A(u) = (k_1, k_2) \Rightarrow (f_1, f_2) + A(u) = (k_1, k_2) \Rightarrow A(u) = (k_1 - f_1, k_2 - f_2) = \text{constant}$. Hence A is a constant function.

Theorem 4.8. Let $G : (A, B)$ be a intuitionistic fuzzy graph on $G^*(V, E)$, a cycle of length n . If B is a constant function, then G is a pseudo regular intuitionistic fuzzy graph.

Proof. If B is a constant function say $B(uv) = (\mu_2(uv), \gamma_2(uv)) = (c_1, c_2)$, for all $uv \in E$. Then $d_a(u) = (2c_1, 2c_2)$, for all $u \in V$. Hence G is a $(2c_1, 2c_2)$ -pseudo regular intuitionistic fuzzy graph.

Remark 4.9. Converse of the above theorem 4.8 need not be true. Converse of the above theorem 4.8 need not be true.

Result 4.10. If v is a pendant vertex, then pseudo degree of v in intuitionistic fuzzy graph is the degree of the vertex in intuitionistic fuzzy graph which is adjacent with v . (or) If v is a pendant vertex, then $d_a(v) = d_G(u)$, where u is the vertex adjacent with v .

Theorem 4.11. If $G(A, B)$ is a regular intuitionistic fuzzy graph on $G^*(V, E)$, an r -regular graph, then $d_a(v) = d_G(v)$, for all $v \in G$.

Proof. Let G is a (k_1, k_2) -regular intuitionistic fuzzy graph on $G^*(V, E)$, an r -regular graph. Then $d_G(v) = (k_1, k_2)$, for all $v \in G$ and $d_G^*(v) = r$, for all $v \in G$. So, $t_G(v) = \sum d_G(v_i)$, where each v_i (for $i=1, 2, \dots, r$) is adjacent with vertex $v \Rightarrow t_G(v) = \sum d_G(v_i) = r(k_1, k_2)$. Also,

$$d_a(v) = \frac{t_G(v)}{d_G^*(v)} \Rightarrow d_a(v) = \frac{t_G(v)}{r} \Rightarrow d_a(v) = \frac{r(k_1, k_2)}{r} \Rightarrow d_a(v) = (k_1, k_2) \Rightarrow d_a(v) = d_G(v).$$

Theorem 4.12. Let $G(A, B)$ be a intuitionistic fuzzy graph on $G^*(V, E)$, an r -regular graph. Then G is a pseudo regular intuitionistic fuzzy graph if G is a regular intuitionistic fuzzy graph.

Proof. Let G be a (k_1, k_2) -regular intuitionistic fuzzy graph on $G^*(V, E)$, an r -regular graph. $\Rightarrow d_a(v) = d_G(v)$, for all $v \in G$. $\Rightarrow d_a(v) = (k_1, k_2)$, for all $v \in G \Rightarrow$ all the vertices have same pseudo degree (k_1, k_2) . Hence G is (k_1, k_2) -pseudo regular intuitionistic fuzzy graph.

V. PSEUDO IRREGULAR INTUITIONISTIC FUZZY GRAPH

Definition 5.1. Let $G = (V, E)$ be an intuitionistic fuzzy graph on $G^*(V, E)$. Then G is said to be a pseudo irregular intuitionistic fuzzy graph if there exists a vertex which is adjacent to the vertices with distinct pseudo degrees

Definition 5.2. Let $G = (V, E)$ be an intuitionistic fuzzy graph on $G^*(V, E)$.

Then G is said to be a pseudo totally irregular intuitionistic fuzzy graph if there exists a vertex which is adjacent to the vertices with distinct total pseudo degrees.

Remark 5.3. A pseudo irregular intuitionistic fuzzy graph need not be a pseudo totally irregular intuitionistic fuzzy graph.

Remark 5.4. A pseudo totally irregular intuitionistic fuzzy graph need not be a pseudo irregular intuitionistic fuzzy graph.

Theorem 5.5. Let $G : (A, B)$ be a intuitionistic fuzzy graph on $G^*(V, E)$. If A is a constant function, then the following conditions are equivalent

(i) G is a pseudo irregular intuitionistic fuzzy graph

(ii) G is a pseudo totally irregular intuitionistic fuzzy graph.

Proof. Assume that $A(v) = (\mu_1(v), \gamma_1(v))$ is a constant function. Let $A(v) = (c_1, c_2)$, for all $v \in V$. Suppose G is a pseudo irregular intuitionistic fuzzy graph. Then at least one vertex of G which is adjacent to the vertices with distinct pseudo degree. Let u_1 and u_2 be the adjacent vertices of u_3 with distinct pseudo degrees (k_1, k_2) and (f_1, f_2) respectively. Then $(k_1, k_2) \neq (f_1, f_2)$. Suppose G is not a pseudo totally irregular intuitionistic fuzzy graph. Then every vertex of G which is adjacent to the vertices with same pseudo total degree $\Rightarrow td_a(u_1) = td_a(u_2) \Rightarrow d_a(u_1) + A(u_1) = d_a(u_2) + A(u_2) \Rightarrow (k_1, k_2) + (c_1, c_2) = (f_1, f_2) + (c_1, c_2) \Rightarrow (k_1 - f_1, k_2 - f_2) = 0 \Rightarrow$

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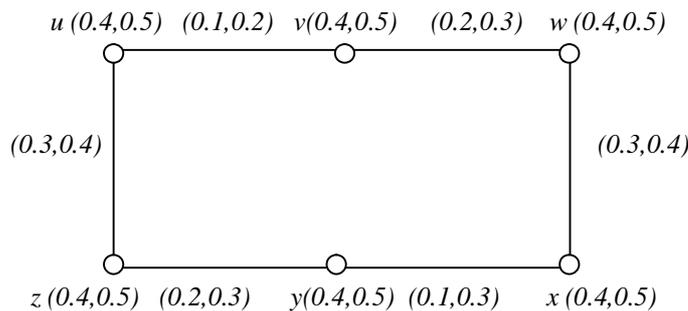
$(k_1, k_2) = (f_1, f_2)$, which is a contradiction to $(k_1, k_2) \neq (f_1, f_2)$. Hence G is pseudo totally irregular intuitionistic fuzzy graph. Thus (i) \Rightarrow (ii) is proved. Now, suppose G is a pseudo totally irregular intuitionistic fuzzy graph. Then atleast one vertex of G which is adjacent to the vertices with distinct pseudo total degree. Let u_1 and u_2 be the adjacent vertices of u_3 with distinct pseudo total degrees (g_1, g_2) and (h_1, h_2) respectively. Now, $td_a(u_1) \neq td_a(u_2) \Rightarrow d_a(u_1) + A(u_1) \neq d_a(u_2) + A(u_2) \Rightarrow (g_1, g_2) + (c_1, c_2) \neq (h_1, h_2) + (c_1, c_2) \Rightarrow (g_1, g_2) \neq (h_1, h_2)$. Hence G is pseudo irregular intuitionistic fuzzy graph. Thus (ii) \Rightarrow (i) is proved. Hence (i) and (ii) are equivalent.

VI. HIGHLY PSEUDO IRREGULAR INTUITIONISTIC FUZZY GRAPH

Definition 6.1. Let $G = (V, E)$ be an intuitionistic fuzzy graph on $G^*(V, E)$. Then G is said to be a highly pseudo irregular intuitionistic fuzzy graph if each vertex of G is adjacent to the vertices with distinct pseudo degrees.

Definition 6.2. Let $G = (V, E)$ be an intuitionistic fuzzy graph on $G^*(V, E)$. Then G is said to be a highly pseudo totally irregular intuitionistic fuzzy graph if each vertex of G is adjacent to the vertices with distinct total pseudo degrees.

Example 6.3. Consider an intuitionistic fuzzy graph $G = (V, E)$ on $G^*(V, E)$.



$d_{a|u}(u) = 0.4; d_{a|v}(u) = 0.6.$

$d_a(u) = (0.4, 0.6), d_a(v) = (0.45, 0.65), d_a(w) = (0.35, 0.6), d_a(x) = (0.4, 0.65), d_a(y) = (0.45, 0.7), d_a(z) = (0.35, 0.6).$ It is noted that each vertex in G is adjacent to the vertices with distinct pseudo degrees. Hence G is highly pseudo irregular intuitionistic fuzzy graph.

Remark 6.3. A highly pseudo irregular intuitionistic fuzzy graph need not be a highly pseudo totally irregular intuitionistic fuzzy graph.

Remark 6.4. A highly pseudo totally irregular intuitionistic fuzzy graph need not be a highly pseudo irregular intuitionistic fuzzy graph.

Theorem 6.5. Let $G: (A, B)$ be an intuitionistic fuzzy graph on $G^*(V, E)$. If A is a constant function, then the following conditions are equivalent.

- (i) G is a highly pseudo irregular intuitionistic fuzzy graph.
- (ii) G is a highly pseudo totally irregular intuitionistic fuzzy graph.

Proof. Proof is similar to the theorem 5.5.

Remark 6.6. The converse of the above theorem need not be true.

Theorem 6.7. Every highly pseudo irregular intuitionistic fuzzy graph is pseudo irregular intuitionistic fuzzy graph.

Proof. Let G be a pseudo highly irregular intuitionistic fuzzy graph. Then every vertex of G is adjacent to the vertices with distinct pseudo degrees \Rightarrow there is a vertex which is adjacent to the vertices with distinct pseudo degrees. Hence the graph G is pseudo irregular intuitionistic fuzzy graph.

Remark: 6.8. The converse of the above theorem need not be true.

VII. NEIGHBOURLY PSEUDO IRREGULAR INTUITIONISTIC FUZZY GRAPH

Definition 7.1. Let $G = (V, E)$ be an intuitionistic fuzzy graph on $G^*(V, E)$. Then G is said to be a neighbourly pseudo irregular intuitionistic fuzzy graph if adjacent vertices of G have distinct pseudo degrees.

Definition 7.2. Let $G = (V, E)$ be an intuitionistic fuzzy graph on $G^*(V, E)$. Then G is said to be a neighbourly pseudo totally irregular intuitionistic fuzzy graph if adjacent vertices of G have distinct total pseudo degrees.

Example 7.3. Consider an intuitionistic fuzzy graph $G = (V, E)$ on $G^*(V, E)$ given in example 6.3. It is noted that adjacent vertices of G have distinct pseudo degrees. Hence G is neighbourly pseudo irregular intuitionistic fuzzy graph.

Remark 7.4. A neighbourly pseudo irregular intuitionistic fuzzy graph need not be a neighbourly pseudo totally irregular intuitionistic fuzzy graph.

Remark 7.5. A neighbourly pseudo totally irregular bipolar fuzzy need not be a pseudo neighbourly irregular intuitionistic fuzzy graph.

Theorem 7.6. Let $G : (A, B)$ be a intuitionistic fuzzy graph on $G^*(V, E)$ and A is a constant function. Then the following are equivalent.

- (i) G is a neighbourly pseudo irregular intuitionistic fuzzy graph.
- (ii) G is a neighbourly pseudo totally irregular intuitionistic fuzzy graph.

Proof. Let $G : (A, B)$ be a connected intuitionistic fuzzy graph on $G^*(V, E)$ and A is a constant function. Then $A(u) = (c_1, c_2)$, for all u in V , where c_1 and c_2 are constant. Assume that G is neighbourly pseudo irregular intuitionistic fuzzy graph. Then every pair of adjacent vertices in G have distinct pseudo degrees. Let u_1 and u_2 be any two adjacent vertices with distinct pseudo degrees (k_1, k_2) and (f_1, f_2) respectively. Then $(k_1, k_2) \neq (f_1, f_2)$. Suppose G is not a neighbourly pseudo totally irregular intuitionistic fuzzy graph. Then atleast one pair of adjacent vertices in G with same total pseudo degree $(tda(u_1) = tda(u_2)) \Rightarrow da(u_1) + A(u_1) = da(u_2) + A(u_2) \Rightarrow (k_1; k_2) + (c_1; c_2) = (f_1; f_2) + (c_1; c_2) \Rightarrow (k_1; k_2) = (f_1; f_2)$, which is a contradiction to $(k_1; k_2) \neq (f_1; f_2)$. Hence G is neighbourly pseudo totally irregular intuitionistic fuzzy graph. Thus (i) \Rightarrow (ii) is proved. Now, suppose G is a neighbourly pseudo totally irregular intuitionistic fuzzy graph. Then every pair adjacent vertices in G have distinct total pseudo degree. Let u_1 and u_2 be any pair of adjacent vertices in G with distinct total pseudo degrees. Now, $tda(u_1) \neq tda(u_2) \Rightarrow da(u_1) + A(u_1) \neq da(u_2) + A(u_2) \Rightarrow da(u_1) + (c_1; c_2) \neq da(u_2) + (c_1; c_2) \Rightarrow da(u_1) \neq da(u_2)$. Hence G is neighbourly pseudo irregular bipolar fuzzy graph. Thus (ii) \Rightarrow (i) is proved.

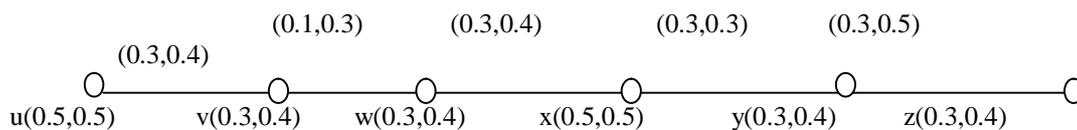
Remark 7.7. If every pseudo regular intuitionistic fuzzy graph G with membership values of all the adjacent vertices are distinct, then G is neighbourly pseudo totally irregular intuitionistic fuzzy graph.

VIII. STRONGLY PSEUDO IRREGULAR INTUITIONISTIC FUZZY GRAPH

Definition 8.1. Let $G = (V, E)$ be an intuitionistic fuzzy graph on $G^*(V, E)$. Then G is said to be a strongly pseudo irregular intuitionistic fuzzy graph if every pair of vertices of G have distinct pseudo degrees.

Definition 8.2. Let $G = (V, E)$ be an intuitionistic fuzzy graph on $G^*(V, E)$. Then G is said to be a neighbourly pseudo totally irregular intuitionistic fuzzy graph if every pair of vertices of G have distinct total pseudo degrees.

Example 8.3. For example consider $G = (V, E)$ be an intuitionistic fuzzy graph such that $G^*(V, E)$, a path on 6 vertices.



$d_a(u) = (0.1, 0.3)$, $d_a(v) = (0.4, 0.7)$, $d_a(w) = (0.6, 0.7)$, $d_a(x) = (0.6, 0.8)$, $d_a(y) = (0.6, 0.9)$, $d_a(z) = (0.3, 0.4)$. So, all vertices have distinct pseudo degrees. Hence G is strongly pseudo irregular intuitionistic fuzzy graph. $td_a(u) = (0.6, 0.8)$, $td_a(v) = (0.7, 1.1)$, $td_a(w) = (0.9, 1.1)$, $td_a(x) = (1.1, 1.3)$, $td_a(y) = (0.9, 1.3)$, $td_a(z) = (0.6, 0.8)$. So, all vertices have distinct total pseudo degrees. Hence G is strongly pseudo totally irregular intuitionistic fuzzy graph.

Remark 8.4. A strongly pseudo irregular intuitionistic fuzzy graph need not be a strongly pseudo totally irregular intuitionistic fuzzy graph.

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Remark 8.5. A strongly pseudo totally irregular intuitionistic fuzzy need not be a strongly pseudo irregular intuitionistic fuzzy graph.

IX. CONCLUSION

We have introduced pseudo regular intuitionistic fuzzy graphs, pseudo irregular intuitionistic fuzzy graphs, highly pseudo irregular intuitionistic fuzzy graphs, neighbourly pseudo irregular intuitionistic fuzzy graphs, Strongly pseudo regular intuitionistic fuzzy graphs in this paper. We can discuss these regularity and irregularity in interval valued fuzzy graphs.

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