

Flexural Torsional Buckling Resistance Design of Steel Circular Arches

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ABSTRACT: The distinguishing characteristics of an arch are the presence of horizontal reactions at the ends and the considerable rise of the axis at the centre span which differ arch from beams. In case of beams supporting uniformly distributed load the bending moment increases with the square of span and hence they become uneconomical for long span structures. Long span structure arches have an advantage as they would develop horizontal reactions which reduce the design bending moment. An arch has tendency to buckle in its plane of the arch loading due to axial compression. Due to curved profile arches have high-efficient in-plane load carrying capacity but steel arches are prone to out-of-plane buckling that controls its design strength rather than in-plane flexural buckling. For pinned arches, in practical use arch ends can be treated as in-plane free to rotate but with respect to out-of-plane bending they are actually semi-rigidly restrained rather than fully restrained. That is why in practical engineering design; much attention needs to be paid to the case of arch with elastic end bending restrained. In practice arches having uniformly distributed vertical loading slightly bending forces and mainly compressive forces are induced on the arch structure. That is why in practical engineering design much attention needs to be paid in the case of arches having uniformly distributed vertical loading, and design method needs to be proposed for this case. The aim of the study is to predict the flexural torsional buckling resistance design of steel circular arch loaded in-plane uniformly distributed vertical loads that buckles out-of-plane in flexural torsional mode before in-plane failure. From this study we can predict the design method for arches with uniformly distributed vertical loads and elastic end bending restraining conditions.

KEYWORDS: Flexural torsional buckling; Out-of-plane buckling; In-plane buckling, uniformly distributed vertical loads.

I. INTRODUCTION

Arch structure is aesthetically pleasant structure; as they represent the primary structural components in important, eye pleasant and expensive structures, many of which are unique. Current trends in modern architectural designer heavily rely on curved building components due to their strengths and architectural appeal. Primarily arch type structures are used in construction of bridges and industrial roofing. The distinguishing characteristics of an arch from beam are the presence of horizontal reactions at the ends and the considerable rise of the axis at the centre span which differ arch from beams. In case of beams supporting uniformly distributed load the bending moment increases with the square of span and hence they become uneconomical for long span structures. Long span structure arches have an advantage as they would develop horizontal reactions which reduce the design bending moment. Horizontal reactions also develop normal thrust at the arch ends, due to these arches may buckle and fail out-of-plane and twist about its longitudinal axis. This out-of-plane buckling controls its design strength rather than in-plane buckling.

Steel arches have in-plane load carrying capacity is higher, due to their curved profile, but it has tendency to buckle out-of-plane that control its design strength rather than in-plane load carrying capacity. Due to this behaviour of steel arches much attention needs to be paid on this problem. Many factors are affecting on out-of-plane buckling and flexural torsional buckling, in which rise-to-span ratio is affecting on flexural torsional buckling behaviour. Due to increase in rise of arch the horizontal reaction induced at support get increased, this horizontal reaction acts as a compressive axial force, and due to increase in axial force structure become unstable and deflects out of its plane. End conditions of supports also affecting on steel arches. For pinned arches, in practical use arch pin ends have to be

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dealing with in-plane free to rotate but with respect to out-of-plane bending they are actually semi-rigidly restrained rather than fully restrained. That is why in practical engineering design; much attention needs to be paid to the case of arch with elastic end bending restrained. Therefore, design method for arches with elastic end bending restraints and having uniformly distributed vertical loads needs to be investigated.

Under the action of uniformly distributed radial loads, circular arches are in ideal status of uniform compression without in-plane bending moment [4]. Several investigations have been carried out into flexural-torsional inelastic buckling and design of circular arches in uniform compressions. Pi and Trahair [1] and Pi and Bradford [2] explored the flexural torsional buckling strength and proposed design equations for pinned and fixed circular arches by using a self-developed finite element (FE) program. Dou and Guo [3] adopted a commercial finite element package for elastoplastic stability analyses to establish design approaches for steel circular arches. The author modelled circular arches with straight beam element (BEAM188) and analysed which shows good argument with practical conditions and previous experimental results. Despite these previous researches were carried out for ideal end boundary conditions. The end boundary conditions are also affecting on steel arches i.e. hinged arches, pinned arches and fixed arches, in which the degree of freedom (dof) at arch end was either fully restrained or fully free. However in practical conditions behaviour of pinned arches are in-plane free to rotate as shown in fig. 1 and out-of-plane buckling is semi-rigidly restrained. The experimental study was conducted by Dou et al. [4] indicate that, for fixed arches all dofs at arch ends can be fabricated rigidly restrained which was close to the assumption in an ideal model. However for pinned arches arch ends can be treated as in-plane free to rotate but out-of-plane it is semi-rigidly restrained that's why much attention needs to be paid on arch end with elastic end bending restrained.

Arches are widely used in bridges, spatial structures and long-span structures because of their high-efficient in-plane load-carrying capacity, especially under full span length uniformly distributed vertical load when the arch ribs are primarily in compression and bending. The arch is always supported in such a way that the spreading of two ends outward is prevented, which introduces compressive and lesser bending action in the arch rib [4]. Similar to axially-loaded straight compression members, buckling resistance design is prevented for circular arches in compression and slightly bending actions are influenced by parameters such as residual stresses, slenderness ratio, end restraint conditions, geometric imperfections, and section types. Apart from these, the rise-to-span ratio which is an important factor, which distinguishes arches from columns. And due to this rise in its elevation horizontal reaction is produced at the arch structure which minimizes bending moments at the arch structure

This research focuses on studying the behaviour of arches with elastic end bending restraints and having uniformly distributed vertical loads on the freestanding arch structure. The aim of the study is to predict the flexural torsional buckling resistance design of steel circular arch loaded in-plane uniformly distributed vertical loads that buckles out-of-plane in flexural torsional mode before in-plane failure.

II. LITERATURE REVIEW

Sakimoto T. and Komatsu S., (1983) This paper mainly focused on effective slenderness ratios for inelastic lateral buckling of through type arches are determined by utilizing an comparative analogy between an arch and a column. He gave a common formula for that designer that when they substitute it into column strength formula with effective slenderness ratio, can use to obtain an ultimate stress for circular arches. The applicability of the proposed formula is examined by comparing with those of computer software simulations. Here he carried out computer simulations for single isolated arches of box section and through-type braced arch bridges composed of twin box-section. The integrity of the diagonal bracing members is assumed in these studies. Lateral bracing systems are considered in more detail in other studies which examine the strength of steel arches under vertical and lateral loads.

Dou C. et.al, (2015) This paper focuses on study with experimental investigations into flexural-torsional ultimate resistance of in-plane-pinned circular steel arches. Author experimented three I-sections arches with the same span but different rise-to span ratio under different loading conditions. These concentrated loads are directed toward fixed points during flexural-torsional buckling of the arches. Also the arches are supported in such a way that their ends in plane are pinned and out of plane are semi-rigidly restrained. The test result shows that when the test arches reach their load-carrying capacity, in the arch rib plastic zones form, and the out-of-plane flexural-torsional deformation is significant. And also It is found that the pinned arches buckle in a shape of asymmetric double-wave S-shaped flexural-torsional failure mode as a result of the semi-rigid restraints for bending out of plane at arch ends, rather than in a one-wave C-

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shaped buckling mode for rigid restraints. The out-of-plane bending rigidity elastic stiffness at the arch ends has a more consistent effect on arches under symmetrical three-point loading over the entire span than on arches under unsymmetrical loading i.e. two-point loading over half the span, and also a more significant effect on arches with a higher rise-to-span ratio than on those with a small rise-to span ratio.

Dou C. et.al, Pi Y. L. (2015)This paper focuses on the study of the flexural-torsional buckling resistance and design of steel circular arches subjected to uniform compression with end conditions are elastic end bending restraints by using finite element (FE) numerical analysis, this type of analysis he considered the geometric and mechanical parameters such as initial imperfections, section type, material properties, slenderness, rise-to-span ratio, and end restrained on flexural-torsional buckling resistant design of circular arches under the loading type of uniformly distributed radial loading on the entire arch rib. And he found out that these effects are eliminated to a large extent by introducing of normalized slenderness ratio. Also he concluded that, the effect of axial force on flexural stiffness for an arch is alike to that for a column but is influenced largely by the rise-to-span ratio. Here author considering the end restraining at arch rib provided by adjacent arch segments and using the normalized slenderness ratio the flexural-torsional buckling resistance of the adverse arch segment was obtained by using analytical equations, with good accordance with results from finite element numerical analyses.

III. METHODOLOGY

To study the behaviour of flexural-torsional buckling of steel section arches, the hot-rolled steel section arches are going to be used. These steel arches having doubly symmetric cross-sections. In this study section type is doubly symmetric welded box section with elastic end bending restraints under the action of uniformly distributed load. Rise-to-span ratio of arch is an important factor which distinguish arch from column. The rise-to-span ratio f/L varies from 0.1 to 0.5, in which f denotes rise of an arch and L denotes the span, all measured from centroid axis of arch cross section. In this study effect of out-of-plane slenderness ratio and end restraints needs to be studied as they are influencing parameters on the flexural torsional buckling of arches. The out-of-plane slenderness $\lambda_y = S/i_y$ varies between 160 and 240, in which S is the developed length of cross-sectional centroid axis of arch and i_y is the gyration radius of cross-section with respect to out-of-plane-bending.

The eigenvalue buckling analysis is carried out for arches with varying rise-to-span ratio, slenderness ratio, and end restraining conditions. From previous study, it was found that in practical use all the degrees of freedom (dofs) at the arch end were close to assumption in an ideal calculation model (either free or fully restrained) except the out-of-plane bending degree of freedom which is actually semi-rigidly restrained rather than fully restrained. Thus in this study, at the arch ends the translation in three direction, torsion along tangent direction and warping deformations of arch axis are fully restrained, and bending in-plane is set free. However the bending out-of-plane is semi-rigidly elastically restrained. The elastic bending rigidity can be equal or unequal to one another.

The finite element package *ANSYS R15.0* is adopted to perform the flexural torsional buckling analysis of arches. The arch is divided into elements and is modelled with *BEAM188* element, and only the global stability is involved in the analysis without local buckling of plates. The arch material is perfectly elastic plastic. For elastically restrained end condition spring element *COMBIN39* is used for analyses, that its in-plane free to rotate and all other dofs are constrained.

IV. VERIFICATION OF NUMERICAL MODEL

In this study of numerical model, FEA model is prepared in *ANSYS*. The I-section model with dimensions of $h \times b \times t_w \times t_f = 200 \text{ mm} \times 100 \text{ mm} \times 8 \text{ mm} \times 12 \text{ mm}$, where h , b , t_w , and t_f are the overall height and width, the thicknesses of web, and the thicknesses of flange of the cross-section, respectively. The span L of these arches was constant ($L = 6.00 \text{ m}$) and two rise-to-span ratios were selected as $f/L = 0.3$. Also geometric imperfections are also included in the numerical model as per the paper. Here geometric imperfections are included is the 1/300 of the actual arc length (S) of the arch used in the modelling. The material properties of steel taken for the analysis are, yield stress of material $f_y = 285 \text{ MPa}$, Modulus of elasticity $E = 202 \text{ GPa}$ and ultimate tensile strength $f_u = 440 \text{ MPa}$ are included in numerical model. For modelling of arch segment *BEAM188* and *COMBIN39* elements are used, properties of these elements are explained above in the report as shown in fig. 1

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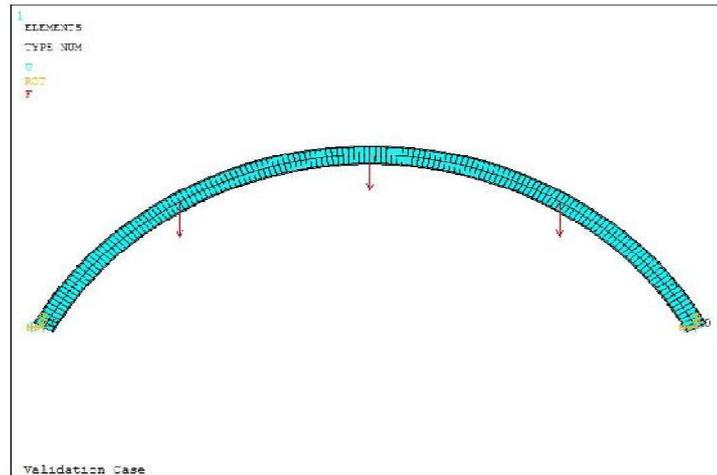


Fig.1. Arch with loading conditions and elastic end restraining conditions

The above figure 1 shows the three points loading of the arch structure with elements *BEAM188* and *COMBIN39* used in the ANSYS. For elastic end bending rigidity the *COMBIN39* element is suitable because in ANSYS it consider a slope of the load deflection diagram and then apply end bending rigidity from that slope.



Fig. 2 Out-of-plane buckling failure of arch section (top view)

These loading conditions are also taken from experimental study conducted by Dou et.al.[4]. The *BEAM188* element is used for modelling. As noted previously *COMBIN39* element used here to formulate elastic end bending restraining conditions. However to formulate the bending rigidity and semi-rigidity at the arch ends, rotational restraints about vertical axis has to be set as a free and bending stiffness of the spring element *COMBIN39* has to be given as shown in fig. 1. In this case the bending stiffness of arch end was taken from [4] and stiffness of the spring element is 81.1 kNm/rad. And the arch section buckles in second mode of linear buckling failure mode and out-of-plane buckling shape of S-shaped buckling as shown in fig. 2.

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Table I. Ultimate Load and Failure mode

Specimen	Ultimate Load per point (kN)			Failure Mode	
	Test	FE model	Error (%)	Test	FE Model
P-3-0.3	91	84.47	7.17	S-shaped	S-shaped

*P-3 = Three point loading, 0.3 = rise to span ratio

The above results shows the good agreement with the experimental work and these two elements are can be used as the further analysis of the problem having arches with uniformly distributed vertical loads and elastic end bending restraints conditions. The further study with these elements can give the accurate and valid results for the practical point of view. The BEAM188 and COMBIN39 elements are used for the further calculations and these elements give the good agreement with the obtained results.

V. ANALYTICAL RESULTS

From Eigenvalue buckling analysis we get the load intensity factor, from that factor we can calculate the ultimate crippling load of steel circular arches. From this crippling load we can found out the reduction factor for semicircular arches in uniformly distributed vertical loading. The linear buckling analysis is carried out for these arches and reduction factor is going to be finding out (ϕ) and ultimate crippling load. The reduction factor for arches under uniform compression is defined as,

$$\phi = N_u / N_y = q_u R / f_y A$$

Where,

N_u = Axial compressive force

N_y = Squash load of cross-section

q_u = uniformly distributed vertical load subjected to ultimate flexural torsional buckling load

R = Radius of arch

f_y = Yield stress

A = Cross sectional area

The analytical results are obtained from the eigenvalue buckling analysis carried out in the ANSYS R15.0. The box section dimensions and the elastic end bending restraining stiffness as shown in table II below, the dimensions of the sections obtained from the out-of-plane slenderness ratio λ_o , the out-of-plane slenderness ratio is defined as the $\lambda_o = S/i_y$ we know the section dimensions from that we get the radius of gyration of the particular section (i_y). After that from trigonometric relations the radius and other dimensions of the section like, included angle, length of arch section and rise-to-span ratio get defined. The detailed dimensions of the arch section are shown in the table II below. The end bending stiffness is depending upon the arc length of the arch structure; here for every rise-to-span ratio the arc length is constant and for particular slenderness ratio it also constant. The end bending rigidity is influencing parameter in the arch structure.

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Table II Box-section Dimensions

Slenderness (λ_y)	Included Angle (Θ)	Rise (f)	Length (L)	Rise-to-span ratio (f/L)	Radius (R)	Stiffness (k) $5EI/S$
240	180	12539.68	25079.36	0.5	12539.68	46700000000
	154.64	11392.18	28480.26	0.4	14596.11	
	123.86	9648.34	32159.58	0.3	18223.34	
	87.22	7141.19	35699.49	0.2	25878.73	
	45.24	3837.94	38379.16	0.1	49892.63	
160	180	8359.78	16719.577	0.5	8359.78	70100000000
	154.64	7594.78	18986.84	0.4	9730.74	
	123.86	6432.23	21439.72	0.3	12148.89	
	87.22	4760.79	23799.66	0.2	17252.48	
	45.24	2558.62	25586.11	0.1	33261.75	

For these dimensions and slenderness ratio the eigenvalue buckling analysis is carried out and the results are obtained. The obtained results are shown that for every change in the slenderness ratio the reduction factor is changed with respect to out-of-plane slenderness ratio. Therefore we cannot predict the design method from the out-of-plane slenderness ratio due to this here, normalized slenderness ratio get introduced in the arch analysis from this we can get the single unified curve from that we can predict the design method. The results obtained with respect to out-of-plane slenderness ratio in the graphical format shown in figure 3 below.

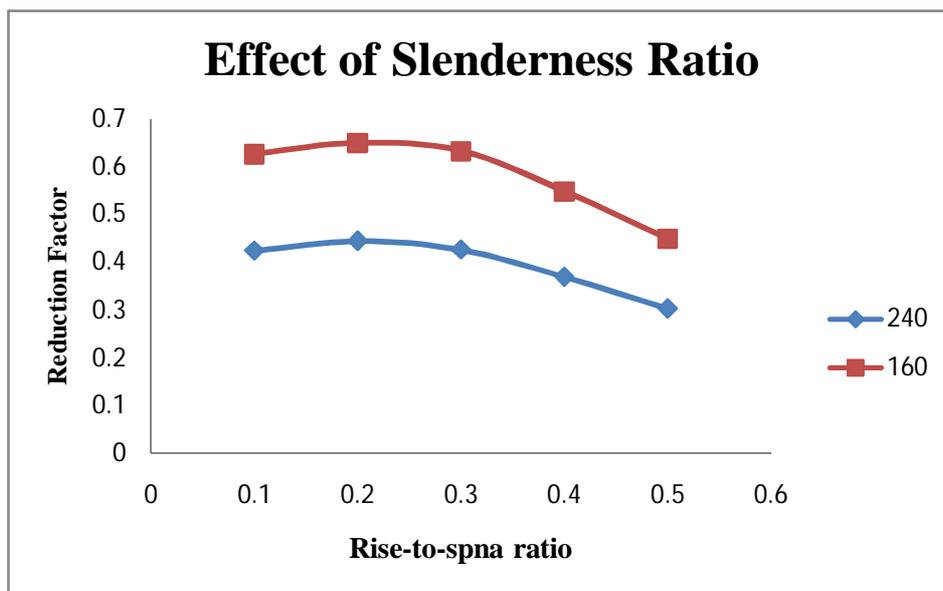


Fig. 3 Effect of slenderness ratio

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Due to introduction of normalized slenderness ratio here, eliminating this effect due to the slenderness ratio and rise-to-span ratio. Then, it is found that, this effect can be eliminated by introducing Normalized slenderness ratio, and to reduce these all effects, the Normalized slenderness ratio is introduced in here. This ratio eliminating all the effects and provide the uniform column curve for the arches.

$$\lambda_a = \sqrt{N_y / N_{acr}}$$

Where, N_y = Squash load of cross-section = $f_y \cdot A$

N_{acr} = Horizontal thrust obtained from EBA analysis carried out in ANSYS R15.0

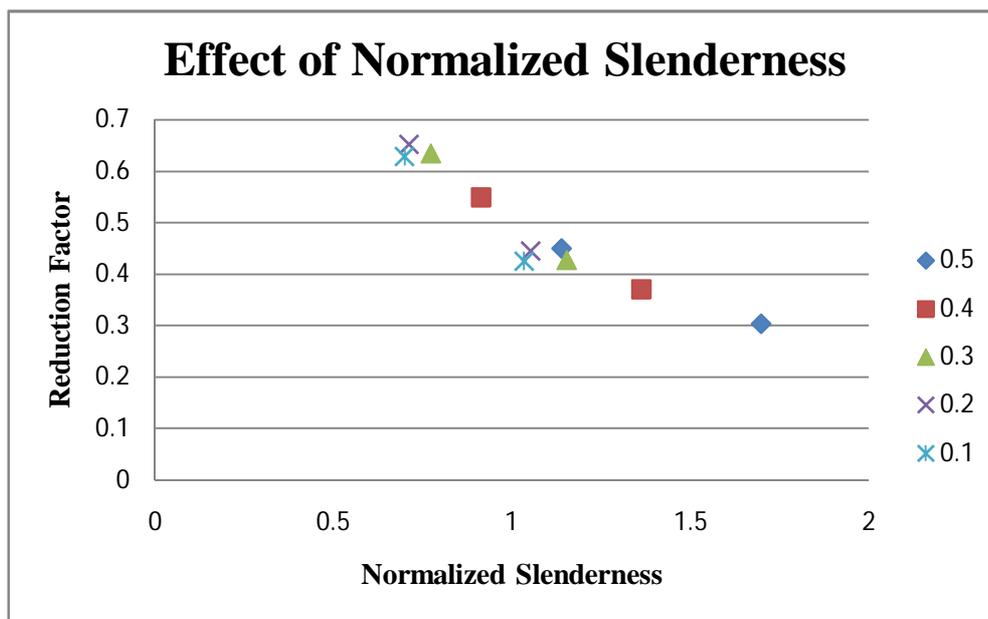


Fig. 4 Effect of normalized slenderness ratio

The fig. 4 shows that the difference in the reduction factor is almost get eliminated, and get the unified curve from that we can predict the design method and obtain the column curve for the arch box-section for compressive actions in the arch rib.

VI. CONCLUSION

The analytical study was carried out to validate the FE model with previous experimental tests for semicircular arches. Following conclusion was drawn from this study.

- The experimental results 91 kN per point load and FE result 84.47 kN per point load are considerably match with 7.17% error, so for flexural-torsional buckling analysis of semicircular arches we can use this FE analysis method.
- *BEAM188* and *COMBIN39* elements are compatible with practical conditions and can be used in buckling analysis of arches with elastic end bending restraining condition.

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- From the normalized slenderness ratio design method can be predicted for the arches with uniformly distributed vertical loads and elastic end bending restrained.

REFERENCES

- [1] Pi Y. L., Trahair N. S., "Out-of-plane inelastic buckling and strength of steel arches". Journal of structural engineering, Vol. 124:No.2, 174-183, 1998.
- [2] Pi Y. L., Bradford M. A., "Out-of-plane strength design of fixed steel I-section arches". Journal of structural engineering, Vol. 131: No. 4, 560-568, 2005.
- [3] Dou C., Guo Y. L., "Out-of-plane inelastic stability and strength design of circular arches in uniform compression". Journal of Building Structure, Vol. 33: No. 1, 104-110, 2012
- [4] Dou C., Guo Y. L., Pi Y. L., Zhao S. Y., "Experimental investigation into flexural-torsional ultimate resistance of circular arches". Journal of Structural Engineering, ISSN 0733-9445 / 04015006, 2015.
- [5] Dou C., Pi Y. L., "Flexural-Torsional Buckling Resistance Design of Circular Arches with Elastic End Restraints". ISSN 0733-9445/04015014, 2015
- [6] Sakimoto T., Kamatsu S., "Ultimate Strength formula for steel arches, Journal of Structural Engineering", Vol. 109: No. 3, 613-627, 1983