

Parallelism and Symmetry in Planar Kinematic Chains

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ABSTRACT: Many methods are reported to test parallelism and symmetry among kinematic chains. A new method for detection of parallelism and symmetry based on theoretic approach, easy to compute and reliable is suggested in this paper. The work presented here is focused on identification of symmetry among kinematic chains.

KEYWORDS: kinematic chains, parallelism, symmetry, distance matrix, mechanisms

I. INTRODUCTION

Structural study and synthesis of kinematic chains is a significant feature of mechanism design. The most essential phase in the study of kinematic structure of mechanisms is the structural synthesis or classification and listing of kinematic chains with a given number of links and degree of freedom.

Chang et al [2] projected a new method based on Eigen vectors and Eigen values to identify Isomorphism among kinematic chains. Kinematic chains are firstly represented by Adjacent Matrices. By comparing the Eigen values and corresponding Eigen vectors of Adjacent Matrices, the Isomorphism of mechanism kinematics chain can be identified.

Cubillo and Wan [3] suggested a new process to identify isomorphism among kinematic chains. With this new procedure, it is only essential to compare Eigen values and numerous Eigen vectors of adjacent matrices of isomorphic kinematic chains to identify the isomorphic chains.

A. C. Rao and Raju [4] suggested a new method for detecting the isomorphism among chains, called the chain loop string is developed for a planar kinematic chain with simple joints to distinguish isomorphism among chains. Another invariant called the link adjacency string is developed, which is a consequence of the same method to detect inversions of a given chain. The planned method is also relevant to know the type of freedom of a chain in case of multi degree of freedom chains.

S.C. Sarkar & Khare [5] proposed a theoretical analysis for detecting isomorphism and effect of uncertainty in 10 bar kinematic chain, using the concept of directed graph. In this case, flow of motion among links is evaluated considering all possible paths for motion transmission instead of only the closest path.

Mruthyunjaya and Balasubramanian [8] proposed a vertex-vertex degree matrix whose ij^{th} entry is the sum of degrees of links i and j if i and j are adjacent and is equal to 1 otherwise. The characteristic polynomial of this matrix effectively identified all the 10-link kinematic chains with up to 3 degrees of freedom.

II. RELATED WORK

A.C. Rao [1] recommended utility of fuzzy logic to examine symmetry among kinematic chains. Fuzzy relationship is assigned to each link of a kinematic chain which develops a fuzzy vector in terms of fits for each link, on the basis of its adjacency, i.e. for every link separate vectors for first adjacency, second adjacency and so on are proposed. Numerical procedures to compare the numerous individual chains with the same number of links and degrees of freedom (d.o.f.) for characteristics like symmetry, parallelism and mobility are projected. Fuzzy entropy is applied in order to compare the chains for the mobility.

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Srinath and Rao [6] presented a method based on correlation concept for detection of isomorphism among kinematic chains and their inversions along with parallelism and type of freedom. Correlation between two links shows the number of links typically connected to them. The method uses the adjacency matrix to attain the correlation matrix.

A. C. Rao, focused on the concept of Hamming number technique and its application to detect isomorphism among kinematic chains. Also, the concept of symmetry in kinematic chains is introduced and its utility in detecting isomorphism among chains.

III. TERMINOLOGY

1) **NODE VALUE**:-A node value is defined as a numeric value assigned to the particular node of a link. Node value is assigned as the ratio of order of link to number of parameters and the joint value is sum of node values connected at the joint as shown in figure (a).

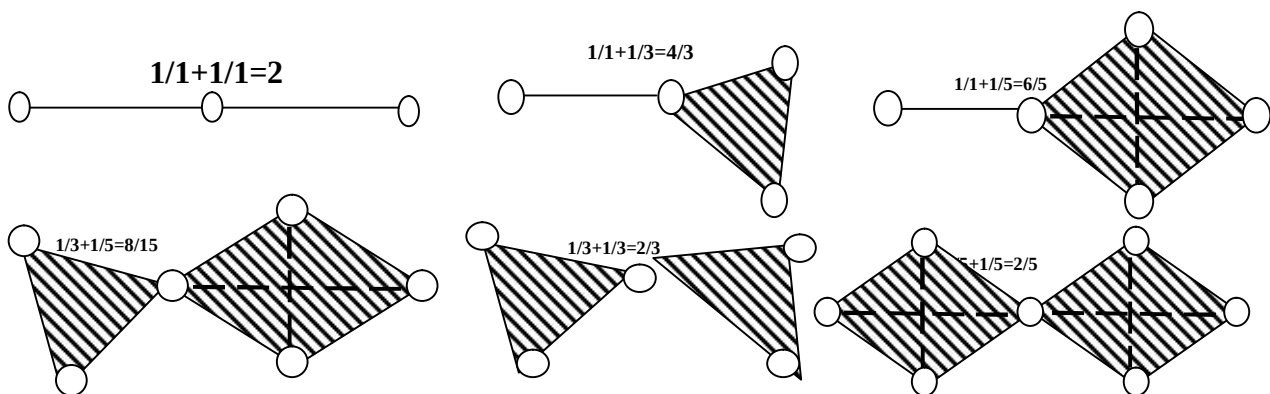


Fig (a) (Various link connectivity's and node values)

- 2) **LEAST DISTANCE**:-It is defined as shortest distance (path) in terms of joint value between the two links.
- 3) **DISTANCE MATRIX (DM)**:-It is a square matrix. It is formed on the basis of sum of least distance between node values of the two links.
- 4) **RELATIVE DISPOSITION MATRIX (RDM)**:- It refers to sum of absolute differences of the elements of particular rows of LDM.
- 5) **RELATIVE DISPOSITION STRING (RDS)**:- It refers to elements of a particular row (or column) of RDM taken in ascending order.
- 6) **RELATIVE DISPOSITION LINK VALUE (RDLV)**:- The sum of the elements of each row of RDM will give a particular value. This value is termed as RDLV for that particular link.
- 7) **RELATIVE DISPOSITION CHAIN STRING (RDCS)**:-The sum of all the RDLV's of a particular chain along with all the elements of the matrix will give the RDCS for that particular chain.

IV. METHODOLOGY

The method involves a novel approach for detection of symmetry and parallelism among kinematic chain. The method is explained using Watt's and Stephenson's chain.

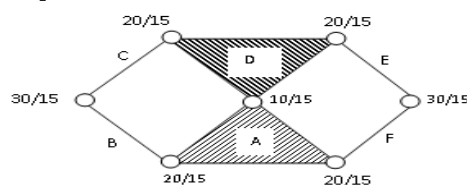


Fig (b) (WATT chain)

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A distance matrix (DM) is formed using the values on the basis of the parameter. Watt chain is shown in fig. (b). Link A & D are ternary link and link B, C, E & F are binary link. Each node of binary link is assigned a joint value “1” and for ternary link joint value “1/3” is assigned. Also quaternary link and pent nary link had the node values 1/5 and 1/7 respectively. Thus at a node joining a binary link and ternary link, a joint value will be assigned as 4/3(20/15) (1/3, joint value of ternary link and 1, joint value of binary link) as shown in figure.

	A	B	C	D	E	F
A	0	20	30	10	30	20
B	20	0	30	30	50	40
C	30	30	0	20	40	50
D	10	30	20	0	20	30
E	30	50	40	20	0	30
F	20	40	50	30	30	0



	A	B	C	D	E	F
A	d ₁₁	d ₁₂	d ₁₃	d ₁₄	d ₁₅	d ₁₆
B	d ₂₁	d ₂₂	d ₂₃	d ₂₄	d ₂₅	d ₂₆
C	d ₃₁	d ₃₂	d ₃₃	d ₃₄	d ₃₅	d ₃₆
D	d ₄₁	d ₄₂	d ₄₃	d ₄₄	d ₄₅	d ₄₆
E	d ₅₁	d ₅₂	d ₅₃	d ₅₄	d ₅₅	d ₅₆
F	d ₆₁	d ₆₂	d ₆₃	d ₆₄	d ₆₅	d ₆₆

As shown in the matrix, for Watt chain, ternary link A is connected to binary link B and the node value is assigned “20”. For connectivity between link A and link C a shortest distance is considered i.e., 20+10=30. Similarly for connectivity between link A and link D the joint value will be “10” and so on. A 6*6 matrix will be formed for a 6 link watt chain.

For interconnectivity between the links a new matrix i.e., RELATIVE DISPOSITION MATRIX (RDM) is introduced. The RELATIVE T DISPOSITION MATRIX (RDM) for Watt chain is given below:-

	A	B	C	D	E	F
A	0	100	120	60	120	100
B	100	0	100	120	140	120
C	120	100	0	100	120	140
D	60	120	100	0	100	120
E	120	140	120	100	0	100
F	100	120	140	120	100	0

As shown in the matrix, the LDM is transformed into RLDM using the formula.

The element in first row, first column i.e., AA (0) is obtained by

$$d_{AA} = (|(d_{11} - d_{11})| + |(d_{12} - d_{12})| + |(d_{13} - d_{13})| + |(d_{14} - d_{14})| + |(d_{15} - d_{15})| + |(d_{16} - d_{16})|)$$

“|” (modulus) is used for absolute value of the difference of the respective element. Similarly for element in first row, second column i.e., AB (100) is obtained by

$$d_{AB} = (|(d_{21} - d_{11})| + |(d_{22} - d_{12})| + |(d_{23} - d_{13})| + |(d_{24} - d_{14})| + |(d_{25} - d_{15})| + |(d_{26} - d_{16})|)$$

For all other elements in first column, the expanded formula is given by:-

$$d_{AC} = (|(d_{31} - d_{11})| + |(d_{32} - d_{12})| + |(d_{33} - d_{13})| + |(d_{34} - d_{14})| + |(d_{35} - d_{15})| + |(d_{36} - d_{16})|) = 120$$

$$d_{AD} = (|(d_{41} - d_{11})| + |(d_{42} - d_{12})| + |(d_{43} - d_{13})| + |(d_{44} - d_{14})| + |(d_{45} - d_{15})| + |(d_{46} - d_{16})|) = 60$$

$$d_{AE} = (|(d_{51} - d_{11})| + |(d_{52} - d_{12})| + |(d_{53} - d_{13})| + |(d_{54} - d_{14})| + |(d_{55} - d_{15})| + |(d_{56} - d_{16})|) = 120$$

$$d_{AF} = (|(d_{61} - d_{11})| + |(d_{62} - d_{12})| + |(d_{63} - d_{13})| + |(d_{64} - d_{14})| + |(d_{65} - d_{15})| + |(d_{66} - d_{16})|) = 100$$

RDS for link A, B, C, D, E & F are: - [0, 60, 2(100), 2(120)] , [0, 2(100), 2(120), 140] , [0, 2(100), 2(120), 140] , [0, 60, 2(100), 2(120)] , [0, 2(100), 2(120), 140] & [0, 2(100), 2(120), 140] respectively. From above values of strings it is clear that link A & D will have the same inversions and link B, C, E & F will have the same inversions.

RDLV for link A, B, C, D, E & F are: - 500, 580, 580, 500, 580, and 580 respectively & RDCS for Watt chain are 3320, 2 [60, 6(100), 6(120), 2(140)].

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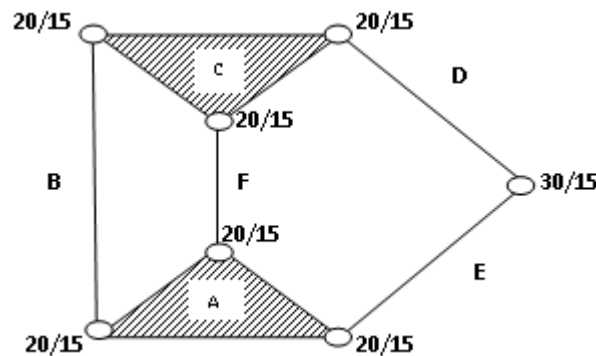


Fig (c) (STEPHENSON's chain)

Figure (c) showing the Stephenson's chain. A distance matrix (DM) & The RELATIVE DISPOSITION MATRIX (RDM) for Stephenson chain are given below:-

	A	B	C	D	E	F
A	0	20	40	50	20	20
B	20	0	20	40	40	40
C	40	20	0	20	50	20
D	50	40	20	0	30	40
E	20	40	50	30	0	40
F	20	40	20	40	40	0

	A	B	C	D	E	F
A	0	110	140	170	110	110
B	110	0	110	120	120	80
C	140	110	0	110	170	110
D	170	120	110	0	120	120
E	110	120	170	120	0	120
F	110	80	110	120	120	0

RDS for link A, B, C, D, E & F are: - $[0, 3(110), 140, 170]$, $[0, 80, 2(110), 2(120)]$, $[0, 3(110), 140, 170]$, $[0, 110, 3(120), 170]$, $[0, 110, 3(120), 170]$, $[0, 80, 2(110), 2(120)]$ respectively.

From above values of strings it is clear that link A & C will have the same inversion, link B & F will have the same inversions, link D & E will have the same inversions.

RDLV for link A, B, C, D, E & F are: - 640, 540, 640, 640, 640 and 540 respectively. Also the sum of all the RDLV's of a particular chain will give the RELATIVE DISPOSITION CHAIN STRING (RDCS) for that particular chain. Hence RDCS for Stephenson chain is $3640, 2[80, 6(110), 5(120), 140, 2(170)]$.

V. PARALLELISM

Parallelism is a vital characteristic of closed kinematic chains mainly for chains with degrees-of-freedom (d.o.f.) greater than one then such chains can be considered for claim as robot manipulators. Parallelism can be detected in kinematic chains with the help of RDCS of a particular kinematic chain. Higher value of RDCS of a kinematic chain will have more parallelism properties. Lower values of RDCS of a kinematic chain will have lower parallelism properties.

Thus Stephenson's chain having RDCS value $3640, 2[80, 6(110), 5(120), 140, 2(170)]$ and Watt's chain having RDCS value $3320, 2[60, 6(100), 6(120), 2(140)]$, it is clear that Stephenson chain is more parallel than Watt chain.

VI. SYMMETRY

Now we know that link A & D will have the same inversions and link B, C, E & F will have the same inversions. For finding symmetry among kinematic links, link A or D is fixed and link B, or C, or E or F is fixed simultaneously.

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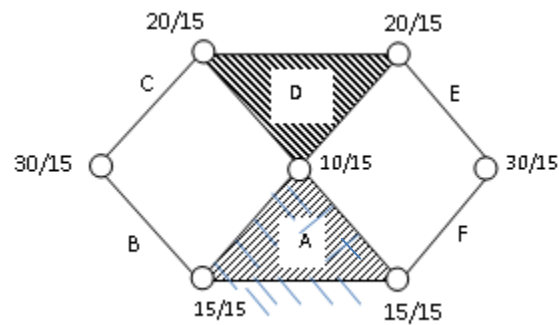


Fig (d) (WATT chain) Link A is fixed

As shown in figure (d), Firstly the link A is fixed. Therefore it's contribution (i.e. 5/15) to the node connecting links B and F is neglected. A matrix will be formed showing the distance between links. The DISTANCE MATRIX (DM) & RELATIVE DISPOSITION MATRIX (RDM) for Watt chain is given below:-

links	A	B	C	D	E	F
A	0	15	25	5	25	15
B	15	0	30	20	40	30
C	25	30	0	20	40	40
D	5	20	20	0	20	20
E	25	40	40	20	0	30
F	15	30	40	20	30	0

	A	B	C	D	E	F
A	0	80	120	30	120	80
B	80	0	80	90	100	80
C	120	80	0	110	100	100
D	30	90	110	0	110	90
E	120	100	100	110	0	80
F	80	80	100	90	80	0

RDS for link A, B, C, D, E & F are: - [0, 30, 2(80), 2(120)] , [0, 3(80), 90, 100] , [0, 80, 2(100), 110, 120] , [0, 30, 2(90), 2(110)] , [0, 80, 2(100), 110, 120] & [0, 3(80), 90, 100] respectively. RDLV for link A, B, C, D, E & F are: - 430, 430, 510, 430, 510, and 430 respectively.

From above values of strings it is clear that when the link A is fixed, link B & F will have symmetrical behavior because of identical values of RDS along with RDLV. Similarly link C & E will have symmetrical behavior.

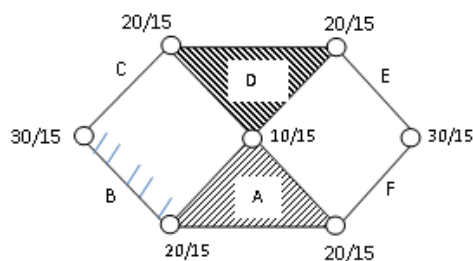


Fig (e) (WATT chain) Link B is fixed

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Now the link B is fixed as shown in figure (e) . The DM and RDM for the chains are:-

	A	B	C	D	E	F
A	0	5	20	10	30	5
B	5	0	15	15	35	10
C	20	15	0	20	40	25
D	10	15	20	0	20	15
E	30	35	40	20	0	30
F	5	10	25	15	30	0

	A	B	C	D	E	F
A	0	30	90	50	145	25
B	30	0	70	60	145	35
C	90	70	0	80	115	85
D	50	60	80	0	115	55
E	145	145	115	115	0	130
F	25	35	85	55	130	0

RDS for link A, B, C, D, E & F are: - [0, 25, 30, 50, 90, (145)] , [0, 30, 35, 60, 70, 145] , [0,70,80,85, 90, 115] , [0, 50, 55,60, 80, 115] , [0, 2(115), 130, 2(145)] & [0, 25, 35, 55, 85, 130] respectively. RDLV for link A, B, C, D, E & F are: - 340, 340, 440, 360, 650, and 330 respectively.

From above values of strings it is clear that when the link B is fixed, No links are having symmetrical behavior. Other links are not required to be fixed since we know that link A & D will have the same inversions and link B, C, E & F will have the same inversions.

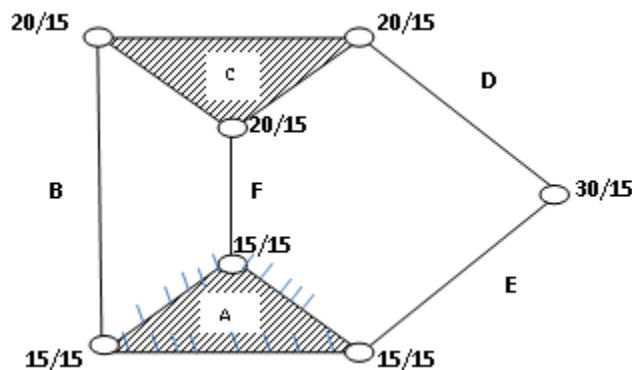


Fig (f) (Stephenson chain) Link A is fixed

Now the link A is fixed as shown in figure (f). The RDS for link A, B, C, D, E & F are: - [0, 2(80), 90,140,170] , [0, 60, 80, 100, 110, 120] , [0, 3(110), 140, 150], [0, 110, 2(120), 140, 170] , [0, 90, 2(100), 140, 150], [0, 60, 80, 100, 110, 120] respectively.

From above values of strings it is clear that when the link A is fixed, link B & F will have symmetrical behavior because of identical values of RDS.

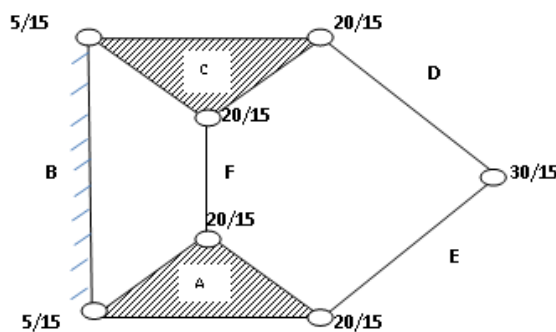


Fig (g) (Stephenson chain) Link B is fixed

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Now the link B is fixed as shown in figure (f). The RDS for link A, B, C, D, E & F are: - $[0, 30, 40, 2(100), 120]$, $[0, 2(30), 3(110)]$, $[0, 30, 40, 2(100), 120]$, $[0, 80, 2(100), 110, 120]$, $[0, 80, 2(100), 110, 120]$, $[0, 4(100), 110]$ respectively.

From above values of strings it is clear that when the link B is fixed, link A & C will have symmetrical behavior because of identical values of RDS. Similarly link D & E will have symmetrical behavior.

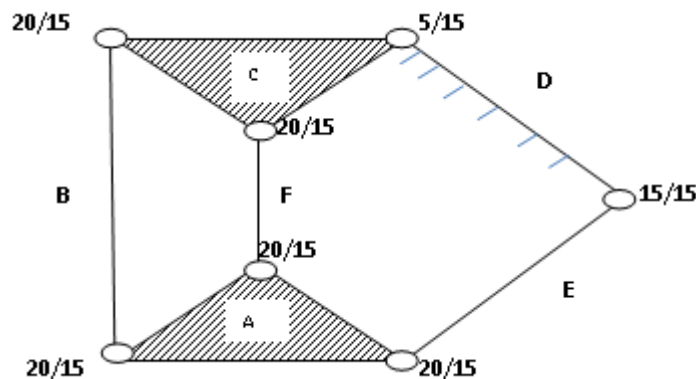


Fig (h) (Stephenson chain) Link D is fixed

Now the link D is fixed as shown in figure (h). The RDS for link A, B, C, D, E & F are: - $[0, 3(110), 2(120)]$, $[0, 80, 90, 110, 2(120)]$, $[0, 30, 2(110), 2(120)]$, $[0, 30, 90, 3(120)]$, $[0, 3(90), 110, 120]$, $[0, 80, 90, 110, 2(120)]$ respectively.

From above values of strings it is clear that when the link D is fixed, link B & F will have symmetrical behavior because of identical values of RDS. Other links are not required to be fixed since we know that A & C, B & F and D & E will have the same inversion.

VII. RESULTS

6 link kinematic Chains	Links to be fixed	Symmetrical links
Watt Chain	A	(B,F) (C,E)
	B	No Symmetrical links
Stephenson chain	A	B & F
	B	(A,C) (D,E)
	D	B & F

Table (1) (symmetry in various 6 link kinematic Chains)

6 link kinematic Chains	RDCS	Parallelism (from most parallel to least parallel)
Stephenson chain	3640	1
Watt Chain	3320	2

Table (2) (parallelism in various 6 link kinematic Chains)

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8 link kinematic Chains	Links to be fixed	Symmetrical links	8 link kinematic Chains	Links to be fixed	Symmetrical links
1	1	(5,7) (6,8)	9	1	(2,4) (7,8)
	6	(3,4)		5	(7,8) (2,4)
2	1	(3,7) (4,8) (5,6)	10	1	No Symmetrical links
	3	(5,6)		2	No Symmetrical links
	5	(1,2) (3,4) (7,8)		3	No Symmetrical links
3	1	(2,3) (5,7) (6,8)		4	No Symmetrical links
	2	No Symmetrical links		5	No Symmetrical links
	4	(2,3) (5,7) (6,8)		6	No Symmetrical links
	5	No Symmetrical links		7	No Symmetrical links
	6	(2,3)		8	No Symmetrical links
4	1	No Symmetrical links	11	1	No Symmetrical links
	2	No Symmetrical links		2	No Symmetrical links
	3	No Symmetrical links		5	No Symmetrical links
	4	No Symmetrical links		7	No Symmetrical links
	5	No Symmetrical links	12	1	(5,6)
	6	No Symmetrical links		2	(5,6)
	7	No Symmetrical links		5	(1,4) (2,3) (7,8)
	8	No Symmetrical links		7	(5,6)
5	1	(5,6)	13	1	No Symmetrical links
	2	(5,6)		2	(1,4) (5,6) (7,8)
	3	(5,6)		3	(1,4) (5,6) (7,8)
	4	(5,6)		5	No Symmetrical links
	5	No Symmetrical links		7	No Symmetrical links
	7	(5,6)	14	1	No Symmetrical links
	8	(5,6)		2	(1,4) (5,6)
6	1	(5,6)		3	(1,4) (5,6)
	2	(5,6)		5	No Symmetrical links
	3	(5,6)		7	(1,4) (5,6)
	4	(5,6)		8	(1,4) (5,6)
	5	No Symmetrical links	15	1	(5,6) (7,8)
	7	(5,6)		5	(1,4) (2,3) (7,8)
	8	(5,6)	16	1	(3,4) (5,7) (6,8)
7	1	(5,6,7,8) (2,3)		5	No Symmetrical links
	2	(5,6) (7,8)			
	4	(5,6,7,8) (2,3)			
	5	(6,7,8)			
8	1	No Symmetrical links			
	5	(2,3)			

Table (4) (Symmetry in various 8 link kinematic Chains)

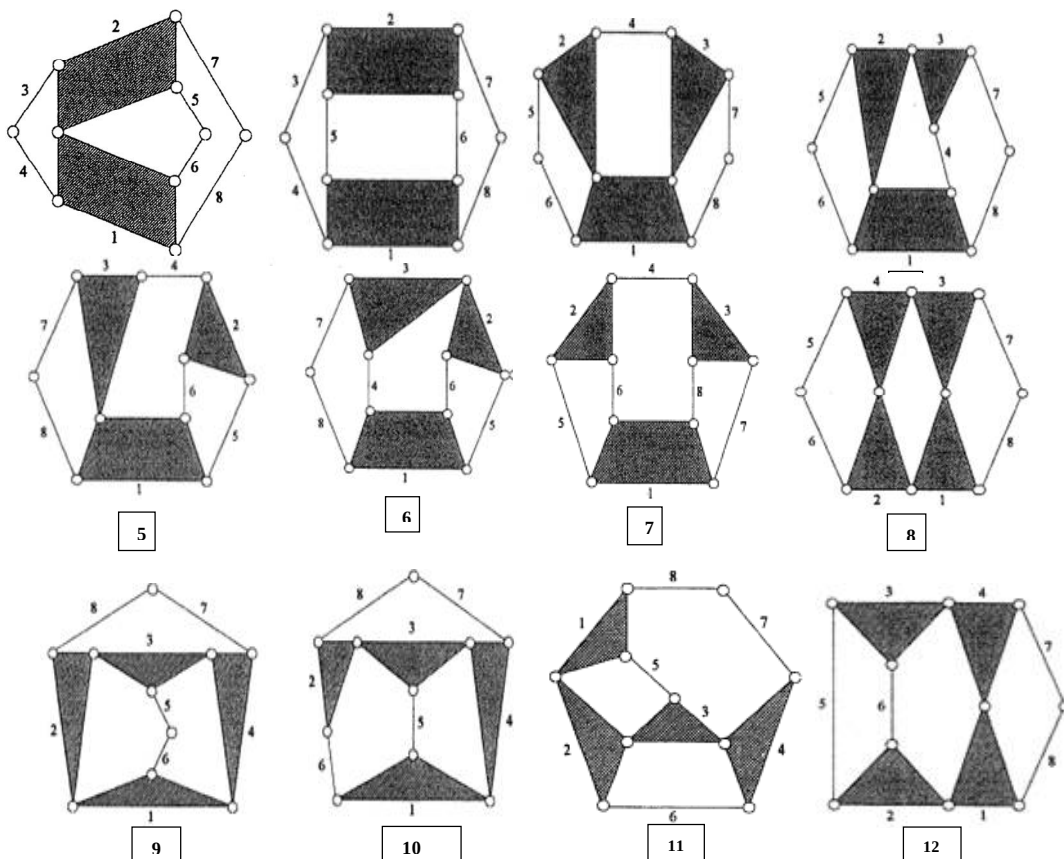
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8 link kinematic Chains	RDCS	Parallelism (from most parallel to least parallel)
7	9264	1
15	8800	2
2	8784	3
8	8320	4
6	8288	5
5	8248	6
11	8080	7
14	8040	8
13	7930	9
16	7840	10
9	7760	11
10	7600	12
3	7488	13
12	7480	14
4	7416	15
1	6576	16

Table (4) (parallelism in various 8 link kinematic Chains)



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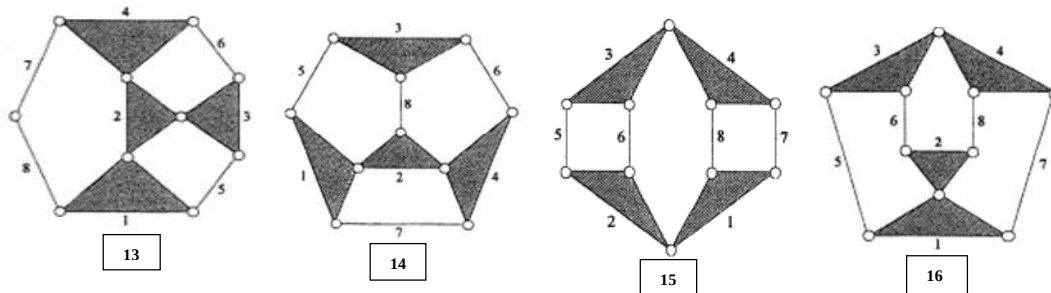


Fig (i) (8link 1 DOF kinematic Chains)

VIII. CONCLUSION

The method is successful for all the chains of 8 links 1 DOF chains. This method can be used for multi degree of freedom kinematic chains. A simple, efficient and reliable method to identify parallelism and symmetry among kinematic chain is developed.

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BIOGRAPHY



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