

Characteristic Polynomial of r -Adjacency Matrix of Path Semigraphs

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ABSTRACT: Let S be a semigraph with n vertices. In this paper, we discuss the characteristic polynomial of r -adjacency matrix for n vertices with $n+1$ edges.

KEYWORDS: Semigraph, Adjacency and Degrees in Semigraph, r -Adjacency Matrix, Path Semigraph.

I. INTRODUCTION

Semigraphs are introduced as a generalization of graphs. Instead of two vertices as in the graph, several vertices are joined by an edge in a semigraph. Semigraphs were introduced by E. Sampathkumar [11], and studied their applications in the year 2000. Since then, the study of semigraphs become an important area in the field of graph theory [1]. N. S. Bhave et.al [2] studied characterization of potentially Hamiltonian graph in terms of dual semigraph. C. M. Deshpande et.al [3] studied matrix representation of semigraphs. Ramaswamy et.al [10] studied Adjacency Matrices of Semigraphs and their Spectral Analysis. Gutman [5] studied Mathematical concepts in organic chemistry. H. Hua and M. Wang [6] studied Unicyclic Graphs with given number of pendent vertices and minimal energy. V. Swaminathan and S. Gomathi [4] studied the domination in semigraphs. V. Nikiforov [9] studied Graphs and Matrices with Maximal Energy. Y. B. Venkatakrishnan [12] discussed bipartite theory of semigraphs. N. Murugesan and D. Narmatha [7] discussed the relationship between various types of adjacencies in semigraphs, and properties of associated graphs of a given semigraph. N. Murugesan and D. Narmatha [8] discussed the e - and n -energies of linear semigraph. In this paper, the characteristic polynomial of r -adjacency matrix of semigraphs have been discussed.

II. PREREQUISITIES

A semigraph S is a pair (V, X) where V is a nonempty set whose elements are called vertices of S and X is a set of ordered n -tuples $n \geq 2$, called edges of S satisfying the following conditions:

- i. the components of an edge E in X are distinct vertices from V .
- ii. any two edges have at most one vertex in common.
- iii. two edges $E_1 = (u_1, u_2, \dots, u_m)$ and $E_2 = (v_1, v_2, \dots, v_n)$ are said to be equal iff
 - a. $m = n$ and
 - b. either $u_i = v_i$ or $u_i = v_{n-i+1}$ for $1 \leq i \leq n$.

The vertices in a semigraph are divided into three types namely end vertices, middle vertices and middle-end vertices, depending upon their positions in an edge. The end vertices are represented by thick dots, middle vertices are represented by small circles, a small tangent is drawn at the small circles to represent middle-end vertices.

There are different types of adjacency of two vertices in a semigraph. Two vertices u and v in a semigraph S are said to be

- i. adjacent if they belong to the same edge.

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- ii. consecutively adjacent if they are adjacent and consecutive in order as well.
- iii. e-adjacent if they are the end vertices of an edge.
- iv. 1e-adjacent if both the vertices u and v belong to the same edge and atleast one of them is an end vertex of that edge.
- v. n -adjacent if they are common to n edges.

Correspondingly different types of adjacent matrices are defined, and they are used to study different kinds of structural properties of the semigraph. In this paper, we consider the most fundamental semigraph and generalize the characteristic polynomial with respect to its r -adjacency matrix.

III. PATH SEMIGRAPHS

A path semigraph $S = (V, X)$ is a semigraph with the following properties.

- i. it has no middle – end vertices, but atleast one middle vertex.
- ii. it has exactly two end vertices each with edge deg one.
- iii. the edge degree of all other end vertices (if they exist) are exactly two.

Let $P_{m,n}$ be a path semigraph with m edges and n vertices. The r -adjacency matrix of $P_{m,n}$ is defined as

$$A = (a_{ij})_{n \times n} = \begin{cases} r & \text{if } r \text{ edges are incident to } v_i \text{ and } v_j \\ 0 & \text{otherwise} \end{cases}$$

For simplicity, we denote the extreme end vertices of $P_{m,n}$ as v_1 and v_n respectively. We know that in a path semigraph $P_{m,n}$, n must be greater than or equal to $m + 1$. First let us assume $n = m + 1$. As an example, for $m = 3$ and $n = 4$ then

$$A_{3,4} = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 1 \end{pmatrix}_{4 \times 4}$$

It can be found that the characteristic polynomial of the above A is $\lambda^4 - 6\lambda^3 + 10\lambda^2 - 4\lambda = 0$. In general, the adjacency matrix of $P_{m,n}$, when $n = m + 1$, is in the following form

$$A_{m,n} = \begin{pmatrix} 1 & 1 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & \dots & 0 & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & 0 & \dots & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 1 & 1 \end{pmatrix}_{(m+1) \times (m+1)}$$

We know that the characteristic polynomial of $A_{m,n}$ is of the form

$$D_{1,m} \lambda^{m+1} - D_{2,m} \lambda^m + D_{3,m} \lambda^{m-1} - \dots + (-1)^{m+1} D_{m+1,m} = 0$$

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Where $D_{s,m}$ is the sum of all s^{th} order minors of $A_{m,n}$, whose principal diagonals lie along the principal diagonal of A . Now we will determine the coefficients of $D's$.

It can be found that the rank $\rho(A) = m$, by applying the elementary operations $R_i \Leftrightarrow R_i - R_{i-1}$ at the $(i-1)^{th}$ step.

Now

$$\begin{vmatrix} 1 & 1 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & \dots & 0 & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & 0 & \dots & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 1 & 1 \end{vmatrix}_{n \times n} =$$

$$\begin{vmatrix} 2 & 1 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & \dots & 0 & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & 0 & \dots & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 1 & 1 \end{vmatrix}_{(n-1) \times (n-1)} -$$

$$\begin{vmatrix} 1 & 1 & 0 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 0 & \dots & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 2 & 1 & \dots & 0 & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & 0 & \dots & 1 & 2 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 0 & 0 & \dots & 0 & 0 & 1 & 1 \end{vmatrix}_{(n-1) \times (n-1)}$$

$$= [(n-2)A - (n-3)B] - [(n-3)A - (n-4)B]$$

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$$= A - B$$

$$= 0$$

where $A = \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix}; B = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix}$

Hence $D_{s,m+1} = 0$.

For $m = 1, 2, 3, \dots$, the coefficients $D'_{m,m+1}s$ from the following triangle.

			1	2					
			1	4	3				
		1	6	10	4				
	1	8	21	20	5				
	1	10	36	56	35	6			
	1	12	55	120	126	56	7		
.....									
.....									

From the above triangle, it can be observed that

- i. $D_{2,m} = 2m$
- ii. $D_{m+1,m} = m + 1$

Let $[1, 2, 3, \dots, n]$ represents determinant of order n of a matrix of order $\geq n$, contains the principal diagonal elements of the rows $1, 2, 3, \dots, n$ respectively in the given order.

$[1, 2, 3, \dots, n]_{-1}$, represents the above determinant in which the last diagonal element is reduced by one. Also $[1, 2, 3, \dots, n] - 1$, represents the above determinant, but in which the value is reduced by one.

Then it can be observed that

$$D_{1,1} + D_{2,1} + D_{3,2} + D_{4,3} + \dots + D_{m-2,m-3} = [1, 2, 3, \dots, m - 1] - 1$$

and $D_{m,m}$ = sum of all $(m - 1) \times (m - 1)$ minors whose diagonal elements are the principal diagonal elements of the matrix of order $(m + 1) \times (m + 1)$

(note that there are $(m + 1)C_{m-1}$ such minors)

$$= [1, 2, 3, \dots, m - 1] + [1, 2, 3, \dots, (m - 2), m]_{-1} + [1, 2, 3, \dots, (m - 3), (m - 1), m]_{-1} + [1, 2, 3, \dots, (m - 4), (m - 2), (m - 1), m]_{-1} + \dots + [1, 3, 4, \dots, (m - 1), m]_{-1} + [2, 3, \dots, (m - 1), m]_{-1}$$

+ the sum of distinct $(m - 2) \times (m - 2)$ minors of

$$\left\{ [1, 2, 3, \dots, m - 1], [1, 2, 3, \dots, (m - 2), m]_{-1}, [1, 2, 3, \dots, (m - 3), (m - 1), m]_{-1}, [1, 2, 3, \dots, (m - 4), (m - 2), (m - 1), m]_{-1}, \dots, [1, 3, 4, \dots, (m - 1), m]_{-1}, [2, 3, 4, \dots, (m - 1), m]_{-1} \right\}$$

+ the sum of distinct $(m - 2) \times (m - 2)$ minors of $[1, 2, 3, \dots, m - 1]_{-1}$

+ the sum of diagonal elements of $[1, 2, 3, \dots, m - 1] - 1$.

(In all cases, only the minors in which the diagonal elements are the principal diagonal elements of the given matrix are considered)

= the sum of $(m - 1) \times (m - 1)$ minors of $A_{m,m}$ + the sum of $(m - 2) \times (m - 2)$ minors of $A_{m,m} +$

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the sum of $(m-2) \times (m-2)$ minors of $A_{m-1,m-1}$ + the sum of diagonal elements of $[1,2,3,\dots,m-1]-1$
i.e., $D_{m,m} = D_{m-1,m-1} + D_{m,m-1} + D_{m-1,m-2} + [D_{m-2,m-3} + \dots + D_{2,1} + D_{1,1}]$

As an example,

$D_{5,5}$ = Sum of minors of order 4×4 in $A_{6,6}$

\ = sum of $\left\{ \begin{matrix} [1234], [1235], [1236], [1245], [1246], [1256], [1345], [1346], [1356], [1456], \\ [2345], [2346], [2356], [2456], [3456] \end{matrix} \right\}$ in $A_{6,6}$.

$$= [1234] + [1235]_{-1} + [1245]_{-1} + [1345]_{-1} + [2345]_{-1}$$

$$+ \text{sum of distinct } 3 \times 3 \text{ minors of } \{[1234] + [1235]_{-1} + [1245]_{-1} + [1345]_{-1} + [2345]_{-1}\}$$

$$+ \text{sum of } 3 \times 3 \text{ minors of } [1234]_{-1}$$

$$+ \text{sum of diagonal elements of } [1234]-1.$$

$$= \left[\begin{matrix} \begin{vmatrix} 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 2 \end{vmatrix}_{-1} + \begin{vmatrix} 1 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{vmatrix}_{-1} + \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{vmatrix}_{-1} + \begin{vmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{vmatrix}_{-1} \right]$$

$$+ \left[\begin{matrix} \begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{vmatrix}_{-1} + \begin{vmatrix} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{vmatrix}_{-1} + \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix}_{-1} + \begin{vmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix}_{-1} + \begin{vmatrix} 2 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix}_{-1} + \begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{vmatrix}_{-1} + \begin{vmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix}_{-1} + \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 0 & 2 \end{vmatrix}_{-1} + \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{vmatrix}_{-1} \right]$$

$$\backslash + \left[\begin{matrix} \begin{vmatrix} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix}_{-1} + \begin{vmatrix} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix}_{-1} + \begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{vmatrix}_{-1} + \begin{vmatrix} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{vmatrix}_{-1} \right]$$

$$+ (1+2+2+1)$$

$$= \left[\begin{matrix} \begin{vmatrix} 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 2 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 1 & 0 & 0 \\ 1 & 2 & 0 & 0 \\ 0 & 0 & 2 & 1 \\ 0 & 0 & 1 & 1 \end{vmatrix} + \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 1 \end{vmatrix} + \begin{vmatrix} 2 & 1 & 0 & 0 \\ 1 & 2 & 1 & 0 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 1 & 1 \end{vmatrix} \right]$$

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$$\begin{aligned}
 & + \left[\begin{array}{c} \left| \begin{array}{ccc} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{array} \right| + \left| \begin{array}{ccc} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 2 \end{array} \right| + \left| \begin{array}{ccc} 2 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{array} \right| + \left| \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 2 \end{array} \right| + \left| \begin{array}{ccc} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{array} \right| + \left| \begin{array}{ccc} 2 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 1 \end{array} \right| + \left| \begin{array}{ccc} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{array} \right| + \\
 & \left| \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 1 \end{array} \right| + \left| \begin{array}{ccc} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{array} \right| + \left| \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{array} \right| \end{array} \right] \\
 & + \left[\begin{array}{c} \left| \begin{array}{ccc} 2 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 1 \end{array} \right| + \left| \begin{array}{ccc} 1 & 0 & 0 \\ 0 & 2 & 1 \\ 0 & 1 & 1 \end{array} \right| + \left| \begin{array}{ccc} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{array} \right| + \left| \begin{array}{ccc} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{array} \right| \end{array} \right] \\
 & + (1 + 2 + 2 + 1)
 \end{aligned}$$

$$\begin{aligned}
 & = [\text{sum of } 4 \times 4 \text{ minors of } A_{5,5}] + [\text{sum of } 3 \times 3 \text{ minors of } A_{5,5}] + \\
 & [\text{sum of } 3 \times 3 \text{ minors of } A_{4,4}] + [\text{sum of } 2 \times 2 \text{ minors of } A_{3,3} + \text{Determinant of } A_{2,2} + 1] \\
 & = D_{5,4} + D_{4,4} + D_{4,3} + (D_{3,2} + D_{2,1} + D_{1,1}) \\
 & = 5 + 20 + 4 + 6 = 35
 \end{aligned}$$

Similarly, for $k = 3, 4, \dots, m-1$.

$$\begin{aligned}
 D_{k,m} & = \text{sum of } (k-1) \times (k-1) \text{ minors of } A_{m+1,m+1}. \\
 & = \text{sum of } \left\{ [1, 2, 3, \dots, k-1], [1, 2, 3, \dots, k-2, k], [1, 2, 3, \dots, k-3, k-1, k], [2, 3, \dots, k-1, k], [2, 3, 4, \dots, k-1, k], \right. \\
 & \left. [2, 3, 4, \dots, k-2, k], \dots, [5, 6, \dots, k+1, k+2, k+3] \right\} \\
 & = \text{the sum of } \left\{ [r_1, r_2, \dots, r_{k-1}], r_i \in \{1, 2, 3, \dots, k\}, \right. \\
 & \left. [r_1, r_2, r_3, \dots, r_{k-2}, r_{k-1}], r = k+1 \right\}
 \end{aligned}$$

$$\begin{aligned}
 & + \text{the sum of distinct } (k-2) \times (k-2) \text{ minors of} \\
 & \left\{ [r_1, r_2, \dots, r_{k-1}], [r_1, r_2, r_3, \dots, r_{k-2}, r_{k-1}], r_i \in \{1, 2, 3, \dots, k\}, r = k+1 \right\}
 \end{aligned}$$

$$\begin{aligned}
 & + \text{the sum of distinct } (k-2) \times (k-2) \text{ minors of} \\
 & \left\{ [r_1, r_2, \dots, r_{k-2}], [r_1, r_2, r_3, \dots, r_{k-3}, r_{k-2}], r_i \in \{1, 2, 3, \dots, k-1\}, r = k \right\}
 \end{aligned}$$

$$\begin{aligned}
 & + \text{the sum of } \left\{ [r_1, r_2, r_3, \dots, r_{k-2}], r_i \in \{1, 2, 3, \dots, k-1\} \right\} - 1
 \end{aligned}$$

$$\begin{aligned}
 & = D_{k,m-1} + D_{k-1,m-1} + D_{k-1,m-2} + \sum_{i=1}^{k-2} D_{i,i+1}
 \end{aligned}$$

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As an example,

$D_{4,5}$ = sum of 3×3 minors whose diagonal elements are the diagonal elements of $A_{6,6}$,

= sum of $\{[1,2,3], [1,2,4], \dots, [4,5,6]\}$

= sum of $\{[1,2,3], [1,2,4], [1,3,4], [2,3,4], [1,2,5]_{-1}, [1,3,5]_{-1}, [1,4,5]_{-1}, [2,3,5]_{-1}, [2,4,5]_{-1}, [3,4,5]_{-1}\}$

of $A_{6,6}$,

+ the sum of distinct 2×2 minors of $\{[1,2], [1,3], [1,4], [2,3], [2,4]_{-1}, [3,4]_{-1}\}$

+ the sum of $\{[1,2], [1,3], [2,3]\} - 1$

$$\begin{aligned}
 &= \left[\begin{array}{c} \left| \begin{array}{ccc} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 1 & 2 \end{array} \right| + \left| \begin{array}{ccc} 1 & 1 & 0 \\ 1 & 2 & 1 \\ 0 & 0 & 2 \end{array} \right| + \left| \begin{array}{ccc} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 2 & 1 \end{array} \right| + \left| \begin{array}{ccc} 1 & 0 & 0 \\ 2 & 1 & 0 \\ 1 & 2 & 1 \end{array} \right| + \left| \begin{array}{ccc} 1 & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 1 \end{array} \right| + \\
 & \left| \begin{array}{ccc} 1 & 0 & 0 \\ 1 & 0 & 0 \\ 2 & 1 & 0 \end{array} \right| + \left| \begin{array}{ccc} 2 & 0 & 0 \\ 2 & 0 & 0 \\ 2 & 1 & 0 \end{array} \right| + \left| \begin{array}{ccc} 0 & 2 & 0 \\ 0 & 2 & 1 \\ 1 & 2 & 1 \end{array} \right| + \left| \begin{array}{ccc} 0 & 2 & 0 \\ 0 & 2 & 1 \\ 1 & 2 & 1 \end{array} \right| + \left| \begin{array}{ccc} 0 & 0 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{array} \right| + \left| \begin{array}{ccc} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{array} \right| \end{array} \right] \\
 &+ \{[1,2] + [1,3] + [1,4] + [2,3] + [2,4] + [3,4] + [1,5]_{-1} + [2,5]_{-1} + [3,5]_{-1} + [4,5]_{-1}\} \\
 &+ \left\{ \left| \begin{array}{cc} 1 & 1 \\ 1 & 2 \end{array} \right| + \left| \begin{array}{cc} 1 & 0 \\ 0 & 2 \end{array} \right| + \left| \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right| + \left| \begin{array}{cc} 2 & 1 \\ 1 & 2 \end{array} \right| + \left| \begin{array}{cc} 2 & 0 \\ 0 & 1 \end{array} \right| + \left| \begin{array}{cc} 2 & 1 \\ 1 & 1 \end{array} \right| \right\} \\
 &+ \left\{ \left| \begin{array}{cc} 1 & 1 \\ 1 & 2 \end{array} \right| + \left| \begin{array}{cc} 1 & 0 \\ 0 & 2 \end{array} \right| + \left| \begin{array}{cc} 2 & 1 \\ 1 & 2 \end{array} \right| - 1 \right\} \\
 &= 20 + \left\{ \left| \begin{array}{cc} 1 & 1 \\ 1 & 2 \end{array} \right| + \left| \begin{array}{cc} 1 & 0 \\ 0 & 2 \end{array} \right| + \left| \begin{array}{cc} 1 & 0 \\ 0 & 2 \end{array} \right| + \left| \begin{array}{cc} 2 & 1 \\ 1 & 2 \end{array} \right| + \left| \begin{array}{cc} 2 & 0 \\ 0 & 2 \end{array} \right| + \left| \begin{array}{cc} 2 & 1 \\ 1 & 2 \end{array} \right| + \left| \begin{array}{cc} 1 & 0 \\ 0 & 1 \end{array} \right| + \left| \begin{array}{cc} 2 & 0 \\ 0 & 1 \end{array} \right| + \left| \begin{array}{cc} 2 & 0 \\ 0 & 1 \end{array} \right| + \left| \begin{array}{cc} 2 & 1 \\ 1 & 1 \end{array} \right| \right\} + 10 + 5 \\
 &= 20 + 21 + 15 \\
 &= 56.
 \end{aligned}$$

From the above observations, we get the following theorem.

Theorem 2.1:

The characteristic equation of an r - adjacency matrix of a path semigraph with m edges and $m + 1$ vertices is

$$\sum_{i=1}^{m+1} D_{i,m} (-\lambda)^{m+2-i} = 0 \text{ where } D_{1,m} = 1 ; D_{m+1,m} = m + 1 ; D_{2,2} = 4 \text{ and}$$

$$D_{i,m} = \begin{cases} D_{i-1,m-1} + \sum_{k=1}^m k, & \text{if } i = m \\ D_{i-1,m-1} + D_{i,m-1} + \sum_{k=1}^{i-1} D_{i-k,m-k-1} & \text{if } i \neq m \end{cases}$$

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IV. CONCLUSION

In this paper, the characteristic polynomial of path semigraphs have been discussed with respect to its r – adjacency matrix.

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