

On Sb^* Homeomorphism in Topological Spaces

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ABSTRACT: In this paper i introduce the concept of Sb^* -closed maps, Sb^* -open maps, Sb^* -Homeomorphisms and also study their properties. Using these new types of maps several characterizations and properties have been obtained

KEYWORDS: Sb^* -closed maps, Sb^* -open maps, Sb^* -Homeomorphism

I. INTRODUCTION

Levine[1] introduced and investigated the concept of generalized closed sets in topological spaces. After this work many of the authors investigated various forms of stronger and weaker forms of closed sets. In later [2] introduced strongly b^* closed sets in topological spaces. K.Balachandran [7] introduced generalized continuous maps in topological spaces. Homeomorphism plays a very important role in topology. By definition, a homeomorphism between two topological spaces X and Y is a bijective map $f: X \rightarrow Y$ when f and f^{-1} are continuous. In 1995, Maki, Devi and Balachandran [3] introduced the concepts of semi-generalized homeomorphisms and generalized semi homeomorphisms and studied some semi topological properties. Devi and Balachandran [4] introduced a generalization of α -homeomorphism in 2001. In this paper I first introduced a new class of closed map called Sb^* -closed map and Sb^* -open map. Moreover I introduce a new class of homeomorphism called Sb^* -homeomorphism which are stronger than homeomorphism and also study their properties

II. PRELIMINARIES

Definition:2.1 A subset (X, τ) is said to be

- (1) Semi-pre closed (β -closed)[6] set if $\text{int}(\text{cl}(\text{int}(A))) \subseteq A$
- (2) g -closed[6] set if $\text{cl}(A) \subseteq U$, whenever $A \subseteq U$ and U is open in X
- (3) w -closed[5] set if $\text{cl}(A) \subseteq U$, whenever $A \subseteq U$ and U is semi-open in X
- (4) α -closed[4] set if $\text{cl}(\text{int}(\text{cl}(A))) \subseteq A$
- (5) wg -closed[5] set if $\text{cl}(\text{int}(A) \subseteq U$, whenever $A \subseteq U$ and U is open in X
- (6) g^* -closed[6] set if $\text{cl}(A) \subseteq U$, whenever $A \subseteq U$ and U is g -open in X
- (7) b -closed[9] set if $(\text{cl}(\text{int}(A)) \cup \text{int}(\text{cl}(A))) \supseteq A$
- (8) b^{**} -closed[2] set if $A \subseteq (\text{int}(\text{cl}(\text{int}(A))) \cup (\text{cl}(\text{int}(\text{cl}(A))))$
- (9) Sb^* -closed[2] set if $\text{cl}(\text{int}(A) \subseteq U$, whenever $A \subseteq U$ and U is b -open in X

The complements of the above mentioned closed sets are their respective open sets

Definition:2.2 A map $f: X \rightarrow Y$ is said to be

- (1) Continuous function if $f^{-1}(V)$ is closed in X for every closed set V in Y
- (2) b -continuous function if $f^{-1}(V)$ is b -closed in X for every closed set V in Y
- (3) g -continuous function if $f^{-1}(V)$ is g -closed in X for every closed set V in Y

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- (4) α -continuous function if $f^{-1}(V)$ is α -closed in X for every closed set V in Y
- (5) w-continuous function if $f^{-1}(V)$ is w-closed in X for every closed set V in Y
- (6) g^* - continuous function if $f^{-1}(V)$ is g^* -closed in X for every closed set V in Y
- (7) Sb^* - continuous function if $f^{-1}(V)$ is Sb^* -closed in X for every closed set V in Y

Definition:2.3 A bijective function $f: (X, \tau) \rightarrow (Y, \sigma)$ is called

- (1) b-homeomorphism if both f and f^{-1} are b-continuous
- (2) w-homeomorphism if both f and f^{-1} are w-continuous
- (3) g-homeomorphism if both f and f^{-1} are g-continuous
- (4) α -homeomorphism if both f and f^{-1} are α -continuous
- (5) g^* - homeomorphism if both f and f^{-1} are g^* -continuous
- (6) Sb^* - homeomorphism if both f and f^{-1} are Sb^* -continuous

III. Sb^* - CLOSED MAPS AND Sb^* - OPEN MAPS

Definition:3.1 Let $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be Strongly b star (briefly Sb^*) closed map (respectively open map) if the image of every closed (open) set in (X, τ) is Sb^* -closed (Sb^* -open) in (Y, σ)

Theorem:3.2 Every closed map is Sb^* -closed map but not conversely

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a closed map . Then F be a closed set in (X, τ) . So $f(F)$ be a closed set in (Y, σ) . Now by the definition of Sb^* -closed, $f(F)$ is a Sb^* -closed set in (Y, σ) . We Know that closed $\Rightarrow Sb^*$ -closed [2] . f is Sb^* -closed map .

The converse of the above theorem need not be true as seen from the following example

Example:3.3 Let $X=Y=\{a,b,c,d\}$, $\tau =\{X, \phi, \{b\}, \{b,c\}, \{a,b,d\}\}$, $\sigma =\{Y, \phi, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\}$. Define a function $f: X \rightarrow Y$ as $f(a)=a$, $f(b)=f(c)=d, f(d)=b$. Let $A=\{a,b,c\}$ be a Sb^* -closed . $f(\{a,b,c\})=\{a,d\}$ which is not closed map

Remark:3.4 Similarly we prove that the below theorems follows from the definition

- (i) Every α -closed map is Sb^* -closed map
- (ii) Every w-closed map is Sb^* -closed map
- (iii) Every g^* -closed map is Sb^* -closed map

The converse of the above theorems need not be true as seen from the following examples.

Example:3.5 (i) Let $X=Y=\{a,b,c,d\}$, $\tau =\sigma =\{X, Y, \phi, \{a\}\}$. Let $f: X \rightarrow Y$ be an identity map such that $f: X \rightarrow Y$. Let $A=\{a\}$ be a Sb^* -closed . $f(\{a\})=\{a\}$ which is not α -closed map

(ii) Let $X=Y=\{a,b,c,d\}$, $\tau =\{X, \phi, \{b\}, \{b,c\}, \{a,b,d\}\}$, $\sigma =\{Y, \phi, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\}$. Define a function $f: X \rightarrow Y$ as $f(a)=a$, $f(b)=c$, $f(c)=b, f(d)=d$. Let $A=\{a\}$ be a Sb^* -closed . $f(\{a\})=\{a\}$ which is not w-closed map

(iii) Let $X=Y=\{a,b,c,d\}$, $\tau =\{X, \phi, \{b\}, \{b,c\}, \{a,b,d\}\}$, $\sigma =\{Y, \phi, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\}$. Let f be an identity map such that $f: X \rightarrow Y$. Let $A=\{b,c\}$ be a Sb^* -closed . $f(\{b,c\})=\{b,c\}$ which is not g^* -closed map

Theorem:3.6 Every Sb^* -closed map is wg-closed map

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be a closed map . Then F be a closed set in (X, τ) . So $f(F)$ be a Sb^* -closed set in (Y, σ) . Now by the definition of wg-closed, $f(F)$ is a wg-closed set in (Y, σ) . We Know that Sb^* -closed \Rightarrow wg-closed [2] . f is wg-closed map .

The converse of the above theorem need not be true as seen from the following example

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Example:3.7 Let $X=Y=\{a,b,c,d\}$, $\tau =\{X, \phi, \{a\},\{a,c\},\{a,c,d\}\}$, $\sigma =\{Y, \phi, \{a\},\{b\},\{a,b\}, \{a,b,c\}\}$. Let f be an identity map such that $f: X \rightarrow Y$. Let $A=\{c,d\}$ be a wg-closed . $f(\{c,d\})=\{c,d\}$ which is not Sb^* -closed map

Remark:3.8 Similarly we prove that the below theorems follows from the definition

- (i) Every Sb^* -closed map is semi pre-closed map
- (ii) Every Sb^* -closed map is b-closed map.

The converse of the above theorems need not be true as seen from the following examples .

Example:3.9 (i)Let $X=Y=\{a,b,c,d\}$, $\tau =\{X, \phi, \{a\},\{a,c\},\{a,c,d\}\}$, $\sigma =\{Y, \phi, \{a\},\{b\},\{a,b\}, \{a,b,c\}\}$. Define a function $f: X \rightarrow Y$ as $f(a)=b, f(b)=a, f(c)=d, f(d)=c$. Let $A=\{c,d\}$ be a semi pre-closed . $f(\{c,d\})=\{d,c\}$ which is not Sb^* -closed map

(ii)Let $X=Y=\{a,b,c\}$, $\tau =\{X, \phi, \{a\},\{a,c\}\}$, $\sigma =\{Y, \phi, \{b\},\{b,c\}\}$. Let f be an identity map such that $f: X \rightarrow Y$. Let $A=\{c\}$ be a b-closed. $f(\{c\})=\{c\}$ which is not Sb^* -closed map

Theorem:3.10 A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is Sb^* -closed map iff for each subset G of (Y, σ) and each open set U containing $f^{-1}(V)$ there is an Sb^* -open set V of (Y, σ) such that $G \subseteq V$ and $f^{-1}(V) \subseteq U$

Proof: Suppose that f is Sb^* -closed set of (X, τ) . Let $G \subseteq Y$ and U be an open set of (X, τ) such that $f^{-1}(G) \subseteq U$. So $X-U$ is a closed set in (X, τ) . Since f is Sb^* -closed in (X, τ) , $f(X-U)$ is an Sb^* -closed set in (Y, σ) . $f^{-1}(X-U)$ is an Sb^* -closed set in (Y, σ) . Then $V=Y-f(X-U)$ is Sb^* -open set in (Y, σ) . $f^{-1}(G) \subseteq U$ implies $G \subseteq V$ and $f^{-1}(V)=X-f^{-1}(f(X-U)) \subseteq X-(X-U)=U$. (ie) $f^{-1}(V) \subseteq U$. Conversely, By the definition of Sb^* -closed , Let Q be a closed set of (X, τ) . Then $f^{-1}(f(Q)^c) \subseteq Q^c$ is an open set in (X, τ) . By hypothesis , there exist an Sb^* -open set V in (Y, σ) such that $f(Q^c) \subseteq V$ and $f^{-1}(V) \subseteq Q$, so $Q \subseteq (f^{-1}(V))^c \Rightarrow V^c \subseteq f(Q) \subseteq f((f^{-1}(V))^c) \subseteq V \Rightarrow f(Q) \subseteq V^c$. Since V^c is Sb^* -closed . $f(Q)$ is Sb^* -closed . (ie) $f(Q)$ is Sb^* -closed in (Y, σ) . Therefore f is Sb^* -closed map.

Theorem:3.11 If $f: (X, \tau) \rightarrow (Y, \sigma)$ is closed map and $g: (Y, \sigma) \rightarrow (Z, \xi)$ is Sb^* -closed map then the composition $g \circ f: (X, \tau) \rightarrow (Z, \xi)$ is Sb^* -closed map .

Proof: By the definition of Sb^* -closed map, Let Q be any closed set in (X, τ) . Since f is a closed map $f(Q)$ is closed set in (Y, σ) . Since g is Sb^* -closed map. $g(f(Q))=g \circ f(Q)$ is Sb^* -closed set in (Z, ξ) . Thus $g \circ f$ is Sb^* -closed map .

Remark:3.12 If $f: (X, \tau) \rightarrow (Y, \sigma)$ is Sb^* closed map and $g: (Y, \sigma) \rightarrow (Z, \xi)$ is closed map then the composition need not be an Sb^* -closed map as seen from the following example

Example:3.13 Let $X=Y=Z=\{a,b,c\}$, $\tau =\{X, \phi, \{a\},\{a,c\}\}$, $\sigma =\{Y, \phi, \{a\},\{b\}, \{b,c\}\}$, $\xi =\{Z, \phi, \{a\},\{b,c\}\}$. Let f be an identity map such that $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ as $g(a)=c, g(b)=a, g(c)=b$. Then f is Sb^* -closed map and g is a closed map. Since $g(c)=\{b\}$ is not an Sb^* -closed map . Let $A=\{c,d\}$ be a wg-closed . $f(\{c,d\})=\{c,d\}$ which is not Sb^* -closed map. Thus $g \circ f$ is not a Sb^* -closed map.

Definition:3.14 A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be Sb^* -closed map is the image of every closed set in (X, τ) is Sb^* -closed in (Y, σ)

Definition: A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is said to be Sb^* -open map is the image of every open set in (X, τ) is Sb^* -open in (Y, σ)

Theorem:3.15 For every bijective function $f: (X, \tau) \rightarrow (Y, \sigma)$ then the following statements are equivalent (i) $f^{-1}: (Y, \sigma) \rightarrow (X, \tau)$ is Sb^* -continuous . (ii) f is an Sb^* -open map . (iii) f is an Sb^* -closed map.

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Proof: (i) implies (ii) Let $f^{-1}: (X, \tau) \rightarrow (Y, \sigma)$ is Sb^* -continuous. Let U be an open map in (X, τ) . By hypothesis, $(f^{-1})(U)$ is Sb^* -open in (Y, σ) . $f(U)$ is Sb^* -open in (Y, σ) . Then by definition of Sb^* -open, $f(U)$ is Sb^* -open. f is Sb^* -open. (ii) implies (iii) Let f is an Sb^* -open map. Let Q be a closed set in (X, τ) . Then Q^c is open. By hypothesis, $(f)(Q)$ is Sb^* -closed in (Y, σ) . $f(Q^c)$ is Sb^* -open in (Y, σ) . Thus $f(Q)$ is Sb^* -closed. f is Sb^* -closed map. (iii) implies (i) Let f is an Sb^* -closed map. Let $f^{-1}: (X, \tau) \rightarrow (Y, \sigma)$ is Sb^* -continuous. Let Q be a closed set in (X, τ) . Then $f(Q)$ is Sb^* -closed. (ie) $(f^{-1})(Q)$ is Sb^* -closed in (Y, σ) . Then by definition of Sb^* -continuous, (f^{-1}) is Sb^* -continuous.

Remark:3.16 Closed maps and Sb^* -closed maps are independent

Example:3.17 Let $X=Y=\{a,b\}$, $\tau = \{X, \phi, \{a\}\}$, $\sigma = \{Y, \phi, \{b\}\}$. Define a function $f: X \rightarrow Y$ as $f(a)=b, f(b)=a$. Let $A=\{a\}$ be a closed map. Since $f(a)=b, f(\{a\})=\{b\}$ which is not Sb^* -closed map

Example :3.18 Let $X=Y=\{a,b,c\}$, $\tau = \{X, \phi, \{a\}, \{a,c\}\}$, $\sigma = \{Y, \phi, \{b\}, \{b,c\}\}$. Define a function $f: X \rightarrow Y$ as $f(a)=b, f(b)=c, f(c)=b$. Let $A=\{a,c\}$ be a Sb^* -closed. $f(\{a,c\})=\{b\}$ which is not closed map

Remark:3.19 In general the composition of two Sb^* -closed maps need not be Sb^* -closed map as shown by the following example

Example:3.20 Let $X=Y=Z=\{a,b,c,d\}$ $\tau = \{X, \phi, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\}$, $\sigma = \{Y, \phi, \{a\}, \{c\}, \{d\}, \{a,c,d\}\}$, $\xi = \{Z, \phi, \{c\}, \{d\}, \{c,d\}, \{b,d\}, \{b,c,d\}\}$ Define a function $f: (X, \tau) \rightarrow (Y, \sigma)$ be an identity map and $g: (Y, \sigma) \rightarrow (Z, \xi)$ as $g(a)=g(b)=a, g(c)=g(d)=b$. Here f and g are Sb^* -closed maps. But $g \circ f$ is not an Sb^* -closed-map. Since $g \circ f(\{b,c,d\})=\{a,b\}$ which is not Sb^* -closed.

Theorem:3.21 Let (X, τ) , (Y, σ) and (Z, ξ) be a topological spaces where every Sb^* -closed set is closed. Then the composition $g \circ f: (X, \tau) \rightarrow (Z, \xi)$ of two Sb^* -closed maps and $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \xi)$ is Sb^* -closed.

Proof: Let Q be a closed set of (X, τ) . Since f is Sb^* -closed, $f(Q)$ is Sb^* -closed in (Y, σ) . By hypothesis $f(Q)$ is closed. Since $g \circ f$ is Sb^* -closed. $g(f(Q))$ is Sb^* -closed. $g(f(Q))=g \circ f(Q)$. $g \circ f$ is Sb^* -closed. Since Q is a closed set and $g \circ f(Q)$ is Sb^* -closed.

Theorem:3.22 If a map $f: (X, \tau) \rightarrow (Y, \sigma)$ is Sb^* -open then for each neighbourhood U of x in (X, τ) there exist Sb^* -neighbourhood W of $f(x)$ in (Y, σ) as $W \subset f(U)$

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an Sb^* -open map. Let $x \in X$ and U be an arbitrary neighbourhood of x in (X, τ) . Then there exist an open set G of (X, τ) such that $x \in G \subset U$. Now $f(x) \in f(G) \subset f(U)$. $f(G)$ is Sb^* -open in (Y, σ) as f is Sb^* -open map. $f(G)$ is Sb^* -neighbourhood of each of its point. Now taking $f(G)=W$. W is a Sb^* -neighbourhood of $f(x)$ in (Y, σ) . Since U be an arbitrary neighbourhood. Implies $W \subset f(U)$

Theorem:3.23 A map $f: (X, \tau) \rightarrow (Y, \sigma)$ is Sb^* -open iff for any subset P of (Y, σ) and any closed set F containing $f^{-1}(P)$ there exist an Sb^* -closed set K of (Y, σ) containing P such that $f^{-1}(K) \subset F$

Proof: Suppose that f is Sb^* -open. Let $P \subset Y$ and F be closed set of (X, τ) such that $f^{-1}(P) \subset F$. Now $X-F$ is open set in (X, τ) . Since f is Sb^* -open map. $f(X-F)$ is Sb^* -open set in (Y, σ) . Then $K=Y-f(X-F)$ is an Sb^* -closed set in (Y, σ) . Since $f^{-1}(P) \subset F, f^{-1}(K)=X-f^{-1}(f(X-F))=X-(X-F)=F, f^{-1}(K)=F, f^{-1}(K) \subset F$. Conversely, Let U be an open set (X, τ) . U^c is closed set in (X, τ) . Then $f^{-1}(f(U^c)) \subset U^c$. By hypothesis, there exist an Sb^* -closed set K of (Y, σ) such that

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$f(U^c) \subset K \Rightarrow f^{-1}(K) \subset U^c \Rightarrow U \subset (f^{-1}(K))^c \Rightarrow K^c \subset f(U) \subset f(f^{-1}(K))^c \subset K^c$. $f(U) \subset K^c$. Since K^c is Sb^* -open $f(U)$ is Sb^* -open in (Y, σ) . f is Sb^* -open map .

IV. Sb^* -HOMEOMORPHISM

Definition:4.1 A bijection $f:(X, \tau) \rightarrow (Y, \sigma)$ is called Strongly b star (briefly Sb^*) homeomorphism if f and f^{-1} are Strongly b star (briefly Sb^*) continuous .

Example:4.2 Consider $X=Y=\{a,b,c,d\}$, $\tau=\{X, \phi, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\}$, $\sigma=\{Y, \phi, \{a\}, \{d\}, \{a,c\}, \{a,c,d\}\}$. Define a function $f: X \rightarrow Y$ as $f(a)=a, f(b)=b, f(c)=c, f(d)=d$. Then f is bijective Sb^* -continuous and f^{-1} is Sb^* -continuous. Hence f is Sb^* -homeomorphism .

Theorem:4.3 Every homeomorphism is Sb^* -homeomorphism but not conversely

Proof: Let $f:(X, \tau) \rightarrow (Y, \sigma)$ be a homeomorphism . Since by the definition f and f^{-1} is Sb^* -continuous . Then f is bijection . We know that every closed set is Sb^* -closed . Then every continuous function is Sb^* -continuous. Then f and f^{-1} is Sb^* -continuous. Then f is Sb^* -homeomorphism.

The converse of the above theorem need not be true as seen from the following example

Example:4.4 Consider $X=Y=\{a,b,c,d\}$, $\tau=\{X, \phi, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\}$, $\sigma=\{Y, \phi, \{a\}, \{d\}, \{a,c\}, \{a,c,d\}\}$ Define a map $f:(X, \tau) \rightarrow (Y, \sigma)$ as $f(a)=f(b)=c$ and $f(c)=f(d)=a$. Then f is Sb^* -homeomorphism. But it is not a homeomorphism. Since the inverse image of the closed set $\{a,c\}$ in Y is not closed in X . f is not a homeomorphism.

Theorem:4.5 Every g^* -homeomorphism is Sb^* -homeomorphism but not conversely

Proof: Let $f:(X, \tau) \rightarrow (Y, \sigma)$ be a g^* -homeomorphism . Since by the definition f and f^{-1} is Sb^* -continuous . Then f is bijection . We know that every g^* -closed set is Sb^* -closed . Then every g^* -continuous function is Sb^* -continuous. Then f and f^{-1} is Sb^* -continuous. Then f is Sb^* -homeomorphism.

The converse of the above theorem need not be true as seen from the following example

Example:4.6 Consider $X=Y=\{a,b,c,d\}$, $\tau=\{X, \phi, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\}$, $\sigma=\{Y, \phi, \{a\}, \{d\}, \{a,c\}, \{a,c,d\}\}$. Let $f:X \rightarrow Y$ be an identity map .Then f is Sb^* -homeomorphism. But it is not a g^* -homeomorphism..

Theorem:4.7 Let $f:(X, \tau) \rightarrow (Y, \sigma)$ be a bijective Sb^* -continuous map, then the following statements are equivalent (i) f is an Sb^* -open map . (ii) f is an Sb^* -homeomorphism . (iii) f is an Sb^* -closed map.

Proof: (i) implies (ii) Let $f^{-1}:(X, \tau) \rightarrow (Y, \sigma)$ be a bijective Sb^* -continuous map .Let F be an closed map in (X, τ) . Then $X-F$ is open in (X, τ) . Since f is Sb^* -open . $f(X-F)$ is Sb^* -open in (Y, σ) . $f(F)$ is Sb^* -closed in (Y, σ) . f is Sb^* -continuous. Now $(f^{-1})^{-1}(F)$ is Sb^* -closed in (Y, σ) . f^{-1} is Sb^* -continuous . Then f and f^{-1} is Sb^* -continuous. f is an Sb^* -homeomorphism (ii) implies (iii) Suppose f is an Sb^* -homeomorphism . By the definition f is bijective , f and f^{-1} are Sb^* -continuous .Let F be an Sb^* - closed set in (X, τ) . Since f and f^{-1} are Sb^* -continuous. Then $(f^{-1})^{-1}(F)=f(F)$ is Sb^* -closed in (Y, σ) . Then f is Sb^* -closed map . (iii) implies (i) Let f is an Sb^* -closed map. Let U is an Sb^* -open in X . Then $X-U$ is Sb^* -closed in Y . Since f is Sb^* -closed. $f(X-U)$ is Sb^* -closed in Y . $Y-f(U)$ is Sb^* -closed in Y . $f(U)$ is Sb^* -open in Y . f is an Sb^* -open map .

Definition:4.8 A bijection $f:(X, \tau) \rightarrow (Y, \sigma)$ is called Sb^* -irresolute if $f^{-1}(V)$ is Sb^* -closed in (X, τ) for every Sb^* -closed V in (Y, σ)

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Example:4.9 Consider $X=Y=\{a,b,c,d\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\}$, $\sigma = \{Y, \phi, \{a\}, \{d\}, \{a,c\}, \{a,c,d\}\}$. Define a function $f: X \rightarrow Y$ as $f(a)=b, f(b)=a, f(c)=c, f(d)=d$. Let $A=\{a,b,c\}$ is Sb^* -closed in Y . Then $f^{-1}(\{a,b,c\})=\{a,b,c\}$ is also Sb^* -closed in X . f is Sb^* -Irresolute.

Theorem:4.10 The composition of two Sb^* -Homeomorphisms need not be an Sb^* -Homeomorphism

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \zeta)$ be an Sb^* -Homeomorphism. By Sb^* -Homeomorphism, f is bijective, both Sb^* -continuous and Sb^* -open. We know that, The composition of two continuous functions need not be a continuous function. Since the composition of two Sb^* -continuous functions need not be a Sb^* -continuous function. Therefore $g \circ f$ is need not be an Sb^* -homeomorphism.

Example:4.11 Let $X=Y=Z=\{a,b,c,d\}$, $\tau = \{X, \phi, \{a\}, \{b\}, \{a,b\}, \{a,b,c\}\}$, $\sigma = \{Y, \phi, \{a\}, \{c\}, \{d\}, \{a,c,d\}\}$, $\zeta = \{Z, \phi, \{c\}, \{d\}, \{c,d\}, \{b,d\}, \{b,c,d\}\}$ Define a function $f: (X, \tau) \rightarrow (Y, \sigma)$ be an identity map and $g: (Y, \sigma) \rightarrow (Z, \zeta)$ as $g(a)=a, g(b)=c, g(c)=b, g(d)=d$. Here f and g are Sb^* -ho. But $g \circ f$ is homeomorphism. But $g \circ f$ is not Sb^* -homeomorphism, Since the inverse image of Sb^* -closed set in $\{a,c,d\}$ in Z is not closed in X .

Theorem:4.12 Let $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \zeta)$ are Sb^* -Homeomorphisms then their composition $g \circ f: (X, \tau) \rightarrow (Z, \zeta)$ is also Sb^* -Homeomorphism

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ and $g: (Y, \sigma) \rightarrow (Z, \zeta)$ are Sb^* -Homeomorphisms. Let U be an Sb^* -open set in (Z, ζ) . $g^{-1}(U)$ is Sb^* -open in (Y, σ) . Now $(g \circ f)^{-1}(U) = g^{-1}(f^{-1}(U))$ is Sb^* -open in (X, τ) . Thus $g \circ f$ is Sb^* -irresolute. Also for an Sb^* -open set G in (X, τ) . $(g \circ f)(G) = g(f(G))$. Since $W=f(G)$. $(g \circ f)(G) = g(W)$. Since $g^{-1}(U)$ is Sb^* -open in (Y, σ) . $f(G)$ is Sb^* -open in (Y, σ) . Thus $g(f(G))$ is Sb^* -open in (Z, ζ) . Hence $(g \circ f)^{-1}$ is irresolute. $g \circ f$ is Sb^* -Homeomorphism

Definition:4.13 A Space X is said to be Sb^* -compact if every cover of X by Sb^* -open sets has a finite sub cover

Definition:4.14 Let x be a point of (X, τ) and V be a subset of X . Then V is called a Sb^* -neighborhood of x in (X, τ) if there exist a Sb^* -open set U of (X, τ) such that $x \in U \subset V$

Definition :4.15 A topological space (X, τ) is called Sb^* -Hausdorff if for each pair x,y of distinct points of X , there exists Sb^* -neighborhoods U_1 and U_2 of x and Y respectively, that are disjoint

Theorem:4.16 Let X be Sb^* -compact and set Y be a Hausdorff space. If $f: (X, \tau) \rightarrow (Y, \sigma)$ is Sb^* -continuous, Sb^* -irresolute and bijective then f is Sb^* -homeomorphism

Proof: Let A be a Sb^* -closed subset of the Sb^* -compact space X . Then A is Sb^* -compact. But f is Sb^* -irresolute. Hence $f(A)$ is Sb^* -compact. Take $g=f^{-1}$. Then $g^{-1}(A)$ is Sb^* -closed. We know that, consequently g is an Sb^* -irresolute map. then f^{-1} is Sb^* -irresolute. f is Sb^* -homeomorphism

Theorem:4.17 If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a Sb^* -Homeomorphism then $Sb^*-cl(f^{-1}(B)) = f^{-1}(Sb^*-cl(B))$ for all $B \subseteq Y$ is Sb^* -closed

Proof: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is a Sb^* -Homeomorphism. Since f is Sb^* -Homeomorphism. f and f^{-1} is both are Sb^* -irresolute. $Sb^*-cl(f(B))$ is closed in (Y, σ) . $f^{-1}(Sb^*-cl(f(B)))$ is Sb^* -closed in (X, τ) . Thus $Sb^*-cl(f^{-1}(B)) \subseteq f^{-1}(Sb^*-cl(B))$. Again f^{-1} is irresolute. $Sb^*-cl(f^{-1}(B))$ is Sb^* -closed in (X, τ) . $((f^{-1})^{-1})Sb^*-cl(f^{-1}(B)) = f(Sb^*-cl(f^{-1}(B)))$ is Sb^* -closed in (X, τ) . $B = f(Sb^*-cl(f^{-1}(B)))$. Hence $Sb^*-cl(f^{-1}(B)) = f^{-1}(Sb^*-cl(B))$.

Theorem:4.18 If $f: (X, \tau) \rightarrow (Y, \sigma)$ is an Sb^* -Homeomorphism then $Sb^*-cl(f(B)) = f(Sb^*-cl(B))$ for all $B \subseteq X$

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Proof: If $f: (X, \tau) \rightarrow (Y, \sigma)$ is an R^* -Homeomorphism. Since $f: (X, \tau) \rightarrow (Y, \sigma)$ is an Sb^* -Homeomorphism. Then $f^{-1}: (Y, \sigma) \rightarrow (X, \tau)$ is also an Sb^* -Homeomorphism. Since f is an Sb^* -Homeomorphism then f and f^{-1} is both are Sb^* -Irresolute. $(Sb^*\text{-cl}(f(B)))$ is Sb^* -closed in (Y, σ) .

$f^{-1} Sb^*\text{-cl}(f(B))$ is Sb^* -closed in (X, τ) . $(Sb^*\text{-int}(A))^c = Sb^*\text{-cl}(A^c)$. $(Sb^*\text{-int}(B))^c = (Sb^*\text{-cl}(B^c))^c$

Then $f(Sb^*\text{-int}(B)) = f((Sb^*\text{-cl}(B^c))^c) = ((Sb^*\text{-cl}(B^c))^c) = Sb^*\text{-cl}(f(B^c))^c = Sb^*\text{-int}(f(B))$. Therefore, $Sb^*\text{-cl}(f(B)) = f(Sb^*\text{-cl}(B))$.

Theorem:4.19 The set $Sb^*\text{-h}(X, \tau)$ is a group under the composition of maps

Proof: Define a binary operation $*$ as follows. $*$: $Sb^*\text{-h}(X, \tau) \times Sb^*\text{-h}(X, \tau) \rightarrow Sb^*\text{-h}(X, \tau)$

$f * g = g \circ f$ for all $f, g \in Sb^*\text{-h}(X, \tau)$. ' \circ ' is the usual operation of composition of maps

$g \circ f \in Sb^*\text{-h}(X, \tau)$. We Know That, the composition of maps is associative and the identity map

$I : (X, \tau) \times (X, \tau) \in Sb^*\text{-h}(X, \tau)$ serves as the identity element. If $f \in Sb^*\text{-h}(X, \tau)$ then $f^{-1} \in Sb^*\text{-h}(X, \tau)$ such that $f \circ f^{-1} = f^{-1} \circ f = I$, and so inverse exists for each element of $Sb^*\text{-h}(X, \tau)$ is a group under composition of maps. $Sb^*\text{-h}(X, \tau)$ is a group under the composition of maps

Theorem:4.20 Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an Sb^* -Homeomorphism. Then f induces an isomorphism from the group $Sb^*\text{-h}(X, \tau)$ onto the group $Sb^*\text{-h}(X, \tau)$

Proof: Let $f: (X, \tau) \rightarrow (Y, \sigma)$ be an Sb^* -Homeomorphism. We define $I_f: Sb^*\text{-h}(X, \tau) \rightarrow Sb^*\text{-h}(X, \tau)$. Now ' f ' induces an isomorphism from the group $I_f(h) = f \circ h \circ f^{-1}$ for every $h \in Sb^*\text{-h}(X, \tau)$. Since I_f is a bijection. Further for every $h_1, h_2 \in Sb^*\text{-h}(X, \tau)$. $I_f(h_1 \circ h_2) = f \circ (h_1 \circ h_2) \circ f^{-1} = (f \circ h_1 \circ f^{-1}) \circ (f \circ h_2 \circ f^{-1}) = I_f(h_1) * I_f(h_2)$. Thus I_f is a Homeomorphism and so it is an isomorphism induced by ' f '. f induces an isomorphism from the group $Sb^*\text{-h}(X, \tau)$ onto the group $Sb^*\text{-h}(X, \tau)$.

V. CONCLUSION

In this paper I have introduced Sb^* -closed maps, Sb^* -open maps, Sb^* -Homeomorphism in topological spaces and studied some properties and this can be extended to other topological spaces like fuzzy and Bi-topological spaces. And these notions can be applied for investigating many other properties.

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