

On $(1,2)^*$ - πgb^* - Closed Sets in Bitopological Spaces

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ABSTRACT: In this Paper we introduce a new class of sets called $(1,2)^*$ - πgb^* -closed set (briefly $(1,2)^*$ - πgb^* -closed) in bitopological spaces and a new class of generalized functions called $(1,2)^*$ - πgb^* -continuous maps and $(1,2)^*$ - πgb^* -irresolute maps in bitopological spaces. Also we obtain basic properties of $(1,2)^*$ - πgb^* -closed sets. Further we introduced a new space $(1,2)^*$ - πgb^* - $T_{1/2}$ space.

Key words: $(1,2)^*$ - πgb^* -closed sets, $(1,2)^*$ - πgb^* -open sets, $(1,2)^*$ - πgb^* -continuous maps, $(1,2)^*$ - πgb^* -irresolute maps, $(1,2)^*$ - πgb^* - $T_{1/2}$ space.

I. INTRODUCTION

The study of bitopological spaces (X, τ_1, τ_2) was first introduced by J.C.Kelly[11] in the year 1963. The bitopological spaces are equipped with two arbitrary topological τ_1 & τ_2 . Levine [13] and Andrijevic[4,5] introduced the concept of generalized open sets and b-open sets respectively in topological spaces. The class of b-open sets is contained in the class of semi pre open sets and contains the class of semi open and the class of preopen sets. Since several researches were done and the notion of generalized semi closed, generalized preclosed and generalized semi-pre open sets were investigated. In 1968 Zaitsev[19] defined π -closed sets. Later Dontchev and Noiri[9] introduced the notion of πgb -closed sets. The idea of πgb -closed sets was introduced by D. Sreeja and C. Janaki [17]. Dhanya. R, A. Parvathi, Introduced " On πgb^* -closed sets in Topological spaces", [8]. This paper is an attempt to highlight a new type of generalized closed sets called $(1,2)^*$ - π generalized b^* -closed, called $(1,2)^*$ - πgb^* -continuous maps and $(1,2)^*$ - πgb^* -irresolute maps in bitopological spaces. These findings results in procuring several characterizations of this class. It analyses its bitopological properties. The application of this set leads to the introduction of a new space called $(1,2)^*$ - πgb^* -space.

II. PRELIMINARIES

Throughout this paper (X, τ_1, τ_2) and (Y, σ_1, σ_2) represents bitopological spaces on which no separation axioms are assumed unless otherwise mentioned.

Definition 2.1 [8]. A subset S of a bitopological space (X, τ_1, τ_2) is said to be $\tau_{1,2}$ open if $S = A \cup B$ where $A \in \tau_1$ and $B \in \tau_2$. A subset S of X is $\tau_{1,2}$ -closed if the complement of S is $\tau_{1,2}$ -open.

Definiton 2.2 [18]. Let S be a subset of X. Then

- (i) The $\tau_1 \tau_2$ -interior of S, denoted by $\tau_1 \tau_2$ -int(S)
- (ii) The $\tau_1 \tau_2$ -closure of S, denoted by $\tau_1 \tau_2$ -cl(S)

Defintion 2.3 A subset A of a bitopological space (X, τ_1, τ_2) is called

- (i) $(1,2)^*$ - **semi-open**[4] if $A \subset \tau_1 \tau_2$ -cl($\tau_1 \tau_2$ -int(A)).
- (ii) $(1,2)^*$ - **pre open**[4] if $A \subset \tau_1 \tau_2$ -int($\tau_1 \tau_2$ -cl(A)).

- (iii) $(1,2)^*$ - α open[4] if $A \subset \tau_1 \tau_2 - \text{int}(\tau_1 \tau_2 - \text{cl}(\tau_1 \tau_2 - \text{int}(A)))$.
- (iv) $(1,2)^*$ -**generalized closed** (briefly $(1,2)^*$ -g-closed)[13] if $\tau_1 \tau_2 - \text{cl}(A) \subset U$ whenever $A \subset U$ and U is $\tau_1 \tau_2$ - open in X .
- (v) $(1,2)^*$ - **regular open**[4] if $A = \tau_1 \tau_2 - \text{int}(\tau_1 \tau_2 - \text{cl}(A))$.
- (vi) $(1,2)^*$ - **semi-generalized-closed** (briefly $(1,2)^*$ -sg-closed) [6] if $(1,2)^*$ -scl(A) $\subset U$ and U is $(1,2)^*$ - semi-open in X .
- (vii) $(1,2)^*$ - **generalized semi-closed** (briefly $(1,2)^*$ -gs-closed)[9] if $(1,2)^*$ -scl(A) $\subset U$, whenever $A \subset U$ and U is $\tau_{1,2}$ - open in X .
- (viii) $(1,2)^*$ - **α -generalized-closed** (briefly $(1,2)^*$ - α g-closed)[10] if $(1,2)^*$ - α cl(A) $\subset U$, whenever $A \subset U$ and U is $\tau_{1,2}$ - open in X .
- (ix) $(1,2)^*$ - **generalized- α -closed** (briefly $(1,2)^*$ -g α -closed)[10] if $(1,2)^*$ - α cl(A) $\subset U$, whenever $A \subset U$ and U is $(1,2)^*$ - α - open in X .
- (x) $(1,2)^*$ - **π generalized-closed** (briefly $(1,2)^*$ - π g-closed)[13] if $(1,2)^*$ - cl(A) $\subset U$, whenever $A \subset U$ and U is $\tau_{1,2}$ - π -open.
- (xi) $(1,2)^*$ - **π generalized α -closed** (briefly $(1,2)^*$ - π g α -closed)[5] if $(1,2)^*$ - α cl(A) $\subset U$, whenever $A \subset U$ and U is $\tau_{1,2}$ - π -open.
- (xii) $(1,2)^*$ - **π generalized semi-closed** (briefly $(1,2)^*$ - π gs-closed)[7] if $(1,2)^*$ - scl(A) $\subset U$, whenever $A \subset U$ and U is $\tau_{1,2}$ - π -open.
- (xiii) $(1,2)^*$ - **π generalized b-closed** (briefly $(1,2)^*$ - π gb-closed)[1] if $(1,2)^*$ - bcl(A) $\subset U$, whenever $A \subset U$ and U is $\tau_{1,2}$ - π -open.
- (xiv) $(1,2)^*$ - **π generalized pre-closed** (briefly $(1,2)^*$ - π gp-closed)[16] if $(1,2)^*$ - pcl(A) $\subset U$, whenever $A \subset U$ and U is $\tau_{1,2}$ - π -open.

The complement of a $(1,2)^*$ -semi-closed (resp. $(1,2)^*$ - α -closed, $(1,2)^*$ -g-closed, $(1,2)^*$ -sg-closed, $(1,2)^*$ -gs-closed, $(1,2)^*$ - α g-closed, $(1,2)^*$ -g α -closed, $(1,2)^*$ - π g-closed, $(1,2)^*$ - π g α -closed, $(1,2)^*$ - π gs-closed, $(1,2)^*$ - π gb-closed, $(1,2)^*$ - π gp-closed) set is called $(1,2)^*$ -semi open (resp. $(1,2)^*$ - α -open, $(1,2)^*$ -g-open, $(1,2)^*$ -sg-open, $(1,2)^*$ -gs-open, $(1,2)^*$ - α g-open, $(1,2)^*$ -g α -open, $(1,2)^*$ - π g-open, $(1,2)^*$ - π gs-open, $(1,2)^*$ - π gb-open, $(1,2)^*$ - π gp- open).

Definition 2.4 The finite union of $(1,2)^*$ -regular open sets is said to be $\tau_{1,2}$ - π -open. The complement of $\tau_{1,2}$ - π -open is said to be $\tau_{1,2}$ - π -closed.

Definition 2.5 A function $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called continuous (resp. $(1,2)^*$ - α -continuous, $(1,2)^*$ -pre-continuous, $(1,2)^*$ -semi-continuous, $(1,2)^*$ -b*-continuous, $(1,2)^*$ -g-continuous, $(1,2)^*$ -g α -continuous, $(1,2)^*$ -gp-continuous, $(1,2)^*$ -gs-continuous, $(1,2)^*$ -gb-continuous) If $f^{-1}(V)$ is closed (resp. $(1,2)^*$ - α -closed, $(1,2)^*$ -pre-closed, $(1,2)^*$ -semi-closed, $(1,2)^*$ -b-closed, $(1,2)^*$ -g-closed, $(1,2)^*$ -gp-closed, $(1,2)^*$ -gs-closed, $(1,2)^*$ -gb-closed) in (X, τ_1, τ_2) for every closed set V in (Y, σ_1, σ_2)

Definition 2.6 A function $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called $(1,2)^*$ - π -continuous (resp. $(1,2)^*$ - $\pi\alpha$ -continuous, $(1,2)^*$ - π g-continuous, $(1,2)^*$ - π gp-continuous, $(1,2)^*$ - π gs-continuous, $(1,2)^*$ - π gb-continuous) if $f^{-1}(V)$ is $(1,2)^*$ - π -closed (resp. $(1,2)^*$ - $\pi\alpha$ -closed, $(1,2)^*$ - π g-closed, $(1,2)^*$ - π gp-closed, $(1,2)^*$ - π gs-closed, $(1,2)^*$ - π gb-closed) in (X, τ_1, τ_2) for every closed set V of (Y, σ_1, σ_2) .

III. $(1,2)^*$ - π gb*- CLOSED SET

We introduce the following definition.

Definition 3.1. A subset A of (X, τ_1, τ_2) is called $(1,2)^*$ - π generalized b*-closed set (briefly $(1,2)^*$ - π gb*-closed.) if $\tau_1 \tau_2 - \text{int}(bcl(A)) \subset U$ whenever $A \subset U$ and U is $\tau_1 \tau_2$ - π -open in X .

Theorem 3.2

- (i) Every $(1,2)^*$ -closed set is $(1,2)^*$ - π gb*-closed.
- (ii) Every $(1,2)^*$ - semi closed set is $(1,2)^*$ - π gb*-closed.
- (iii) Every $(1,2)^*$ - pre closed set is $(1,2)^*$ - π gb*-closed
- (iv) Every $(1,2)^*$ - α closed set is $(1,2)^*$ - π gb*- closed.
- (v) Every $(1,2)^*$ -b closed set is $(1,2)^*$ - π gb*-closed.
- (vi) Every $(1,2)^*$ - g-closed set is $(1,2)^*$ - π gb*-closed.

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- (vii) Every $(1,2)^*$ -gp-closed set is $(1,2)^*$ - πgb^* -closed.
- (viii) Every $(1,2)^*$ -gs-closed set is $(1,2)^*$ - πgb^* -closed.
- (ix) Every $(1,2)^*$ -g α -closed set is $(1,2)^*$ - πgb^* -closed.
- (x) Every $(1,2)^*$ -gb-closed set is $(1,2)^*$ - πgb^* -closed.
- (xi) Every $(1,2)^*$ -b^{*}-closed set is $(1,2)^*$ - πgb^* -closed.
- (xii) Every $(1,2)^*$ - πgp -closed set is $(1,2)^*$ - πgb^* -closed.
- (xiii) Every $(1,2)^*$ - πgs -closed set is $(1,2)^*$ - πgb^* -closed.

Proof: Straight forward.

Converse of the above need not be true as seen in the following examples.

Example 3.3 Let $X = \{a, b, c, d\}$ $\tau_1 = \{X, \Phi, \{b\}\}$ $\tau_2 = \{X, \Phi, \{a\}\}$. Here $A = \{a, \{b\}, \{c\}, \{d\}, \{b, c\}, \{a, d\}, \{b, d\}, \{a, c\}, \{a, b, c\}, \{a, b, d\}\}$ are in $(1,2)^*$ - πgb^* -closed but it is not $(1,2)^*$ -closed.

Example 3.4 Let $X = \{a, b, c, d\}$ $\tau_1 = \{X, \Phi, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$ $\tau_2 = \{X, \Phi, \{a\}\}$. Here $A = \{a, \{b\}, \{b, c\}\}$ are in $(1,2)^*$ - πgb^* -closed, but it is not $(1,2)^*$ -semi-closed.

Example 3.5 Let $X = \{a, b, c, d\}$ $\tau_1 = \{X, \Phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ $\tau_2 = \{X, \Phi, \{c\}\}$. Here $A = \{a, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$ are in $(1,2)^*$ - πgb^* -closed, but it is not in $(1,2)^*$ -pre-closed.

Example 3.6 Let $X = \{a, b, c, d\}$ $\tau_1 = \{X, \Phi, \{a\}, \{a, c\}\}$ $\tau_2 = \{X, \Phi, \{b\}\}$. Here $A = \{a, \{b\}, \{a, b\}, \{b, c\}, \{d, a\}, \{a, b, c\}, \{a, b, d\}\}$ are in $(1,2)^*$ - πgb^* -closed, but it is not $(1,2)^*$ - α -closed.

Example 3.7 Let $X = \{a, b, c, d\}$ $\tau_1 = \{X, \Phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$ $\tau_2 = \{X, \Phi, \{c\}\}$. Here $A = \{a, \{b\}, \{a, b\}, \{a, b, c\}\}$ are in $(1,2)^*$ - πgb^* -closed, but it is not $(1,2)^*$ -b-closed.

Example 3.8 Let $X = \{a, b, c, d\}$ $\tau_1 = \{X, \Phi, \{c\}, \{a, c\}, \{a, b, c\}\}$ $\tau_2 = \{X, \Phi, \{a\}\}$. Here $A = \{c, \{a, b\}, \{b, c\}, \{a, c\}, \{a, b, c\}\}$ are in $(1,2)^*$ - πgb^* -closed, but it is not $(1,2)^*$ -g-closed.

Example 3.9 Let $X = \{a, b, c, d\}$ $\tau_1 = \{X, \Phi, \{a, b\}, \{b\}\}$ $\tau_2 = \{X, \Phi, \{a, c\}\}$. Here $A = \{a, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{d, a\}, \{b, c, d\}, \{a, c\}, \{a, b, d\}\}$ are in $(1,2)^*$ - πgb^* -closed, but it is not $(1,2)^*$ -gp-closed.

Example 3.10 Let $X = \{a, b, c, d\}$ $\tau_1 = \{X, \Phi, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$ $\tau_2 = \{X, \Phi, \{a\}\}$. Here $A = \{a\}$ is $(1,2)^*$ - πgb^* -closed, but not in $(1,2)^*$ -gs-closed.

Example 3.11 Let $X = \{a, b, c, d\}$ $\tau_1 = \{X, \Phi, \{a\}, \{a, c\}\}$ $\tau_2 = \{X, \Phi, \{b\}\}$. Here $A = \{a, c\}$ is $(1,2)^*$ - πgb^* -closed, but it is not in $(1,2)^*$ -g α -closed.

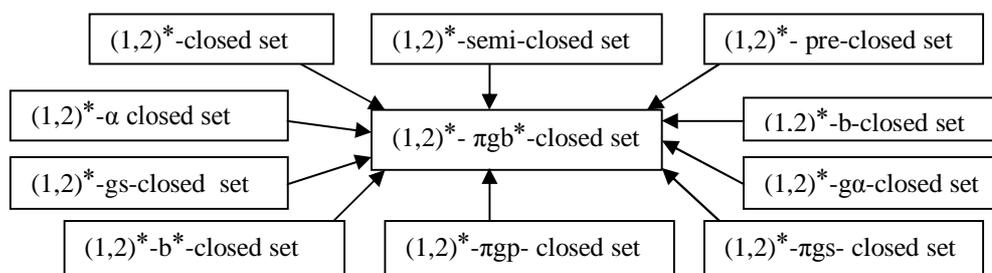
Example 3.12 Let $X = \{a, b, c, d\}$ $\tau_1 = \{X, \Phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}, \{a, b, d\}\}$ $\tau_2 = \{X, \Phi, \{c\}\}$. Here $A = \{a, b\}$ is $(1,2)^*$ - πgb^* -closed, but it is not in $(1,2)^*$ -gb-closed.

Example 3.13 Let $X = \{a, b, c, d\}$ $\tau_1 = \{X, \Phi, \{a\}, \{a, b\}, \{a, c\}\}$ $\tau_2 = \{X, \Phi, \{c\}\}$. Here $A = \{b, d\}$ is $(1,2)^*$ - πgb^* -closed, but not in $(1,2)^*$ -b^{*}-closed.

Example 3.14 Let $X = \{a, b, c, d\}$ $\tau_1 = \{X, \Phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}\}$ $\tau_2 = \{X, \Phi, \{c\}\}$. Here $A = \{a, b\}$ is $(1,2)^*$ - πgb^* -closed, but it is not in $(1,2)^*$ - πgp -closed.

Example 3.15 Let $X = \{a, b, c, d\}$ $\tau_1 = \{X, \Phi, \{b\}, \{c\}, \{a, b\}, \{b, c\}\}$ $\tau_2 = \{X, \Phi, \{c\}\}$. Here $A = \{a, b\}$ is $(1,2)^*$ - πgb^* -closed, but it is not in $(1,2)^*$ - πgs -closed.

From the above results, we have the following implications diagram.



IV. CHARACTERISTICS OF $(1,2)^*$ - πgb^* -CLOSED SETS

Remark 4.1 Finite union of $(1,2)^*$ - πgb^* -closed sets need not be $(1,2)^*$ - πgb^* -closed which can be seen from the following example.

Example 4.1 Let $X = \{a,b,c,d\}$ $\tau_1 = \{X, \Phi, \{a\}, \{b\}, \{a,b\}\}$, $\tau_2 = \{X, \Phi, \{a,b,d\}\}$. Let $A = \{a\}$ and $B = \{b\}$ then both A and B are $(1,2)^*$ - πgb^* -closed. But, $A \cup B = \{a,b\}$ is not $(1,2)^*$ - πgb^* -closed.

Remark 4.2 Finite intersection of $(1,2)^*$ - πgb^* -closed sets need not be $(1,2)^*$ - πgb^* -closed which can be seen from the following example.

Example 4.2 Let $X = \{a,b,c,d\}$ $\tau_1 = \{X, \Phi, \{a\}, \{d\}, \{a,d\}\}$, $\tau_2 = \{X, \Phi, \{c,d\}, \{a,c,d\}\}$. Let $A = \{a,b,c\}$ and $B = \{a,c,d\}$. Then both A and B are $(1,2)^*$ - πgb^* -closed. But $A \cap B = \{a,c\}$ is not $(1,2)^*$ - πgb^* -closed.

Theorem 4.1 Let (X, τ_1, τ_2) be a bitopological space if $A \subset X$ is $(1,2)^*$ - πgb^* -closed set then $\tau_{1,2} - \text{int}(\text{bcl}(A)) - A$ does not contain any non empty $(1,2)^*$ - π -closed set.

Proof Let A be a $(1,2)^*$ - πgb^* -closed set in (X, τ_1, τ_2) and $F \subset \tau_{1,2} - \text{int}(\text{bcl}(A)) \subset A$ such that F is π -closed in X . Then $(X - F)$ is π -open in X and $A \subseteq (X - F)$. Since A is $(1,2)^*$ - πgb^* -closed, $\tau_{1,2} - \text{int}(\text{bcl}(A)) \subset (X - F) \Rightarrow F \subset (X - \tau_{1,2} - \text{int}(\text{bcl}(A)))$. Therefore $F \subset \tau_{1,2} - (\text{int}(\text{bcl}(A)) - A) \cap (X - \tau_{1,2} - (\text{int}(\text{bcl}(A)) - A)) \Rightarrow F = \Phi$. Therefore $\tau_{1,2} - (\text{int}(\text{bcl}(A)) - A)$ does not contain any non empty π -closed set.

Theorem 4.2 If A is a $(1,2)^*$ - πgb^* -closed and B is any set such that $A \subseteq B \subseteq \tau_{1,2} - \text{int}(\text{bcl}(A))$, then B is a $(1,2)^*$ - πgb^* -closed.

Proof Let $B \subseteq U$ and U be π -open. Since $A \subseteq B \subseteq U$ and A is $(1,2)^*$ - πgb^* -closed, $\tau_{1,2} - \text{int}(\text{bcl}(A)) \subseteq U$. Now $\tau_{1,2} - \text{int}(\text{bcl}(B)) \subseteq \tau_{1,2} - \text{int}(\text{bcl}(A)) \subseteq U$. Therefore B is a $(1,2)^*$ - πgb^* -closed.

Definition 4.1 A set $A \subseteq X$ is called $(1,2)^*$ - πgb^* -open if its complement is $(1,2)^*$ - πgb^* -closed in X .

Theorem 4.1(a) If $\tau_{1,2} - \text{cl}(\text{bint}(A)) \subseteq B \subseteq A$ and A is $(1,2)^*$ - πgb^* -open, then B is $(1,2)^*$ - πgb^* -open.

Proof Let F be a $(1,2)^*$ - π -closed set such that $F \subseteq B$. Since $B \subseteq A$ we get $F \subseteq A$. Given A is $(1,2)^*$ - πgb^* -open thus $F \subseteq \tau_{1,2} - \text{cl}(\text{bint}(A)) \subseteq \tau_{1,2} - \text{cl}(\text{bint}(B))$. Therefore B is $(1,2)^*$ - πgb^* -open.

Definition 4.2 A space (X, τ_1, τ_2) is called a $(1,2)^*$ - πgb^* - $T_{1/2}$ space if every $(1,2)^*$ - πgb^* -closed set is b^* -closed.

Theorem 4.2(a) For a bitopological space (X, τ_1, τ_2) the following are equivalent

- 1) X is $(1,2)^*$ - πgb^* - $T_{1/2}$ space
- 2) \forall subset $A \subseteq X$, A is $(1,2)^*$ - πgb^* -open if A is b^* -open.

Proof (1) \Rightarrow (2) Let $A \subseteq X$ be $(1,2)^*$ - πgb^* -open. Then $(X-A)$ is $(1,2)^*$ - πgb^* -closed and by (1) $(X-A)$ is $(1,2)^*$ - b^* -closed $\Rightarrow A$ is $(1,2)^*$ - b^* -open. Conversely assume A is b^* -open. Then $(X-A)$ is $(1,2)^*$ - b^* -closed. As every $(1,2)^*$ - b^* -closed set is $(1,2)^*$ - πgb^* -closed. $(X-A)$ is $(1,2)^*$ - πgb^* -closed $\Rightarrow (1,2)^*$ - πgb^* -open.

(2) \Rightarrow (1) Let A be a $(1,2)^*$ - πgb^* -closed set in X . Then $(X-A)$ is $(1,2)^*$ - πgb^* -open. Hence by (2) $(X-A)$ is $(1,2)^*$ - b^* -open $\Rightarrow A$ is $(1,2)^*$ - b^* -closed. Hence X is $(1,2)^*$ - πgb^* - $T_{1/2}$.

V. $(1,2)^*$ - πgb^* -CONTINUITY

Definition 5.1 A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called $(1,2)^*$ - πgb^* -continuous if $f^{-1}(V)$ is $(1,2)^*$ - πgb^* -closed in (X, τ_1, τ_2) for every closed set V of (Y, σ_1, σ_2) .

Definition 5.2 A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is called $(1,2)^*$ - πgb^* -irresolute if $f^{-1}(V)$ is $(1,2)^*$ - πgb^* -closed in (X, τ_1, τ_2) for every $(1,2)^*$ - πgb^* -closed set V in (Y, σ_1, σ_2)

Theorem 5.3 Every continuous function is $(1,2)^*$ - πgb^* -continuous.

Proof Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a continuous function. Let V be a closed set in Y . Since f is continuous $f^{-1}(V)$ is closed in X . As every closed set is $(1,2)^*$ - πgb^* -closed, $f^{-1}(V)$ is $(1,2)^*$ - πgb^* -closed. Hence f is $(1,2)^*$ - πgb^* -continuous.

Remark 5.4 The converse of the above theorem need not be true as seen from the following example.

Example 5.5 Consider $X = \{a, b, c\}$, $\tau_1 = \{X, \{a, \{c\}, \{a, c\}, \emptyset\}$, $\tau_2 = \{X, \{c\}, \emptyset\}$ and $Y = \{a, b, c\}$ with the topology $\sigma_1 = \{\emptyset, Y, \{b, a\}, \{a\}, \{b\}\}$, $\sigma_2 = \{\emptyset, Y, \{b\}\}$. Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be defined by $f(a) = b$, $f(b) = c$, $f(c) = a$. Then f is $(1,2)^*$ - πgb^* -continuous but it is not continuous.

Theorem 5.6 Every $(1,2)^*$ -pre-continuous function is $(1,2)^*$ - πgb^* -continuous.

Proof Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a pre-continuous function. Let V be a closed subset of Y . Since f is $(1,2)^*$ -pre-continuous $f^{-1}(V)$ is $(1,2)^*$ -pre-closed in X . As every $(1,2)^*$ -pre-closed set is $(1,2)^*$ - πgb^* -closed, $f^{-1}(V)$ is $(1,2)^*$ - πgb^* -closed. Hence f is $(1,2)^*$ - πgb^* -continuous.

Remark 5.7 The converse of above theorem need not be true which can be shown by the following example.

Example 5.8 Let $X = \{a, b, c\}$ with topology $\tau_1 = \{\emptyset, \{a\}, \{c\}, \{a, c\}, X\}$, $\tau_2 = \{\emptyset, \{a\}, X\}$ and $Y = \{a, b, c\}$ with the topology $\sigma_1 = \{\emptyset, Y, \{a\}, \{b\}, \{a, b\}\}$, $\sigma_2 = \{\emptyset, Y, \{a\}\}$. $(1,2)^*$ -pre-continuous = $\{X, \emptyset, \{a, c\}\}$. Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ as $f(a) = a$, $f(b) = c$, $f(c) = b$. Then f is $(1,2)^*$ - πgb^* -continuous but it is not $(1,2)^*$ -pre-continuous.

Theorem 5.9 Every $(1,2)^*$ -semi-continuous function is $(1,2)^*$ - πgb^* -continuous.

Proof Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a semi-continuous function. Let V be a closed subset of Y , since f is semi-continuous $f^{-1}(V)$ is $(1,2)^*$ -semi-closed in X . As every $(1,2)^*$ -semi-closed set is $(1,2)^*$ - πgb^* -closed, $f^{-1}(V)$ is $(1,2)^*$ - πgb^* -closed. Hence f is $(1,2)^*$ - πgb^* -continuous.

Remark 5.10 The converse of the above theorem need not be true as seen from the following example.

Example 5.11 Let $X = \{a, b, c, d\}$ with topology $\tau_1 = \{\emptyset, \{a\}, \{b\}, \{a, b\}, \{b, c\}, \{a, b, c\}, X\}$, $\tau_2 = \{\emptyset, \{a\}, \{b\}, X\}$ and $Y = \{a, b, c, d\}$ with topology $\sigma_1 = \{\emptyset, \{b\}, \{d\}, \{b, d\}, \{b, c\}, \{a, b, c\}\}$, $\sigma_2 = \{\emptyset, Y, \{b\}, \{d\}\}$. $(1,2)^*$ -semi-continuous = $\{X, \emptyset, \{a, b, c\}\}$. Define $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be the identity function. Then f is $(1,2)^*$ - πgb^* -continuous but not it is $(1,2)^*$ -semi-continuous.

Theorem 5.12 Every $(1,2)^*$ -b-continuous function is $(1,2)^*$ - πgb^* -continuous.

Proof Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a $(1,2)^*$ -b-continuous function. Let V be a closed subset of Y . Since f is $(1,2)^*$ -b-continuous $f^{-1}(V)$ is $(1,2)^*$ -b-closed in X . As every $(1,2)^*$ -b-closed set is $(1,2)^*$ - πgb^* -closed, $f^{-1}(V)$ is $(1,2)^*$ - πgb^* -closed. Hence f is $(1,2)^*$ - πgb^* -continuous.

Remark 5.13 The converse of the above theorem need not be true which can be seen from the following example.

Example 5.14 Let $X = \{a, b, c, d\}$ with topology $\tau_1 = \{\emptyset, \{c\}, \{d\}, \{c, d\}, X\}$ and $\tau_2 = \{\emptyset, \{d\}, X\}$ and $Y = \{a, b, c, d\}$ with topology $\sigma_1 = \{\emptyset, Y, \{a\}, \{b\}, \{a, b\}\}$, $\sigma_2 = \{\emptyset, Y, \{b\}\}$. $(1,2)^*$ - πgb^* -continuous = $\{X, \emptyset, \{b, c, d\}\}$. Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be defined as a identity function. $(1,2)^*$ -b-continuous = $\{X, \emptyset, \{b, c, d\}, \{c, d\}\}$. Then f is $(1,2)^*$ - πgb^* -continuous but it is not $(1,2)^*$ -b-continuous.

Theorem 5.15 Every $(1,2)^*$ -g-continuous function is $(1,2)^*$ - πgb^* -continuous.

Proof Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a $(1,2)^*$ -g-continuous function. Let V be a closed subset of Y . Since f is $(1,2)^*$ -g-continuous $f^{-1}(V)$ is $(1,2)^*$ -g-closed in X . As every $(1,2)^*$ -g-closed set is $(1,2)^*$ - πgb^* -closed, $f^{-1}(V)$ is $(1,2)^*$ - πgb^* -closed. Hence f is $(1,2)^*$ - πgb^* -continuous.

Remark 5.16 The converse of the above theorem need not be true which can be seen from the following example.

Example 5.17 Let $X = \{a, b, c, d\}$ with topology $\tau_1 = \{\emptyset, \{c\}, \{d\}, \{c, d\}, X\}$, $\tau_2 = \{\emptyset, \{d\}, X\}$ and $Y = \{a, b, c, d\}$ with topology $\sigma_1 = \{\emptyset, Y, \{a\}, \{b\}, \{a, b\}\}$, $\sigma_2 = \{\emptyset, Y, \{b\}\}$. $(1,2)^*$ -g-continuous = $\{X, \emptyset, \{c, d\}\}$. Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be defined as the identity function. $(1,2)^*$ -g-continuous = $\{\emptyset, \{c, d\}, X\}$. Then f is $(1,2)^*$ - πgb^* -continuous but not $(1,2)^*$ -g-continuous.

Theorem 5.18 Every $(1,2)^*$ -gp-continuous function is $(1,2)^*$ - πgb^* -continuous.

Proof Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a $(1,2)^*$ -gp-continuous function. Let V be a closed subset of Y . Since f is $(1,2)^*$ -gp-continuous $f^{-1}(V)$ is $(1,2)^*$ -gp-closed in X . As every $(1,2)^*$ -gp-closed set is $(1,2)^*$ - πgb^* -closed, $f^{-1}(V)$ is $(1,2)^*$ - πgb^* -closed. Hence f is $(1,2)^*$ - πgb^* -continuous.

Remark 5.19 The converse of the above theorem need not be true as seen from the following example.

Example 5.20 Let $X = \{a, b, c\}$ with topology $\tau_1 = \{\emptyset, \{a\}, \{c\}, \{a, c\}, X\}$, $\tau_2 = \{\emptyset, X, \{c\}\}$ and let $Y = \{a, b, c\}$ with topology $\sigma_1 = \{\emptyset, Y, \{a\}, \{b\}, \{b, a\}\}$, $\sigma_2 = \{\emptyset, Y, \{b\}\}$. Let $f: (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be defined as $f(a) = b$, $f(b) = c$, $f(c) = a$. $(1,2)^*$ -gp-continuous = $\{\emptyset, \{c\}, \{a, c\}, X\}$. Then f is $(1,2)^*$ - πgb^* -continuous but it is not $(1,2)^*$ -gp-continuous.

Theorem 5.21 Every $(1,2)^*$ -gs-continuous function is $(1,2)^*$ - πgb^* -continuous.

Proof Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a $(1,2)^*$ -gs-continuous function. Let V be a closed subset of Y . Since f is gs-continuous $f^{-1}(V)$ is $(1,2)^*$ -gs-closed in X . As every $(1,2)^*$ -gs-closed set is $(1,2)^*$ - π gb*-closed, $f^{-1}(V)$ is $(1,2)^*$ - π gb*-closed. Hence f is $(1,2)^*$ - π gb*-continuous.

Remark 5.22 The converse of the above theorem need not be true it can be seen from the following example.

Example 5.23 Let $X = \{a,b,c,d\}$ with topology $\tau_1 = \{\emptyset, \{a\}, \{c\}, \{a,c\}, X\}$, $\tau_2 = \{\emptyset, \{c\}, X\}$ and $Y = \{a,b,c,d\}$ with topology $\sigma_1 = \{\emptyset, Y, \{b\}, \{a\}, \{a,b\}\}$, $\sigma_2 = \{\emptyset, Y, \{a,b\}\}$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be defined as identity function. $(1,2)^*$ -gs-continuous = $\{\emptyset, \{a,c,d\}, X\}$. Then f is $(1,2)^*$ - π gb*-continuous. But is not $(1,2)^*$ -gs-continuous.

Theorem 5.24 Every $(1,2)^*$ -gb-continuous function is $(1,2)^*$ - π gb*-continuous.

Proof Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a $(1,2)^*$ -gb-continuous function. Let V be a closed subset of Y . Since f is $(1,2)^*$ -gb-continuous $f^{-1}(V)$ is $(1,2)^*$ -gb-closed in X . As every $(1,2)^*$ -gb-closed set is $(1,2)^*$ - π gb*-closed, $f^{-1}(V)$ is $(1,2)^*$ - π gb*-closed. Hence f is $(1,2)^*$ - π gb*-continuous.

Remark 5.25 The converse of above theorem need not be true as seen from the following example.

Example 5.26 Let $X = \{a,b,c,d\}$ with topology $\tau_1 = \{\emptyset, \{a\}, \{c\}, \{a,c\}, X\}$, $\tau_2 = \{\emptyset, \{a\}, X\}$ and $Y = \{a,b,c,d\}$ with topology $\sigma_1 = \{\emptyset, Y, \{b\}, \{a\}, \{a,b\}\}$, $\sigma_2 = \{\emptyset, Y, \{a,b\}\}$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be defined as $f(a) = d, f(b) = c, f(c) = a, f(d) = b$. $(1,2)^*$ -gb-continuous = $\{\emptyset, \{c,d\}, \{a,c,d\}, X\}$. Then f is $(1,2)^*$ - π gb*-continuous but is not $(1,2)^*$ -gb-continuous.

Theorem 5.27 Every $(1,2)^*$ - π g-continuous function is $(1,2)^*$ - π gb*-continuous.

Proof Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a $(1,2)^*$ - π g-continuous function. Let V be a closed subset of Y . Since f is $(1,2)^*$ - π g-continuous $f^{-1}(V)$ is $(1,2)^*$ - π g-closed in X . As every $(1,2)^*$ - π g-closed set is $(1,2)^*$ - π gb*-closed, $f^{-1}(V)$ is $(1,2)^*$ - π gb*-closed. Hence f is $(1,2)^*$ - π gb*-continuous.

Remark 5.28 The converse of the above theorem need not be true it can be seen from the following example.

Example 5.29 Let $X = \{a,b,c,d\}$ with topology $\tau_1 = \{\emptyset, \{a\}, \{b\}, \{a,b\}, X\}$, $\tau_2 = \{\emptyset, \{a\}, X\}$ and $Y = \{a,b,c,d\}$ with topology $\sigma_1 = \{\emptyset, Y, \{b\}, \{d\}, \{c\}, \{b,d\}, \{b,c\}, \{c,d\}, \{b,c,d\}\}$, $\sigma_2 = \{\emptyset, Y, \{c,b\}\}$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ as follows $f(a) = d, f(b) = b, f(c) = c, f(d) = a$. $(1,2)^*$ - π g-continuous = $\{\emptyset, \{a,d\}, X\}$. Then f is $(1,2)^*$ - π gb*-continuous but it is not $(1,2)^*$ - π g-continuous.

Theorem 5.3 Every $(1,2)^*$ - π gp-continuous function is $(1,2)^*$ - π gb*-continuous.

Proof Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a $(1,2)^*$ - π gp-continuous function. Let V be a closed subset of Y . since f is $(1,2)^*$ - π gp-continuous $f^{-1}(V)$ is $(1,2)^*$ - π gp-closed in X . As every $(1,2)^*$ - π gp-closed set is $(1,2)^*$ - π gb*-closed, $f^{-1}(V)$ is $(1,2)^*$ - π gb*-closed. Hence f is $(1,2)^*$ - π gb*-continuous.

Remark 5.31 The converse of the above theorem need not be true it can be seen from the following example.

Example 5.32 Let $X = \{a,b,c\}$ with topology $\tau_1 = \{\emptyset, \{a\}, \{c\}, \{a,c\}, X\}$, $\tau_2 = \{\emptyset, X, \{c\}\}$ and let $Y = \{a,b,c\}$ with topology $\sigma_1 = \{\emptyset, Y, \{a\}, \{b\}, \{b,a\}, \{\emptyset, Y, \{b\}\}\}$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be defined as $f(a) = b, f(b) = c, f(c) = a$. $(1,2)^*$ - π gp-continuous = $\{\emptyset, \{a\}, \{c\}, \{a,c\}, X\}$. Then f is $(1,2)^*$ - π gb*-continuous but it is not $(1,2)^*$ - π gp-continuous.

Theorem 5.33 Every $(1,2)^*$ - π gs-continuous function is $(1,2)^*$ - π gb*-continuous.

Proof Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a $(1,2)^*$ - π gs-continuous function. Let V be a closed subset of Y . since f is $(1,2)^*$ - π gs-continuous $f^{-1}(V)$ is $(1,2)^*$ - π gs-closed in X . As every $(1,2)^*$ - π gs-closed set is $(1,2)^*$ - π gb*-closed, $f^{-1}(V)$ is $(1,2)^*$ - π gb*-closed. Hence f is $(1,2)^*$ - π gb*-continuous.

Remark 5.34 The converse of the above theorem need not be true as seen from the following example.

Example 5.35 Let $X = \{a,b,c,d\}$ with topology $\tau_1 = \{\emptyset, \{a\}, \{c\}, \{a,c\}, X\}$, $\tau_2 = \{\emptyset, \{c\}, X\}$ and $Y = \{a,b,c,d\}$ with topology $\sigma_1 = \{\emptyset, Y, \{b\}, \{a\}, \{b,a\}\}$, $\sigma_2 = \{\emptyset, Y, \{b\}\}$. Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be defined as identity function. $(1,2)^*$ - π gs-continuous = $\{\emptyset, \{a,c,d\}, X\}$. Then f is $(1,2)^*$ - π gb*-continuous but it is not $(1,2)^*$ - π gs-continuous.

VI. $(1,2)^*$ - π gb*-CONTINUITY AND ITS CHARACTERISTICS

Theorem 6.1 Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a function. Then the following statements are equivalent:

- (1) f is $(1,2)^*$ - π gb*-continuous;
- (2) The inverse image of every open set in Y is $(1,2)^*$ - π gb*-open in X .

Proof (1) \Rightarrow (2) Let U be open subset of Y . Then $(Y-U)$ is closed in Y . Since f is $(1,2)^*$ - π gb*-continuous, $f^{-1}(Y-U) = X - f^{-1}(U)$ is $(1,2)^*$ - π gb*-closed in X . Hence $f^{-1}(U)$ is $(1,2)^*$ - π gb*-open in X .

(2) \Rightarrow (1) Let V be a closed subset of Y . Then $(Y - V)$ is open in Y hence by hypothesis (2) $f^{-1}(Y - V) = X - f^{-1}(V)$ is $(1,2)^*$ - πgb^* -open in X . Hence $f^{-1}(V)$ is $(1,2)^*$ - πgb^* -closed in X . Therefore, f is $(1,2)^*$ - πgb^* -continuous.

Theorem 6.2 Every $(1,2)^*$ - πgb^* -irresolute function is $(1,2)^*$ - πgb^* -continuous.

Proof Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be $(1,2)^*$ - πgb^* -irresolute function. Let V be closed set in Y , then V is $(1,2)^*$ - πgb^* -closed in Y . Since f is $(1,2)^*$ - πgb^* -irresolute $f^{-1}(V)$ is $(1,2)^*$ - πgb^* -closed in X . Hence f is $(1,2)^*$ - πgb^* -continuous.

Theorem 6.3 Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a function. Then the following statements are equivalent: (1) For each $x \in X$ and each open set V containing $f(x)$ there exists a $(1,2)^*$ - πgb^* -open set U containing x such that $f(U) \subset V$. (2) $f((1,2)^*\text{-}\pi gb^*\text{-cl}(A)) \subset \text{cl}(f(A))$ for every subset A of X .

Proof (1) \Rightarrow (2) Let $y \in f((1,2)^*\text{-}\pi gb^*\text{-cl}(A))$ then, there exists an $x \in (1,2)^*\text{-}\pi gb^*\text{-cl}(A)$ such that $y = f(x)$. We claim that $y \in \text{cl}(f(A))$ and let V be any open neighbourhood of y . Since $x \in (1,2)^*\text{-}\pi gb^*\text{-cl}(A)$ there exists an $(1,2)^*$ - πgb^* -open set U such that $x \in U$ and $U \cap A \neq \emptyset$, $f(U) \subset V$. Since $U \cap A \neq \emptyset$, $f(A) \cap V \neq \emptyset$. Therefore, $y = f(x) \in \text{cl}(f(A))$. Hence $f((1,2)^*\text{-}\pi gb^*\text{-cl}(A)) \subset \text{cl}(f(A))$.

(2) \Rightarrow (1) Let $x \in X$ and V be any open set containing $f(x)$. Let $A = f^{-1}(Y - V)$, Since $f((1,2)^*\text{-}\pi gb^*\text{-cl}(A)) \subset \text{cl}(f(A)) \subset (Y - V) \Rightarrow (1,2)^*\text{-}\pi gb^*\text{-cl}(A) \subset f^{-1}(Y - V) = A$. Hence $(1,2)^*\text{-}\pi gb^*\text{-cl}(A) = A$. Since $f(x) \in V \Rightarrow x \in f^{-1}(V) \Rightarrow x \notin A \Rightarrow x \notin (1,2)^*\text{-}\pi gb^*\text{-cl}(A)$. Thus there exists an open set U containing x such that $U \cap A = \emptyset \Rightarrow f(U) \cap f(A) = \emptyset$. Therefore $f(U) \subset V$.

Definition 6.4 A bitopological space (X, τ_1, τ_2) is a $(1,2)^*$ - πgb^* -space if every $(1,2)^*$ - πgb^* -closed set is closed.

Theorem 6.4(a) Every $(1,2)^*$ - πgb^* -space is $(1,2)^*$ - πgb^* - $T_{1/2}$ space.

Proof Let (X, τ_1, τ_2) be a $(1,2)^*$ - πgb^* -space and let $A \subset X$ be $(1,2)^*$ - πgb^* -closed set in X . Then A is closed $\Rightarrow A$ is $(1,2)^*$ - b^* -closed $\Rightarrow (X, \tau)$ is a $(1,2)^*$ - πgb^* - $T_{1/2}$ space.

Theorem 6.5 Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be a function then,

- (1) If f is $(1,2)^*$ - πgb^* -irresolute and X is $(1,2)^*$ - πgb^* - $T_{1/2}$ space, then f is $(1,2)^*$ - b^* -irresolute.
- (2) If f is $(1,2)^*$ - πgb^* -continuous and X is $(1,2)^*$ - πgb^* - $T_{1/2}$ space, then f is $(1,2)^*$ - b^* -continuous.

Proof

(1) Let V be b^* -closed in (Y, σ_1, σ_2) , then V is $(1,2)^*$ - πgb^* -closed in Y . Since f is $(1,2)^*$ - πgb^* -irresolute, $f^{-1}(V)$ is $(1,2)^*$ - πgb^* -closed in X . Since X is $(1,2)^*$ - πgb^* - $T_{1/2}$ space, $f^{-1}(V)$ is b^* -closed in X . Hence f is $(1,2)^*$ - b^* -irresolute.

(2) Let V be closed in Y . Since f is $(1,2)^*$ - πgb^* -continuous, $f^{-1}(V)$ is $(1,2)^*$ - πgb^* -closed in X . Since X is $(1,2)^*$ - πgb^* - $T_{1/2}$ space, $f^{-1}(V)$ is $(1,2)^*$ - b^* -closed. Therefore f is $(1,2)^*$ - b^* -continuous.

Definition 6.6 A function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is said to be almost $(1,2)^*$ - πgb^* -continuous if $f^{-1}(V)$ is $(1,2)^*$ - πgb^* -closed in (X, τ_1, τ_2) for every regular Closed set V of (Y, σ_1, σ_2)

Theorem 6.7 For a function $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$, the following statements are equivalent:

- (1) f is almost $(1,2)^*$ - πgb^* -continuous.
- (2) $f^{-1}(V)$ is $(1,2)^*$ - πgb^* -open in X for every regular open set V of Y .
- (3) $f^{-1}(\text{int}(\text{cl}(V)))$ is $(1,2)^*$ - πgb^* -open in X for every open set V of Y .
- (4) $f^{-1}(\text{cl}(\text{int}(V)))$ is $(1,2)^*$ - πgb^* -closed in X for every closed set V of Y .

Proof (1) \Rightarrow (2) Suppose f is almost $(1,2)^*$ - πgb^* -continuous. Let V be a regular open subset of Y . Since $(Y - V)$ is regular closed and f is almost $(1,2)^*$ - πgb^* -continuous, $f^{-1}(Y - V) = X - f^{-1}(V)$ is $(1,2)^*$ - πgb^* -closed in X . Hence $f^{-1}(V)$ is $(1,2)^*$ - πgb^* -open in X .

(2) \Rightarrow (1) Let V be a regular closed subset of Y . Then $(Y - V)$ is regular open. By the hypothesis, $f^{-1}(Y - V) = X - f^{-1}(V)$ is $(1,2)^*$ - πgb^* -open in X . Hence $f^{-1}(V)$ is $(1,2)^*$ - πgb^* -closed.

Thus f is $(1,2)^*$ - πgb^* -continuous. Therefore, $f^{-1}(\text{int}(\text{cl}(V)))$ is $(1,2)^*$ - πgb^* -open in X .

(2) \Rightarrow (3) Let V be an open subset of Y . Then $\text{int}(\text{cl}(V))$ is regular open in Y . By the hypothesis, $f^{-1}(\text{int}(\text{cl}(V)))$ is $(1,2)^*$ - πgb^* -open in X .

(3) \Rightarrow (2) Let V be a regular open subset of Y . Since $V = \text{int}(\text{cl}(V))$ and every regular open set is open then $f^{-1}(V)$ is $(1,2)^*$ - πgb^* -open in X .

(3) \Rightarrow (4) Let V be a closed subset of Y . Then $(Y - V)$ is open in Y . By the hypothesis, $f^{-1}(\text{int}(\text{cl}(Y - V))) = f^{-1}(Y - \text{cl}(\text{int}(V))) = X - f^{-1}(\text{cl}(\text{int}(V)))$ is $(1,2)^*$ - πgb^* -open in X . Therefore $f^{-1}(\text{cl}(\text{int}(V)))$ is $(1,2)^*$ - πgb^* -closed in X .

(4) \Rightarrow (3) Let V be an open subset of Y . Then $(Y - V)$ is closed. By the hypothesis $f^{-1}(\text{cl}(\text{int}(Y - V))) = X - f^{-1}(\text{int}(\text{cl}(V)))$ is $(1,2)^*$ - πgb^* -closed in X .

Theorem 6.8 Every $(1,2)^*$ - πgb^* -continuous function is almost $(1,2)^*$ - πgb^* -continuous.

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Proof Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be $(1,2)^*$ - πgb^* -continuous function. Let V be regular closed set in Y , then V is closed in Y . Since f is $(1,2)^*$ - πgb^* -continuous function $f^{-1}(V)$ is $(1,2)^*$ - πgb^* -closed in X . Therefore f is almost $(1,2)^*$ - πgb^* -continuous.

Theorem 6.9 Every almost $(1,2)^*$ - b^* -continuous function is almost $(1,2)^*$ - πgb^* -continuous.

Proof Let $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ be almost $(1,2)^*$ - b^* -continuous function and let V be regular closed set in Y . Then $f^{-1}(V)$ is $(1,2)^*$ - b^* -closed in X , hence $f^{-1}(V)$ is $(1,2)^*$ - πgb^* -closed in X . Therefore f is almost $(1,2)^*$ - πgb^* -continuous.

Theorem 6.10 Let X be a $(1,2)^*$ - πgb^* - $T_{1/2}$ space. Then $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is almost $(1,2)^*$ - πgb^* -continuous if and only if f is almost $(1,2)^*$ - b^* -continuous.

Proof Suppose $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is almost $(1,2)^*$ - πgb^* -continuous. Let A be a regular closed subset of Y . Then $f^{-1}(A)$ is $(1,2)^*$ - πgb^* -closed in X . Since X is $(1,2)^*$ - πgb^* - $T_{1/2}$ space, $f^{-1}(A)$ is $(1,2)^*$ - b^* -closed in X . Hence f is almost $(1,2)^*$ - b^* -continuous. Conversely, suppose that $f : (X, \tau_1, \tau_2) \rightarrow (Y, \sigma_1, \sigma_2)$ is almost $(1,2)^*$ - b^* -continuous and A be a regular closed subset of Y . Then $f^{-1}(A)$ is $(1,2)^*$ - b^* -closed in X . Since every $(1,2)^*$ - b^* -closed set is $(1,2)^*$ - πgb^* -closed, $f^{-1}(A)$ is $(1,2)^*$ - πgb^* -closed. Therefore, f is almost $(1,2)^*$ - πgb^* -continuous.

VII. CONCLUSION

Through the above findings, this paper has attempted to compare $(1,2)^*$ - πgb^* -closed with other closed sets in bitopological spaces. An attempt of this paper is to state that the definitions and results that shown in this paper, will result in obtaining several characterizations and enable to study various properties as well.

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