

# On Topological $sb\hat{g}$ – quotient mappings

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**ABSTRACT:** In this paper, we define new class of mappings namely  $sb\hat{g}$ -quotient mappings and we prove some of their basic properties. Also, we introduce some new mappings namely strongly  $sb\hat{g}$ -open map, strongly  $sb\hat{g}$ -quotient map,  $sb\hat{g}^*$ -quotient map and strongly  $sb\hat{g}^*$ -quotient map and we discuss some of their relationship among other mappings.

**KEYWORDS:**  $sb\hat{g}$  – closed set,  $sb\hat{g}$ -continuous function,  $sb\hat{g}$ -irresolute function,  $sb\hat{g}$ -quotient mapping, strongly  $sb\hat{g}$ -open map, strongly  $sb\hat{g}$ -quotient map,  $sb\hat{g}^*$ -quotient map, strongly  $sb\hat{g}^*$ -quotient map.

## I. INTRODUCTION

In 1963, N.Levine[8] introduced semi-open sets in Topology and studied their properties. Andrijevic[1] introduced one such new version called b-open sets in 1996. N.Levine[9] introduced the concept of generalized closed sets and studied their properties in 1970. By considering the concept of g-closed sets many concepts of topology have been generalized and interesting results have been obtained by several mathematician. M.K.R.S.Veerakumar[13] defined  $\hat{g}$ -closed sets in 2003. Also, R.Subasree and M.Mariasingam[10] introduced  $b\hat{g}$ -closed sets in 2013. We introduced  $sb\hat{g}$ -closed sets[2] and studied their properties in 2015.

In this paper, we define new class of mappings namely  $sb\hat{g}$ -quotient mappings and we prove some of their basic properties. Also, we introduce some new mappings namely strongly  $sb\hat{g}$ -open map, strongly  $sb\hat{g}$ -quotient map,  $sb\hat{g}^*$ -quotient map and strongly  $sb\hat{g}^*$ -quotient map and we discuss some of their relationship among other mappings.

## II. PRELIMINARIES

Throughout this paper  $(X, \tau)$  (or simply  $X$ ) represents topological spaces in which no separation axioms are assumed unless otherwise mentioned. For a subset  $A$  of  $(X, \tau)$ ,  $Cl(A)$ ,  $Int(A)$  and  $A^c$  denote the closure of  $A$ , interior of  $A$  and the complement of  $A$  respectively. We are giving some definitions.

**Definition 2.1:** A subset  $A$  of a topological space  $(X, \tau)$  is called a

- (i) semi-open set[8] if  $A \subseteq Cl(Int(A))$ .
- (ii)  $\alpha$ -open set[7] if  $A \subseteq Int(Cl(Int(A)))$ .
- (iii) b-open set[1] if  $A \subseteq Cl(Int(A)) \cup Int(Cl(A))$ .

**Definition 2.2:** A subset  $A$  of a topological space  $(X, \tau)$  is called a

- (i)  $b\hat{g}$ -closed set[10] if  $bCl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $\hat{g}$ -open in  $X$ .
- (ii)  $sb\hat{g}$ -closed set[2] if  $sCl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $b\hat{g}$ -open in  $X$ .

The complements of the above mentioned closed sets and their respective open sets.

**Remark 2.3:** The family of  $sb\hat{g}$ -closed sets and  $sb\hat{g}$ -open sets are denoted by  $sb\hat{g}-C(X)$  and  $sb\hat{g}-O(X)$ .

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**Definition 2.4:** A map  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called a

- (i) continuous[6] if  $f^{-1}(V)$  is closed in  $X$  for every closed set  $V$  in  $Y$ .
- (ii)  $\alpha$ -continuous[6] if  $f^{-1}(V)$  is  $\alpha$ -closed in  $X$  for every closed set  $V$  in  $Y$ .
- (iii)  $sb\hat{g}$ -continuous[11] if  $f^{-1}(V)$  is  $sb\hat{g}$ -closed in  $X$  for every closed set  $V$  in  $Y$ .
- (iv)  $sb\hat{g}$ -continuous[3] if  $f^{-1}(V)$  is  $sb\hat{g}$ -closed in  $X$  for every closed set  $V$  in  $Y$ .
- (v)  $sb\hat{g}$  – irresolute map[3] if  $f^{-1}(V)$  is  $sb\hat{g}$ -closed in  $(X, \tau)$  for every  $sb\hat{g}$ -closed set  $V$  in  $(Y, \sigma)$ .
- (vi)  $sb\hat{g}$  - open map[3] if  $f(V)$  is  $sb\hat{g}$  – open in  $(Y, \sigma)$  for every open set  $V$  in  $(X, \tau)$ .
- (vii) Contra  $sb\hat{g}$ -continuous map[4] if  $f^{-1}(V)$  is  $sb\hat{g}$ -closed in  $(X, \tau)$  for every open set  $V$  in  $(Y, \sigma)$ .
- (viii)  $sb\hat{g}$  – homeomorphism[3] if  $f$  is both  $sb\hat{g}$  – continuous map and  $sb\hat{g}$  – open map.
- (ix) Strongly  $sb\hat{g}$ -continuous[3] if  $f^{-1}(V)$  is closed in  $(X, \tau)$  for every  $sb\hat{g}$ -closed set  $V$  in  $(Y, \sigma)$ .
- (x) Quotient[12] if  $f$  is continuous and  $f^{-1}(V)$  is open in  $X$  implies  $V$  is open in  $Y$ .
- (xi)  $\alpha$ -quotient[12] if  $f$  is  $\alpha$ -continuous and  $f^{-1}(V)$  is open in  $X$  implies  $V$  is  $\alpha$ -open in  $Y$ .
- (xii)  $sb\hat{g}$ -quotient[12] if  $f$  is  $sb\hat{g}$ -continuous and  $f^{-1}(V)$  is open in  $X$  implies  $V$  is  $sb\hat{g}$ -open in  $Y$ .

**Definition 2.5:** A space  $X$  is called a  $T_{sb\hat{g}}$ -space[2] if every  $sb\hat{g}$ -closed set in it is closed.

### III. $sb\hat{g}$ – QUOTIENT MAP

**Definition 3.1:** A surjective map  $f: (X, \tau) \rightarrow (Y, \sigma)$  is said to be a  $sb\hat{g}$ -quotient map if  $f$  is  $sb\hat{g}$ -continuous and  $f^{-1}(V)$  is open in  $(X, \tau)$  implies  $V$  is  $sb\hat{g}$ -open in  $(Y, \sigma)$ .

**Example 3.2:** Let  $X = \{a, b, c, d\}$  and  $Y = \{p, q, r\}$  with topologies

$\tau = \{X, \Phi, \{a\}, \{a, c\}, \{a, b, d\}\}$  and  $\sigma = \{Y, \{p\}, \{p, q\}, \{p, r\}\}$

Define a function  $f: (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = p, f(b) = r, f(c) = q, f(d) = r$ .

$sb\hat{g}\text{-}O(X) = \{X, \Phi, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, b, d\}, \{a, c, d\}\}$

$sb\hat{g}\text{-}O(Y) = \{Y, \Phi, \{p\}, \{p, q\}, \{p, r\}\}$

Clearly,  $f$  is a  $sb\hat{g}$ -quotient map.

**Definition 3.3:** A function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is said to be a strongly  $sb\hat{g}$ -open map if  $f(V)$  is  $sb\hat{g}$ -open in  $Y$  for each  $sb\hat{g}$ -open set  $V$  in  $X$ .

**Example 3.4:** Let  $X = \{a, b, c, d\}$  and  $Y = \{p, q, r, s\}$  with topologies

$\tau = \{X, \Phi, \{b\}, \{a, b\}, \{b, c, d\}\}$  and  $\sigma = \{Y, \{p\}, \{p, r\}, \{p, q, s\}\}$

Define a function  $f: (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = s, f(b) = p, f(c) = r, f(d) = q$ .

$sb\hat{g}\text{-}O(X) = \{X, \Phi, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}$

$sb\hat{g}\text{-}O(Y) = \{Y, \Phi, \{p\}, \{p, q\}, \{p, r\}, \{p, s\}, \{p, q, r\}, \{p, q, s\}, \{p, r, s\}\}$

Clearly,  $f$  is a strongly  $sb\hat{g}$ -open map.

**Theorem 3.5:** If a function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is surjective  $sb\hat{g}$ -continuous and  $sb\hat{g}$ -open map then  $f$  is  $sb\hat{g}$ -quotient map.

**Proof:** Let  $f^{-1}(V)$  be open in  $(X, \tau)$ . Since  $f$  is  $sb\hat{g}$ -open map,  $f(f^{-1}(V))$  is  $sb\hat{g}$ -open in  $(Y, \sigma)$ . Since  $f$  is surjective,  $V$  is  $sb\hat{g}$ -open in  $(Y, \sigma)$ . Hence,  $f$  is  $sb\hat{g}$ -quotient map.

**Theorem 3.6:** If a function  $f: (X, \tau) \rightarrow (Y, \sigma)$  is surjective  $sb\hat{g}$ -continuous and strongly  $sb\hat{g}$ -open map then  $f$  is  $sb\hat{g}$ -quotient map.

**Proof:** Let  $f^{-1}(V)$  be open in  $(X, \tau)$ . By Proposition 3.4 in [2],  $f^{-1}(V)$  is  $sb\hat{g}$ -open in  $(X, \tau)$ . Since  $f$  is strongly  $sb\hat{g}$ -open,  $f(f^{-1}(V))$  is  $sb\hat{g}$ -open in  $(Y, \sigma)$ . Since  $f$  is surjective,  $V$  is  $sb\hat{g}$ -open in  $(Y, \sigma)$ . Hence,  $f$  is  $sb\hat{g}$ -quotient map.

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**Corollary 3.7:** If a map  $f:(X,\tau) \rightarrow (Y,\sigma)$  is surjective  $sb\hat{g}$ -continuous and  $f^{-1}$  is  $sb\hat{g}$ -irresolute then  $f$  is  $sb\hat{g}$ -quotient map.

**Proof:** The proof follows from the fact that  $f^{-1}$  is  $sb\hat{g}$ -irresolute and  $f$  is strongly  $sb\hat{g}$ -open is equivalent.

**Theorem 3.8:** If a map  $f:(X,\tau) \rightarrow (Y,\sigma)$  is  $sb\hat{g}$ -homeomorphism then  $f$  is  $sb\hat{g}$ -quotient map.

**Proof:** Since  $f$  is  $sb\hat{g}$ -homeomorphism,  $f$  is bijective and  $f$  is  $sb\hat{g}$ -continuous. Let  $f^{-1}(V)$  be open in  $(X,\tau)$ . Since  $f^{-1}$  is  $sb\hat{g}$ -continuous,  $f(f^{-1}(V)) = V$  is  $sb\hat{g}$ -open in  $(Y,\sigma)$ . Hence,  $f$  is  $sb\hat{g}$ -quotient map.

**Theorem 3.9:** Every quotient map is  $sb\hat{g}$ -quotient map

**Proof:** Let  $f:(X,\tau) \rightarrow (Y,\sigma)$  be a quotient map. By proposition 3.5(a) in [3] and proposition 3.4 in [2], the proof follows from the definition.

The following example shows that the converse of the above theorem need not be true.

**Example 3.10:** Let  $X=\{a,b,c,d\}$  and  $Y = \{p,q,r,s\}$  with topologies

$\tau = \{X, \Phi, \{a\}, \{a,c\}, \{a,b,d\}\}$  and  $\sigma = \{Y, \{q\}, \{p,q\}, \{q,r,s\}\}$

Define a function  $f:(X,\tau) \rightarrow (Y,\sigma)$  by  $f(a) = q, f(b) = p, f(c) = r, f(d) = s$ .

$sb\hat{g}\text{-}O(X) = \{X, \Phi, \{a\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$

$sb\hat{g}\text{-}O(Y) = \{Y, \Phi, \{q\}, \{p,q\}, \{q,r\}, \{q,s\}, \{p,q,r\}, \{p,q,s\}, \{q,r,s\}\}$

Clearly,  $f$  is  $sb\hat{g}$ -quotient map. Since the inverse image of open sets  $\{p,q\}, \{q,r,s\}$  in  $Y$  are  $\{a,b\}, \{a,c,d\}$  which are not open in  $X$ ,  $f$  is not continuous. Therefore,  $f$  is not a quotient map.

**Theorem 3.11:** Every  $\alpha$ -quotient map is  $sb\hat{g}$ -quotient map

**Proof:** Let  $f:(X,\tau) \rightarrow (Y,\sigma)$  be a  $\alpha$ -quotient map. By proposition 3.5(b) in [3] and proposition 3.7 in [2], the proof follows from the definition.

The following example shows that the converse of the above theorem need not be true.

**Example 3.12:** Let  $X=\{a,b,c\}$  and  $Y = \{p,q,r\}$  with topologies

$\tau = \{X, \Phi, \{a\}, \{b\}, \{a,b\}\}$  and  $\sigma = \{Y, \{p\}, \{q\}, \{p,q\}, \{p,r\}\}$

Define a function  $f:(X,\tau) \rightarrow (Y,\sigma)$  by  $f(a) = p, f(b) = q, f(c) = r$

$\alpha\text{-}O(X) = \{X, \Phi, \{a\}, \{b\}, \{a,b\}\}$

$\alpha\text{-}O(Y) = \{Y, \Phi, \{p\}, \{q\}, \{p,q\}, \{p,r\}\}$

$sb\hat{g}\text{-}O(X) = \{X, \Phi, \{a\}, \{b\}, \{a,b\}, \{a,c\}, \{b,c\}, \{a,b,d\}, \{a,c,d\}\}$

$sb\hat{g}\text{-}O(Y) = \{Y, \Phi, \{p\}, \{q\}, \{p,q\}, \{p,r\}\}$

Clearly,  $f$  is  $sb\hat{g}$ -quotient map. Since the inverse image of open set  $\{p,r\}$  in  $Y$  is  $\{a,c\}$  which is not  $\alpha$ -open in  $X$ ,  $f$  is not  $\alpha$ -continuous. Therefore,  $f$  is not  $\alpha$ -quotient map.

**Theorem 3.13:** Every  $sb\hat{g}$ -quotient map is  $b\hat{g}$ -quotient map

**Proof:** Let  $f:(X,\tau) \rightarrow (Y,\sigma)$  be a  $sb\hat{g}$ -quotient map. By proposition 3.7(f) in [3] and proposition 3.23 in [2], the proof follows from the definition.

The following example shows that the converse of the above theorem need not be true.

**Example 3.14:** Let  $X=\{a,b,c\}$  and  $Y = \{p,q,r\}$  with topologies

$\tau = \{X, \Phi, \{a,c\}\}$  and  $\sigma = \{Y, \{p\}, \{q\}, \{p,q\}, \{p,r\}\}$

Define a function  $f:(X,\tau) \rightarrow (Y,\sigma)$  by  $f(a) = p, f(b) = r, f(c) = q$

$b\hat{g}\text{-}O(X) = \{X, \Phi, \{a\}, \{c\}, \{a,b\}, \{a,c\}, \{b,c\}\}$

$b\hat{g}\text{-}O(Y) = \{Y, \Phi, \{p\}, \{q\}, \{p,q\}, \{p,r\}\}$

$sb\hat{g}\text{-}O(X) = \{X, \Phi, \{a,c\}\}$

$sb\hat{g}\text{-}O(Y) = \{Y, \Phi, \{p\}, \{q\}, \{p,q\}, \{p,r\}\}$

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Clearly,  $f$  is  $\text{sb}\hat{g}$ -quotient map. Since the inverse image of open sets  $\{p\}, \{q\}$  and  $\{p,r\}$  in  $Y$  are  $\{a\}, \{c\}$  and  $\{a,b\}$  which are not  $\text{sb}\hat{g}$ -open in  $X$ ,  $f$  is not  $\text{sb}\hat{g}$ -continuous. Therefore,  $f$  is not  $\text{sb}\hat{g}$ -quotient map.

### IV. STRONG FORM OF $\text{sb}\hat{g}$ – QUOTIENT MAP

**Definition 4.1:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a surjective map. Then  $f$  is called strongly  $\text{sb}\hat{g}$ -quotient map provided a set  $V$  of  $(Y, \sigma)$  is open in  $Y$  if and only if  $f^{-1}(V)$  is  $\text{sb}\hat{g}$ -open in  $X$ .

**Example 4.2:** Let  $X = \{a, b, c, d\}$  and  $Y = \{p, q, r\}$  with topologies  $\tau = \{X, \Phi, \{d\}, \{a, b, c\}\}$  and  $\sigma = \{Y, \Phi, \{r\}, \{p, q\}\}$ . Define a function  $f: (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = f(b) = p, f(c) = q, f(d) = r$ .  
 $\text{sb}\hat{g}\text{-O}(X) = \{X, \Phi, \{d\}, \{a, b, c\}\}$   
Clearly,  $f$  is strongly  $\text{sb}\hat{g}$ -quotient map.

**Theorem 4.3:**

- (i) Every strongly  $\text{sb}\hat{g}$ -quotient map is  $\text{sb}\hat{g}$ -open map.
- (ii) Every strongly  $\text{sb}\hat{g}$ -quotient map is strongly  $\text{sb}\hat{g}$ -open map.

**Proof:**

- (i) Let  $f$  be a strongly  $\text{sb}\hat{g}$ -quotient map and  $V$  be any open set in  $X$ . By Theorem 3.4 in [2],  $V$  is a  $\text{sb}\hat{g}$ -open set in  $(X, \tau)$ . That is,  $f^{-1}(f(V))$  is  $\text{sb}\hat{g}$ -open in  $(X, \tau)$ . Since  $f$  is strongly  $\text{sb}\hat{g}$ -quotient,  $f(V)$  is open in  $Y$  and hence  $\text{sb}\hat{g}$ -open in  $Y$ . Therefore,  $f$  is a  $\text{sb}\hat{g}$ -open map.
- (ii) Let  $f$  be a strongly  $\text{sb}\hat{g}$ -quotient map and  $V$  be any  $\text{sb}\hat{g}$ -open set in  $X$ . That is  $f^{-1}(f(V))$  is  $\text{sb}\hat{g}$ -open in  $(X, \tau)$ . Since  $f$  is strongly  $\text{sb}\hat{g}$ -quotient,  $f(V)$  is open in  $Y$  and hence  $\text{sb}\hat{g}$ -open in  $Y$ . Hence,  $f$  is a strongly  $\text{sb}\hat{g}$ -open map.

The following example shows that the converse of the above theorem need not be true.

**Example 4.4:**

- (i) Let  $X = Y = \{a, b, c, d\}$  with topologies  $\tau = \{X, \Phi, \{a\}, \{a, c\}, \{a, b, d\}\}$  and  $\sigma = \{Y, \Phi, \{a\}, \{b, c\}, \{a, b, c\}\}$ . Define a function  $f: (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = a, f(b) = b, f(c) = d, f(d) = c$ .  
 $\text{sb}\hat{g}\text{-O}(X) = \{X, \Phi, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, c, d\}, \{a, b, d\}\}$   
 $\text{sb}\hat{g}\text{-O}(Y) = \{Y, \Phi, \{a\}, \{b, c\}, \{a, d\}, \{a, b, c\}, \{b, c, d\}\}$   
Clearly,  $f$  is a  $\text{sb}\hat{g}$ -open map. Since the inverse image of an open set  $\{b, c\}$  in  $Y$  is  $\{b, d\}$  which is not  $\text{sb}\hat{g}$ -open set in  $X$ ,  $f$  is not  $\text{sb}\hat{g}$ -continuous. Therefore,  $f$  is not strongly  $\text{sb}\hat{g}$ -quotient map.
- (ii) Let  $X = \{a, b, c\}$  and  $Y = \{p, q, r\}$  with topologies  $\tau = \{X, \Phi, \{a\}, \{a, b\}\}$  and  $\sigma = \{Y, \Phi, \{p\}\}$ . Define a function  $f: (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = p, f(b) = q, f(c) = r$ .  
 $\text{sb}\hat{g}\text{-O}(X) = \{X, \Phi, \{a\}, \{a, b\}, \{a, c\}\}$   
 $\text{sb}\hat{g}\text{-O}(Y) = \{Y, \Phi, \{p\}, \{p, q\}, \{p, r\}\}$   
Clearly,  $f$  is a strongly  $\text{sb}\hat{g}$ -open map. Since the inverse image of  $\text{sb}\hat{g}$ -open set  $\{p, q\}$  and  $\{p, r\}$  in  $Y$  are  $\{a, b\}$  and  $\{a, c\}$  which are not open in  $Y$ ,  $f$  is not strongly  $\text{sb}\hat{g}$ -quotient map.

**Definition 4.5:** A surjective map  $f: (X, \tau) \rightarrow (Y, \sigma)$  is said to be a  $\text{sb}\hat{g}^*$ -quotient map, if  $f$  is  $\text{sb}\hat{g}$ -irresolute and  $f^{-1}(V)$  is  $\text{sb}\hat{g}$ -open in  $X$  implies  $V$  is open in  $Y$ .

**Example 4.6:** Let  $X = \{a, b, c, d\}$  and  $Y = \{p, q, r\}$  with topologies  $\tau = \{X, \Phi, \{b\}, \{a, b\}, \{b, c, d\}\}$  and  $\sigma = \{Y, \Phi, \{p\}, \{p, q\}, \{p, r\}\}$ . Define a function  $f: (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = r, f(b) = p, f(c) = q, f(d) = p$ .  
 $\text{sb}\hat{g}\text{-O}(X) = \{X, \Phi, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}\}$

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$sb\hat{g}\text{-}O(Y) = \{Y, \Phi, \{p\}, \{p,q\}, \{p,r\}\}$   
Clearly,  $f$  is  $sb\hat{g}^*$ -quotient map.

**Theorem 4.7:**

- (i) Every  $sb\hat{g}^*$ -quotient map is  $sb\hat{g}$ -irresolute.
- (ii) Every  $sb\hat{g}^*$ -quotient map is strongly  $sb\hat{g}$ -open map.

**Proof:**

- (i) Proof follows from definitions 4.5 and proposition 3.4 in [2].
- (ii) Let  $f$  be a  $sb\hat{g}^*$ -quotient map. Let  $V$  be any  $sb\hat{g}$ -open set in  $X$ . Then  $f^{-1}(f(V))$  is  $sb\hat{g}$ -open in  $(X, \tau)$ . Since  $f$  is  $sb\hat{g}^*$ -quotient,  $f(V)$  is open in  $Y$  and hence from proposition 3.4 in [2],  $f(V)$  is  $sb\hat{g}$ -open in  $Y$ . Hence,  $f$  is a strongly  $sb\hat{g}$ -open map.

The following example shows that the converse of the above theorem need not be true.

**Example 4.8:**

- (i) Let  $X = \{a,b,c,d\}$  and  $Y = \{p,q,r\}$  with topologies  
 $\tau = \{X, \Phi, \{a\}, \{a,c\}, \{a,b,d\}\}$  and  $\sigma = \{Y, \Phi, \{p\}, \{p,r\}\}$   
 Define a function  $f: (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = p, f(b) = q, f(c) = f(d) = r$ .  
 $sb\hat{g}\text{-}O(X) = \{X, \Phi, \{a\}, \{a,b\}, \{a,c\}, \{a,d\}, \{a,b,c\}, \{a,b,d\}, \{a,c,d\}\}$   
 $sb\hat{g}\text{-}O(Y) = \{Y, \Phi, \{p\}, \{p,r\}\}$   
 Clearly,  $f$  is  $sb\hat{g}$ -irresolute map. Since the inverse image of  $sb\hat{g}$ -open sets  $\{a,b\}$  in  $X$  is  $\{p,q\}$  which is not open in  $Y$ ,  $f$  is not  $sb\hat{g}^*$ -quotient map.
- (ii) Let  $X=Y = \{a,b,c,d\}$  with topologies  
 $\tau = \{X, \Phi, \{a\}, \{b,c,d\}\}$  and  $\sigma = \{Y, \Phi, \{a\}, \{b,c\}, \{a,b,c\}\}$   
 Define a function  $f: (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = a, f(b) = b, f(c) = c, f(d) = d$ .  
 $sb\hat{g}\text{-}O(X) = \{X, \Phi, \{a\}, \{b,c,d\}\}$   
 $sb\hat{g}\text{-}O(Y) = \{Y, \Phi, \{a\}, \{a,d\}, \{b,c\}, \{a,b,c\}, \{b,c,d\}\}$   
 Clearly,  $f$  is a strongly  $sb\hat{g}$ -open map. Since the inverse image of the  $sb\hat{g}$ -open sets  $\{a,d\}, \{b,c\}$  and  $\{a,b,c\}$  in  $Y$  are  $\{a,d\}, \{b,c\}$  and  $\{a,b,c\}$  which are not  $sb\hat{g}$ -open in  $X$ ,  $f$  is not  $sb\hat{g}^*$ -quotient map.

**Definition 4.9:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a surjective map. Then  $f$  is called strongly  $sb\hat{g}^*$ -quotient if a set  $V$  is  $sb\hat{g}$ -open in  $Y$  if and only if  $f^{-1}(V)$  is  $sb\hat{g}$ -open in  $X$ .

**Example 4.10:** Let  $X = \{a,b,c\}$  and  $Y = \{p,q,r\}$  with topologies

$\tau = \{X, \Phi, \{a\}\}$  and  $\sigma = \{Y, \Phi, \{p\}, \{p,q\}\}$   
 Define a function  $f: (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = p, f(b) = q, f(c) = r$ .  
 $sb\hat{g}\text{-}O(X) = \{X, \Phi, \{a\}, \{a,b\}, \{a,c\}\}$   
 $sb\hat{g}\text{-}O(Y) = \{Y, \Phi, \{p\}, \{p,q\}, \{p,r\}\}$   
 Clearly,  $f$  is strongly  $sb\hat{g}^*$ -quotient map.

**Theorem 4.11:** Every  $sb\hat{g}^*$ -quotient map is strongly  $sb\hat{g}^*$ -quotient map.

**Proof:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be  $sb\hat{g}^*$ -quotient map. Let  $V$  be a  $sb\hat{g}$ -open set in  $Y$ . Since  $f$  is  $sb\hat{g}^*$ -irresolute,  $f^{-1}(V)$  is  $sb\hat{g}$ -open in  $X$ . Conversely, Let  $f^{-1}(V)$  be  $sb\hat{g}$ -open in  $X$ . Since  $f$  is  $sb\hat{g}^*$ -quotient,  $V$  is open in  $Y$ . By Theorem 3.4 in [2],  $V$  is  $sb\hat{g}$ -open in  $Y$ . Therefore,  $f$  is strongly  $sb\hat{g}^*$ -quotient map.

The following example shows that the converse of the above theorem need not be true.

**Example 4.12:** Let  $X = \{a,b,c\}$  and  $Y = \{p,q,r\}$  with topologies

$\tau = \{X, \Phi, \{a\}, \{a,b\}, \{a,c\}\}$  and  $\sigma = \{Y, \Phi, \{p\}, \{p,q\}\}$   
 Define a function  $f: (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = p, f(b) = q, f(c) = r$ .  
 $sb\hat{g}\text{-}O(X) = \{X, \Phi, \{a\}, \{a,b\}, \{a,c\}\}$

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$$sb\hat{g}\text{-}O(Y) = \{Y, \Phi, \{p\}, \{p,q\}, \{p,r\}\}$$

Clearly,  $f$  is a strongly  $sb\hat{g}^*$ -quotient map. Since  $f^{-1}(\{p, r\}) = \{a, c\}$  which is  $sb\hat{g}$ -open in  $X$  but  $\{p,r\}$  is not open in  $Y$ ,  $f$  is not  $sb\hat{g}^*$ -quotient map.

### V. COMPARISONS

**Theorem 5.1:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a surjective map. Then  $f$  is strongly  $sb\hat{g}$ -quotient map if and only if  $f$  is  $sb\hat{g}^*$ -quotient.

**Proof:** Let  $V$  be any  $sb\hat{g}$ -open set in  $Y$ . As  $f$  is surjective,  $f(f^{-1}(V))$  is  $sb\hat{g}$ -open in  $Y$ . By Theorem 4.3(ii),  $f$  is strongly  $sb\hat{g}$ -open map. Hence,  $f(f^{-1}(V))$  is  $sb\hat{g}$ -open in  $Y$  implies  $f^{-1}(V)$  is  $sb\hat{g}$ -open in  $X$ . Thus,  $f$  is a  $sb\hat{g}$ -irresolute map. Now if  $f^{-1}(V)$  is  $sb\hat{g}$ -open in  $X$  and since  $f$  is strongly  $sb\hat{g}$ -quotient map,  $V$  is open in  $Y$ . This means that  $f^{-1}(V)$  is  $sb\hat{g}$ -open in  $X$  implies  $V$  is open in  $Y$ . Hence,  $f$  is  $sb\hat{g}^*$ -quotient map.

Conversely, Let  $V$  be any open set in  $Y$ . By Theorem 3.4 in [2],  $V$  is  $sb\hat{g}$ -open in  $Y$ . Since  $f$  is  $sb\hat{g}$ -irresolute,  $f^{-1}(V)$  is  $sb\hat{g}$ -open in  $X$ . This means that  $V$  is open in  $Y$  implies  $f^{-1}(V)$  is  $sb\hat{g}$ -open in  $X$ . Now if  $f^{-1}(V)$  is  $sb\hat{g}$ -open in  $X$  and since  $f$  is  $sb\hat{g}^*$ -quotient map,  $V$  is open set in  $Y$ . This means that  $f^{-1}(V)$  is  $sb\hat{g}$ -open in  $X$  implies  $V$  is open in  $Y$ . Hence,  $f$  is strongly  $sb\hat{g}$ -quotient map.

**Theorem 5.2:**

- (i) Every strongly  $sb\hat{g}$ -quotient map is  $sb\hat{g}$ -quotient map.
- (ii) Every  $sb\hat{g}^*$ -quotient map is  $sb\hat{g}$ -quotient map.

**Proof:**

- (i) Let  $V$  be any open set in  $Y$ . Since  $f$  is strongly  $sb\hat{g}$ -quotient map,  $f^{-1}(V)$  is  $sb\hat{g}$ -open in  $X$ . This means that  $f$  is  $sb\hat{g}$ -continuous. Now let  $f^{-1}(V)$  be any open set in  $X$ . By proposition 3.4 in [2],  $f^{-1}(V)$  is  $sb\hat{g}$ -open in  $X$  and since  $f$  is strongly  $sb\hat{g}$ -quotient map,  $V$  is an open set in  $Y$ . Therefore,  $V$  is  $sb\hat{g}$ -open in  $Y$ . This means that  $f^{-1}(V)$  is open in  $X$  implies  $V$  is  $sb\hat{g}$ -open in  $Y$ . Hence,  $f$  is  $sb\hat{g}$ -quotient map.
- (ii) Let  $V$  be any open set in  $Y$ . By proposition 3.4 in [2],  $V$  is  $sb\hat{g}$ -open in  $Y$ . Since  $f$  is  $sb\hat{g}^*$ -quotient map,  $f^{-1}(V)$  is  $sb\hat{g}$ -open in  $X$ . This means that for every open set  $V$  in  $Y$ ,  $f^{-1}(V)$  is  $sb\hat{g}$ -open in  $X$ . Now let  $f^{-1}(V)$  be any open set in  $X$ . Again by proposition 3.4 in [2],  $f^{-1}(V)$  is  $sb\hat{g}$ -open in  $X$  and since  $f$  is  $sb\hat{g}^*$ -quotient map,  $V$  is open in  $Y$ . Therefore,  $V$  is  $sb\hat{g}$ -open in  $Y$ . This means that  $f^{-1}(V)$  is open in  $X$  implies  $V$  is  $sb\hat{g}$ -open in  $Y$ . Hence,  $f$  is  $sb\hat{g}$ -quotient map.

The following example shows that the converse of the above theorem need not be true.

**Example 5.3:**

- (i) Let  $X = \{a, b, c\}$  and  $Y = \{p, q, r\}$  with topologies  
 $\tau = \{X, \Phi, \{a\}, \{b\}, \{a, b\}\}$  and  $\sigma = \{Y, \Phi, \{p\}, \{q\}, \{p, q\}, \{p, r\}\}$   
 Define a function  $f: (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = p, f(b) = q, f(c) = r$ .  
 $sb\hat{g}\text{-}O(X) = \{X, \Phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$   
 $sb\hat{g}\text{-}O(Y) = \{Y, \Phi, \{p\}, \{q\}, \{p, q\}, \{p, r\}\}$   
 Clearly,  $f$  is  $sb\hat{g}$ -quotient map. Since  $f^{-1}(\{q, r\}) = \{b, c\}$  which is  $sb\hat{g}$ -open in  $X$ , but  $\{q, r\}$  is not open in  $Y$ ,  $f$  is not strongly  $sb\hat{g}$ -quotient map.
- (ii) Let  $X = \{a, b, c, d\}$  and  $Y = \{p, q, r, s\}$  with topologies  
 $\tau = \{X, \Phi, \{a\}, \{a, c\}, \{a, b, d\}\}$  and  $\sigma = \{Y, \Phi, \{q\}, \{p, q\}, \{q, r, s\}\}$   
 Define a function  $f: (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = q, f(b) = p, f(c) = r, f(d) = s$ .  
 $sb\hat{g}\text{-}O(X) = \{X, \Phi, \{a\}, \{a, b\}, \{a, c\}, \{a, d\}, \{a, b, c\}, \{a, c, d\}, \{a, b, d\}\}$   
 $sb\hat{g}\text{-}O(Y) = \{Y, \Phi, \{q\}, \{p, q\}, \{q, r\}, \{q, s\}, \{p, q, r\}, \{p, q, s\}, \{q, r, s\}\}$   
 Clearly,  $f$  is  $sb\hat{g}$ -quotient map. Since  $f^{-1}(\{q, r\}) = \{a, c\}$  which is  $sb\hat{g}$ -open in  $X$ , but  $\{q, r\}$  is not open in  $Y$ ,  $f$  is not strongly  $sb\hat{g}$ -quotient map.

**Theorem 5.4:** Every strongly  $sb\hat{g}^*$ -quotient map is  $sb\hat{g}$ -quotient.

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**Proof:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be strongly  $sb\hat{g}^*$ -quotient map. Let  $V$  be an open set in  $Y$ . By Theorem 3.4 in [2],  $V$  is  $sb\hat{g}$ -open set in  $Y$ . Since  $f$  is strongly  $sb\hat{g}^*$ -quotient,  $f^{-1}(V)$  is  $sb\hat{g}$ -open in  $X$ . Hence,  $f$  is  $sb\hat{g}$ -continuous. Let  $f^{-1}(V)$  be an open set in  $X$ . Then,  $f^{-1}(V)$  is  $sb\hat{g}$ -open in  $X$ . Since  $f$  is strongly  $sb\hat{g}^*$ -quotient map,  $V$  is  $sb\hat{g}$ -open in  $Y$ . Therefore,  $f$  is  $sb\hat{g}$ -quotient map.

The following example shows that the converse of the above theorem need not be true.

**Example 5.5:** Let  $X = \{a, b, c\}$  and  $Y = \{p, q, r\}$  with topologies

$\tau = \{X, \Phi, \{a\}, \{b\}, \{a, b\}\}$  and  $\sigma = \{Y, \Phi, \{p\}, \{q\}, \{p, q\}, \{p, r\}\}$

Define a function  $f: (X, \tau) \rightarrow (Y, \sigma)$  by  $f(a) = p, f(b) = q, f(c) = r$ .

$sb\hat{g}\text{-}O(X) = \{X, \Phi, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$

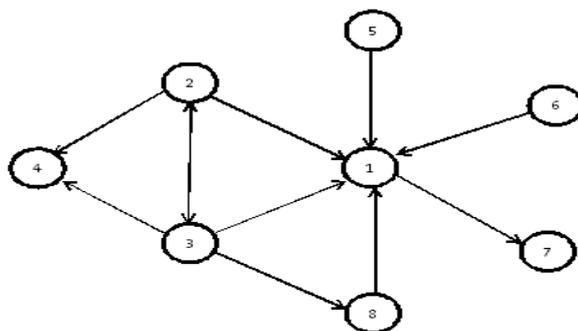
$sb\hat{g}\text{-}O(Y) = \{Y, \Phi, \{p\}, \{q\}, \{p, q\}, \{p, r\}\}$

Clearly,  $f$  is  $sb\hat{g}$ -quotient. Since  $f^{-1}(\{q, r\}) = \{b, c\}$  which is  $sb\hat{g}$ -open in  $X$ , but  $\{q, r\}$  is not  $sb\hat{g}$ -open in  $Y$ ,  $f$  is not strongly  $sb\hat{g}^*$ -quotient map.

**Theorem 5.6:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be strongly  $sb\hat{g}^*$ -quotient map and  $Y$  is  $T_{sb\hat{g}}$ -space. Then  $f$  is  $sb\hat{g}^*$ -quotient map.

**Proof:** Let  $U$  be  $sb\hat{g}$ -open in  $Y$ . Since  $f$  is strongly  $sb\hat{g}^*$ -quotient map,  $f^{-1}(U)$  is  $sb\hat{g}$ -open in  $X$ . Hence,  $f$  is  $sb\hat{g}$ -irresolute function. Let  $f^{-1}(U)$  be  $sb\hat{g}$ -open in  $X$ . Since  $f$  is strongly  $sb\hat{g}^*$ -quotient map,  $U$  is  $sb\hat{g}$ -open in  $Y$ . Since  $Y$  is  $T_{sb\hat{g}}$ -space,  $U$  is open in  $Y$ . Hence,  $f$  is  $sb\hat{g}^*$ -quotient map.

**Remark 5.7:** From the above discussions and known results, we have the following diagram. In this diagram,  $A \rightarrow B$  means  $A$  implies  $B$  but not conversely.  $A \leftrightarrow B$  means  $A$  and  $B$  are independent of each other.



- |                                   |   |                                |
|-----------------------------------|---|--------------------------------|
| 1. $sb\hat{g}$ -quotient map      | 2. Strongly $sb\hat{g}$ -quotient map   | 3. $sb\hat{g}^*$ -quotient map |
| 4. Strongly $sb\hat{g}$ -open map | 5. Quotient map                         | 6. $\alpha$ -quotient map.     |
| 7. $b\hat{g}$ -quotient map       | 8. Strongly $sb\hat{g}^*$ -quotient map |                                |

## VI. APPLICATIONS OF $sb\hat{g}$ -QUOTIENT MAPS

**Theorem 6.1:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be an open surjective,  $sb\hat{g}$ -irresolute map and  $g: (Y, \sigma) \rightarrow (Z, \eta)$  be a  $sb\hat{g}$ -quotient map. Then  $g \circ f: (X, \tau) \rightarrow (Z, \eta)$  is a  $sb\hat{g}$ -quotient map.

**Proof:** Let  $V$  be any open set in  $(Z, \eta)$ . Since  $g$  is a  $sb\hat{g}$ -quotient map,  $g^{-1}(V)$  is  $sb\hat{g}$ -open in  $Y$  and since  $f$  is  $sb\hat{g}$ -irresolute,  $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$  is a  $sb\hat{g}$ -open set in  $(X, \tau)$ . This means that for every open set  $V$  in  $(Z, \eta)$ ,  $(g \circ f)^{-1}(V)$  is  $sb\hat{g}$ -open in  $(X, \tau)$ . Hence,  $g \circ f$  is a  $sb\hat{g}$ -continuous map.

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Now assume that  $(gof)^{-1}(V)$  is open in  $(X, \tau)$  for  $V \subseteq Z$ , that is  $f^{-1}(g^{-1}(V))$  is open in  $X$ . Since  $f$  is an open map,  $f[f^{-1}(g^{-1}(V))]$  is open in  $Y$ . It follows that  $g^{-1}(V)$  is open in  $Y$ , because  $f$  is surjective. Since  $g$  is a sbg-quotient map,  $V$  is sbg-open in  $Z$ . Hence,  $gof: (X, \tau) \rightarrow (Z, \eta)$  is a sbg-quotient map.

**Theorem 6.2:** Let  $h: (X, \tau) \rightarrow (Y, \sigma)$  is a sbg-quotient map and  $g: (X, \tau) \rightarrow (Z, \eta)$  is a continuous map that is constant on each set  $h^{-1}(y)$  for  $y \in Y$ , then  $g$  induces a sbg-Continuous map  $f: (Y, \sigma) \rightarrow (Z, \eta)$  such that  $f \circ h = g$ .

**Proof:** To prove:  $f$  is sbg-continuous. Since  $g$  is constant on  $h^{-1}(y)$  for each  $y \in Y$ , the set  $g[h^{-1}(y)]$  is a one point set in  $(Z, \eta)$ . If  $f(y)$  denote this point, then it is clear that  $f$  is well-defined and for each  $x \in X$ ,  $f(h(x)) = g(x)$ . We claim that  $f$  is sbg-continuous. For, if we let  $V$  be any open set in  $(Z, \eta)$  then  $g^{-1}(V)$  is an open set as  $g$  is continuous. But  $g^{-1}(V) = h^{-1}(f^{-1}(V))$  is open in  $(X, \tau)$ . Since  $h$  is a sbg-quotient map,  $f^{-1}(V)$  is a sbg-open set. Hence,  $f$  is sbg-Continuous.

**Theorem 6.3:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a strongly sbg-open, surjective and sbg-irresolute map and  $g: (Y, \sigma) \rightarrow (Z, \eta)$  be a strongly sbg-quotient map. Then,  $gof: (X, \tau) \rightarrow (Z, \eta)$  is a strongly sbg-quotient map.

**Proof:** To Prove:  $gof: (X, \tau) \rightarrow (Z, \eta)$  is a strongly sbg-quotient map. Let  $V$  be any open set in  $(Z, \eta)$ . Since  $g$  is a strongly sbg-quotient map,  $g^{-1}(V)$  is sbg-open set in  $Y$  and since  $f$  is sbg-irresolute,  $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$  is a sbg-open set in  $(X, \tau)$ . This means that for every open set  $V$  in  $(Z, \eta)$ ,  $(gof)^{-1}(V)$  is sbg-open in  $(X, \tau)$ . Hence,  $gof$  is a sbg-continuous map.

Now assume that  $(gof)^{-1}(V)$  is sbg-open in  $(X, \tau)$  for  $V \subseteq Z$ , that is  $f^{-1}(g^{-1}(V))$  is sbg-open in  $X$ . Since  $f$  is strongly sbg-open,  $f[f^{-1}(g^{-1}(V))]$  is sbg-open in  $Y$ . Since  $f$  is surjective, it follows that,  $g^{-1}(V)$  is sbg-open in  $Y$ . This gives that,  $V$  is an open set in  $Y$ , since  $g$  is strongly sbg-quotient. This implies that, if  $(gof)^{-1}(V)$  is sbg-open in  $X$  implies  $V$  is open in  $Y$ . Hence,  $gof$  is strongly sbg-quotient map.

**Theorem 6.4:** Let  $h: (X, \tau) \rightarrow (Y, \sigma)$  be a sbg-quotient map where  $(X, \tau)$  and  $(Y, \sigma)$  are  $T_{sbg}$ -spaces. Then  $f: (Y, \sigma) \rightarrow (Z, \eta)$  is strongly sbg-continuous if and only if the composite map  $f \circ h: (X, \tau) \rightarrow (Z, \eta)$  is strongly sbg-continuous.

**Proof:** Let  $V$  be any open set in  $(Z, \eta)$ . Then  $f^{-1}(V)$  is open in  $Y$ , since  $f$  is strongly sbg-continuous. Now,  $h^{-1}(f^{-1}(V)) = (f \circ h)^{-1}(V)$  is sbg-open in  $X$ , since  $h$  is a sbg-quotient map. Since  $X$  is a  $T_{sbg}$ -space,  $h^{-1}(f^{-1}(V))$  is open in  $X$ . This implies that for every sbg-open set  $V$  in  $Z$ ,  $h^{-1}(f^{-1}(V)) = (f \circ h)^{-1}(V)$  is open in  $X$ . Thus,  $f \circ h$  is strongly sbg-continuous.

Conversely, let  $f \circ h$  is strongly sbg-continuous, then for every sbg-open set  $V$  in  $Z$ ,  $h^{-1}(f^{-1}(V)) = (f \circ h)^{-1}(V)$  is open in  $X$ . Since  $h$  is a sbg-quotient map,  $f^{-1}(V)$  is sbg-open in  $Y$ . Since  $Y$  is a  $T_{sbg}$ -space,  $f^{-1}(V)$  is open in  $Y$ . Hence,  $f$  is a strongly sbg-continuous map.

**Theorem 6.5:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a surjective, strongly sbg-open and sbg-irresolute map and  $g: (Y, \sigma) \rightarrow (Z, \eta)$  be a sbg\*-quotient map. Then  $gof: (X, \tau) \rightarrow (Z, \eta)$  is sbg\*-quotient map.

**Proof:** Let  $V$  be a sbg-open set in  $(Z, \eta)$ . Since  $g$  is a sbg\*-quotient map,  $g^{-1}(V)$  is sbg-open in  $Y$ . Since  $f$  is sbg-irresolute,  $f^{-1}(g^{-1}(V)) = (gof)^{-1}(V)$  is a sbg-open set in  $(X, \tau)$ . This means that for every sbg-open set  $V$  in  $(Z, \eta)$ ,  $(gof)^{-1}(V)$  is sbg-open in  $(X, \tau)$ . Hence,  $gof$  is a sbg-irresolute map.

Now, suppose that  $(gof)^{-1}(V)$  is sbg-open in  $(X, \tau)$  for  $V \subseteq Z$ , that is  $f^{-1}(g^{-1}(V))$  is sbg-open in  $X$ . Since  $f$  is strongly sbg-open,  $f[f^{-1}(g^{-1}(V))]$  is sbg-open in  $Y$ . Since  $f$  is surjective, it follows that,  $g^{-1}(V)$  is sbg-open in  $Y$ . This gives that  $V$  is an open set in  $Y$ , since  $g$  is sbg\*-quotient. This implies that, if  $(gof)^{-1}(V)$  is sbg-open in  $X$  implies  $V$  is open in  $Y$ . Hence,  $gof$  is a sbg\*-quotient map.

**Theorem 6.6:** The composition of two sbg\*-quotient maps is also sbg\*-quotient.

**Proof:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  and  $g: (Y, \sigma) \rightarrow (Z, \eta)$  be two sbg\*-quotient maps. To Prove their composition  $gof: (X, \tau) \rightarrow (Z, \eta)$  is a sbg\*-quotient map. Let  $V$  be a sbg-open set in  $(Z, \eta)$ . Since  $g$  is sbg\*-quotient,  $g^{-1}(V)$  is

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$sb\hat{g}$ -open in  $Y$ . Since  $f$  is  $sb\hat{g}^*$ -quotient,  $f^{-1}(g^{-1}(V))$  is  $sb\hat{g}$ -open in  $X$ . That is,  $(gof)^{-1}(V)$  is  $sb\hat{g}$ -open in  $X$ . Hence,  $gof$  is  $sb\hat{g}$ -irresolute. Let  $(gof)^{-1}(V)$  is  $sb\hat{g}$ -open in  $X$ . That is  $f^{-1}(g^{-1}(V))$  is  $sb\hat{g}$ -open in  $X$ . Since  $f$  is  $sb\hat{g}^*$ -quotient,  $g^{-1}(V)$  is open in  $Y$ , then  $g^{-1}(V)$  is  $sb\hat{g}$ -open in  $Y$ . Since  $g$  is  $sb\hat{g}^*$ -quotient,  $V$  is open in  $Z$ . This implies that,  $(gof)^{-1}(V)$  is  $sb\hat{g}$ -open in  $X$  implies  $V$  is open in  $Z$ . Hence,  $gof$  is  $sb\hat{g}^*$ -quotient map.

**Theorem 6.7:** The composition of two strongly  $sb\hat{g}$ -open map is also strongly  $sb\hat{g}$ -open map.

**Proof:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  and  $g: (Y, \sigma) \rightarrow (Z, \eta)$  be two strongly  $sb\hat{g}$ -open maps. To Prove their composition  $gof: (X, \tau) \rightarrow (Z, \eta)$  is a strongly  $sb\hat{g}$ -open map. Let  $U$  be  $sb\hat{g}$ -open set in  $X$ . Since  $f$  is strongly  $sb\hat{g}$ -open map,  $f(U)$  is  $sb\hat{g}$ -open set in  $Y$ . Since  $g$  is strongly  $sb\hat{g}$ -open map,  $g(f(U))$  is  $sb\hat{g}$ -open set in  $Z$ . Therefore,  $gof$  is strongly  $sb\hat{g}$ -open map.

**Theorem 6.8:** The composition of two strongly  $sb\hat{g}$ -quotient map is also strongly  $sb\hat{g}$ -quotient map.

**Proof:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  and  $g: (Y, \sigma) \rightarrow (Z, \eta)$  be two strongly  $sb\hat{g}$ -quotient maps. By theorem 5.1,  $f$  and  $g$  are  $sb\hat{g}^*$ -quotient map. By theorem 6.6,  $gof$  is  $sb\hat{g}^*$ -quotient map. By the converse part of theorem 5.1,  $gof$  is strongly  $sb\hat{g}$ -quotient map.

**Theorem 6.9:** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a strongly  $sb\hat{g}$ -quotient map and  $g: (Y, \sigma) \rightarrow (Z, \eta)$  be  $sb\hat{g}^*$ -quotient map and  $Y$  be  $T_{sb\hat{g}}$ -space. Then  $gof: (X, \tau) \rightarrow (Z, \eta)$  is  $sb\hat{g}^*$ -quotient.

**Proof:** Let  $V$  be a  $sb\hat{g}$ -open set in  $Z$ . Since  $g$  is  $sb\hat{g}^*$ -quotient map,  $g^{-1}(V)$  is  $sb\hat{g}$ -open in  $Y$ . Since  $Y$  is  $T_{sb\hat{g}}$ -space,  $g^{-1}(V)$  is open in  $Y$ . Since  $f$  is strongly  $sb\hat{g}$ -quotient map,  $f^{-1}(g^{-1}(V))$  is  $sb\hat{g}$ -open in  $X$ . That is,  $(gof)^{-1}(V)$  is  $sb\hat{g}$ -open in  $X$ . Hence,  $gof: (X, \tau) \rightarrow (Z, \eta)$  is  $sb\hat{g}$ -irresolute. Let  $(gof)^{-1}(V)$  be  $sb\hat{g}$ -open in  $X$ . That is,  $f^{-1}(g^{-1}(V))$  is  $sb\hat{g}$ -open in  $X$ . Since  $f$  is strongly  $sb\hat{g}$ -quotient map,  $g^{-1}(V)$  is open in  $Y$ . By Proposition 3.4 in [2],  $g^{-1}(V)$  is  $sb\hat{g}$ -open in  $Y$ . Since  $g$  is  $sb\hat{g}^*$ -quotient map,  $V$  is open in  $Z$ . Hence,  $gof$  is  $sb\hat{g}^*$ -quotient.

### VII. CONCLUSION

The properties of  $sb\hat{g}$ -quotient map,  $sb\hat{g}^*$ -quotient map and strong form of  $sb\hat{g}$ -quotient map have been studied. Further the relation between strong and weak form of  $sb\hat{g}$ -quotient map have been investigated. Based on composite function, several theorems have been proved by using  $sb\hat{g}$ -quotient map.

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