

Analytical, Experimental Determination of Deflection of Curved Beams and its Validation

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ABSTRACT: To use a beam in any application it is necessary to choose a beam which is strong enough to carry its loads. An important factor to notice is the deflection of beam. It should not be excessive that leads to failure. One of the types of beams are curved semicircular beams, that are widely used in precision mechanisms, where space available for mechanism is very less and motion should cover all possible range of distance. Compliant mechanisms exploit bending of flexible members to achieve their motion. In this paper vertical and horizontal deflections of the curved beams as hinges in planar compliant mechanisms are calculated with the help of Castigliano's theorem. Analytical expressions are used to calculate deflection of curved beams for different values of loads from 1N to 5N. Accordingly an experimental setup is designed and developed to obtain deflections from experiment. The results obtained from experimentation are compared, which are in closely match with analytical and ansys. The deviation between them is only 4-10%.

KEYWORDS: Semicircular beam hinge, Castigliano's theorem, horizontal deflection, vertical deflection.

I. INTRODUCTION

Curved beams have many important applications in various engineering fields. Due to load acting on them stresses developed are less and that is why they have more deflection. Determination of the deflection of beam is very important to know deflection range. This range of deflection is actually the range of motion that can be covered for required applications. Deflection is caused by various sources of loads. It is important to calculate the deflection within possible limit so as to know the stresses caused by loads and hence prevent structural damage. The conventional hinge designs in planar compliant mechanisms had a limited deformation range due to high stresses induced in it during deflection. To improve the range of motion of these mechanisms, semicircular beam hinges are used that allowed for large displacements. To calculate the deflection a hinge consisting of two semicircular beams with a rigid member in between them is used and its deflection is found for some range of load and compared with different methods.

II. LITERATURE REVIEW

Horacio Ahuett-Garzaa, et.al [1] defines conventional hinge designs in planar compliant mechanisms have a limited deformation range because of the high stresses induced during deflection. To improve the range of motion of these mechanisms, hinges that allow for large displacement are highly desirable. Overall, the studies showed that semicircular beam hinges present favourable characteristics in applications where large deflections are desired. Clive L.Dym, et.al [2] examines the linear response of thin, shallow curved beams to an axially-directed compressive load and explored that the curved beam behavior ranges from compression of a straight bar, for very shallow arch-like structures, to a bending state for steep curved beams. L.L Howell, et.al [3] proposed a concept of pseudo rigid body model and method of approximation of deflection path of end loaded large deflection cantilever beam and the geometric nonlinearity analysis is studied. Tadashi Horibke, et.al[4] In this paper, analytical expressions are obtained

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for both in-plane and out-of-plane displacement of a J-shaped beam subjected to a concentrated load at free end by using the Castigliano's theorem. Tore Dahlberg [5] in this the Castigliano's theorem and a numerical integration algorithm from the MATLAB are used for elliptically curved beams. The beams are either statically determinate or statically indeterminate. A.Saxena et.al [6] proposed a pseudo-rigid-body model and solved for the tip deflection of flexible beams for combined end loads and numerical integration technique using quadrature formulae has been employed to solve the large deflection Bernoulli-Euler beam equation for the tip deflection. Gere and Timoshenko [7] proposed the process of finding displacement by a direct application of Castigliano's second theorem and is efficient if more than one load acts on the structure. Yogesh Gangamwar, et.al [8] proposed analytical method for finding deflection of curved beam by using Castigliano's theorem. Now in this paper vertical and horizontal deflection of hinge with two semicircular beams connected by straight rigid member is found by analytically and experimentally and validated in ansys.

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|--|--|
| <p>Nomenclature A= Cross section area (mm²) E= Modulus of elasticity(N/mm²) f= load (N) I= Moment of inertia (mm⁴) L=Length(mm) m=Bending moment p= Generalized force r= Radius of semicircular beam(mm) u= Strain energy</p> | <p>Subscripts: h= horizontal v= vertical Symbols: δ= displacement θ=angle δ₁, δ₂, δ₃=Deflections of members 1,2,3</p> |
|--|--|

III. ANALYTICAL METHOD

The analytical method for finding horizontal and vertical deflection using Castigliano's theorem is described as follows:

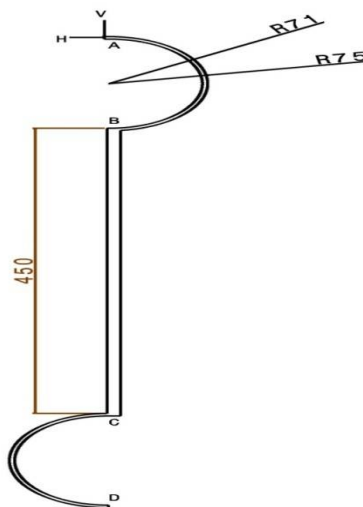


Fig (1): Hinge using semicircular beams

As shown in fig(1) The outer radius of both semicircular beams is 75 mm and inner radius of both semicircular

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beam is 71 mm, length of straight member is 450 mm. For all members thickness is 6 mm and width is 4 mm. Lower end of bottom end semicircular beam is fixed to the base.

The calculation of deflection must be broken into three parts;

- 1) Integral for semicircular beam A-B
- 2) Integral for straight member B-C
- 3) Integral for semicircular beam C-D

The integration of semicircular beam is bound by 0- π because it is semicircular and integration of straight member is bound by zero at base C and till length L.

III.1 Vertical deflection -

According to Castigliano's theorem, the deflection in the direction of force p is found as,

$$\delta_V = \frac{\partial u}{\partial p} = \frac{\partial u_1}{\partial p} + \frac{\partial u_2}{\partial p} + \frac{\partial u_3}{\partial p} \quad \dots\dots\dots (1)$$

The total strain energy in the part consists of three components;

- 1) Strain energy due to bending in AB, where $m = pr \sin\theta$;

$$u_1 = \int_0^\pi \frac{m_1^2}{2EI} d\theta \quad \dots\dots\dots 2(a)$$

- 2) Strain energy due to axial load in BC, where $f = p$;

$$u_2 = \int_0^L \frac{p^2}{2AE} dx \quad \dots\dots\dots 2(b)$$

- 3) Strain energy due to bending in CD, where $m = pr \sin\theta$;

$$u_3 = \int_0^\pi \frac{m_2^2}{2EI} r d\theta \quad \dots\dots\dots 2(c)$$

$$\text{now the total deflection, } \delta_V = \delta_1 + \delta_2 + \delta_3 \quad \dots\dots\dots (3)$$

By simplifying we get,

$$\delta_1 = \frac{\pi pr^3}{2EI}, \quad \delta_2 = \frac{pL}{AE}, \quad \delta_3 = \frac{\pi pr^3}{2EI} \quad \dots\dots\dots (4)$$

$$\text{Hence, } \delta_V = \frac{\pi pr^3}{2EI} + \frac{pL}{AE} + \frac{\pi pr^3}{2EI}$$

$$\Delta V = \frac{\pi pr^3}{EI} + \frac{pL}{AE} \quad \dots\dots\dots (5)$$

III.2 Horizontal deflection -

Now according to Castigliano's theorem, deflection in direction of force H is found as,

$$\delta_H = \frac{\partial u}{\partial H} = \frac{\partial u_1}{\partial H} + \frac{\partial u_2}{\partial H} + \frac{\partial u_3}{\partial H} \quad \dots\dots\dots (6)$$

The total strain energy in the part consists of three components;

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1) Strain energy due to bending in AB due to load p and H,

Where $m_1 = Hr (1 - \cos\theta)$

$$u_1 = \int_0^\pi \frac{m_1^2}{2EI} d\theta \quad \dots\dots\dots 7(a)$$

2) Strain energy due to bending in BC due to load H, where $m_2 = H (2r + y)$ and due to axial load in BC, where $f = p$
Now for dummy load H at a point in desired direction, obtain an expression for total strain due to all loads acting on member including dummy load.

$$\text{Total } u_2 = \int_0^L \frac{p^2}{2AE} dx + \int_0^\pi \frac{m_2^2}{2EI} d\theta \quad \dots\dots\dots 7(b)$$

3) Strain energy due to bending in CD due to load p and H,

Where $m_3 = Hr (1 - \cos\theta) + pr \sin\theta$

$$u_3 = \int_0^\pi \frac{m_3^2}{2EI} d\theta \quad \dots\dots\dots 7(c)$$

$$\text{Now the total deflection, } \delta_H = \delta_1 + \delta_2 + \delta_3 \quad \dots\dots\dots (8)$$

By simplifying we get,

$$\delta_1 = -\frac{2pr^3}{EI}, \quad \delta_2 = 0, \quad \delta_3 = \frac{2pr^2 [3r+L]}{EI} \quad \dots\dots\dots (9)$$

$$\text{Hence, } \delta_H = -\frac{2pr^3}{EI} + 0 + \frac{2pr^2 [3r+L]}{EI}$$

$$\Delta H = \frac{2pr^2(3r+L)}{EI} - \frac{2pr^3}{EI} \quad \dots\dots\dots(10)$$

III.3 Analytical calculations

From equation (5) & (10), for the data $r = 75 \text{ mm}$, $L = 450 \text{ mm}$, $E = 70 \times 10^3 \text{ MPa}$, $I = 32 \text{ mm}^4$, $A = 24 \text{ mm}^2$, For 1N and 4N load the vertical deflections are 0.59 mm and 2.36 mm respectively. For 1N and 4N load the horizontal deflections are 3.01 mm and 12.05 mm respectively.

IV. EXPERIMENTAL SETUP

Compliant mechanisms are manufactured as one part hence there is no assembly of any parts. The material selected for semicircular beam hinge is aluminium. Therefore, the semicircular beam hinge needs to be manufactured as a one part, and hence it needs to be cut from aluminium sheet of dimensions $200 \times 800 \text{ mm}$ and thickness 6 mm. As per the dimensions shown in fig. no. 1, An L-section member is attached to tip end of semicircular beam, so that dial gauge can be placed at the surface of L-section. Surface is flat, used for continuous contact to dial gauge plunger. Vertical dial gauge with accuracy 0.1 mm is used to measure vertical deflection while measuring scale with accuracy 1 mm is used to measure horizontal deflection. The upper end of top end beam is used for hanging weight using hanger. The weight hanger is welded at L-section. The weight hanger has an arrangement for weight to be increased from 50 gm to 500 gm. A vertical rod, fixed to the base is used to support dial gauge. Vertical deflections is measured using dial gauge of range 0.1mm to 10mm and horizontal deflections measured using measuring scale clamped on vertical support stand. Semicircular beam hinge is fixed to aluminium sheet using welding process at bottom end beam. The base is made of rectangular wooden block $500 \times 250 \text{ mm}$ and along its sides; support stand is erected made of mild steel plates. The semicircular beam hinge is placed on base vertically parallel in accordance with vertical rod used for dial gauge mounting. The weight in a step of 50 gm was put on weight hanger and deflections were observed.

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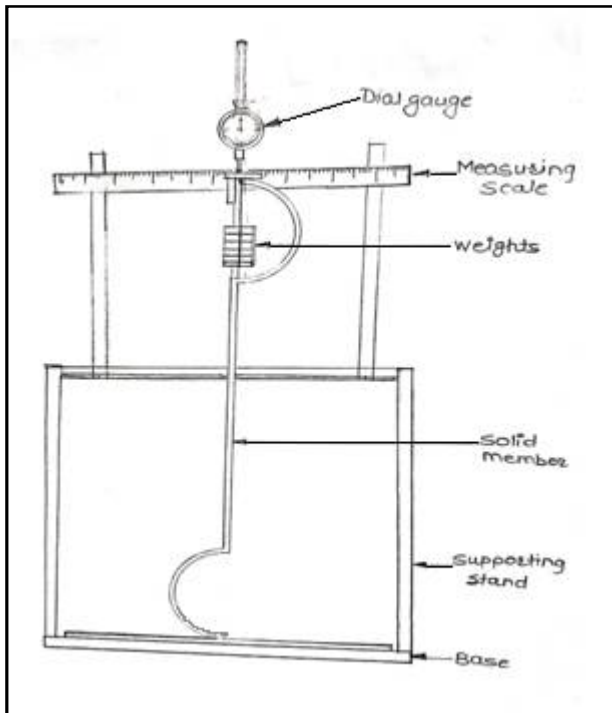


Fig (2): Experimental Setup



Fig (3): Actual Setup

Actual experimental setup is designed as per the dimensions shown in fig (1).

IV.1 EXPERIMENTAL RESULTS

Table (1) Observation Table (1)

| Sr.No | Weight (gm) | Horizontal Deflection (mm) | Vertical Deflection (mm) |
|-------|-------------|----------------------------|--------------------------|
| 1 | 100 | 4 | 0.6 |
| 2 | 200 | 7 | 1.3 |
| 3 | 300 | 10.1 | 1.8 |
| 4 | 400 | 15 | 2.5 |
| 5 | 500 | 18 | 3.1 |

The table (1) shows the horizontal and vertical deflection obtained from experimental setup after successive increase in weight starting from 100 grams to the 500grams.

V. RESULTS AND DISCUSSION

V.1 ANSYS VALIDATION

1) Ansys results for 1N force –

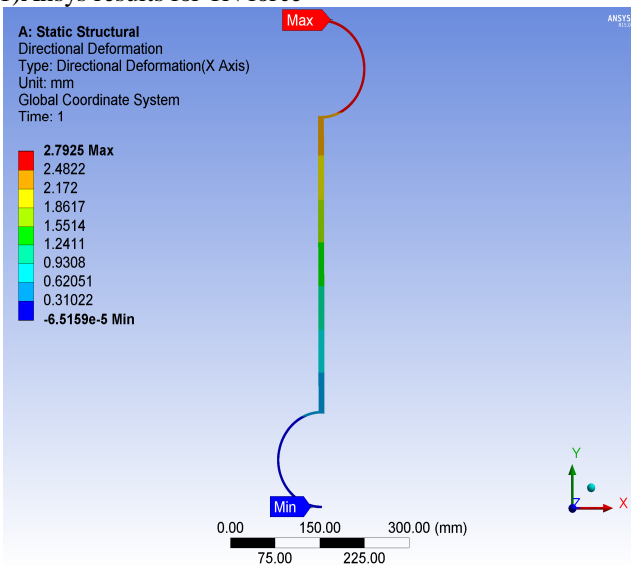


Fig (4): Horizontal Deflection for 1N force

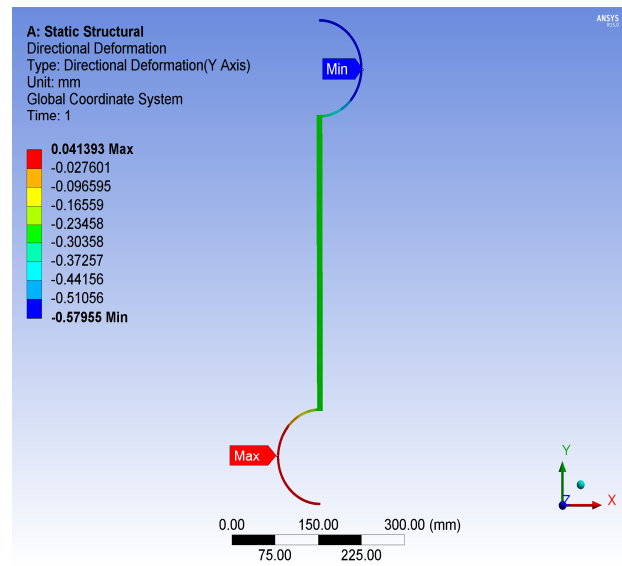


Fig (5): Vertical Deflection for 1N force

From fig (4) and (5), For 1N force the deflection of top end of beam is 2.79 mm in horizontal direction and 0.53 mm in vertical direction. Type of mesh used is coarse mesh (free mesh), minimum element size is 3 mm, minimum edge length is 4 mm, elements=1518, node=10770.

2) For 4N force –

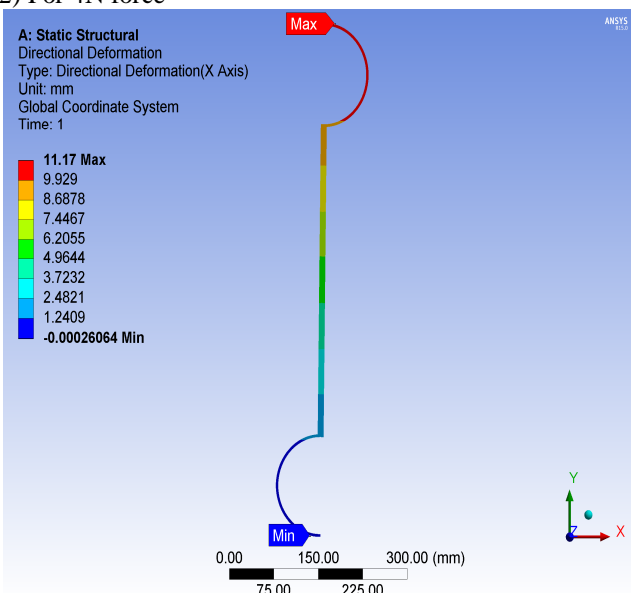


Fig (6): Horizontal Deflection for 4N force

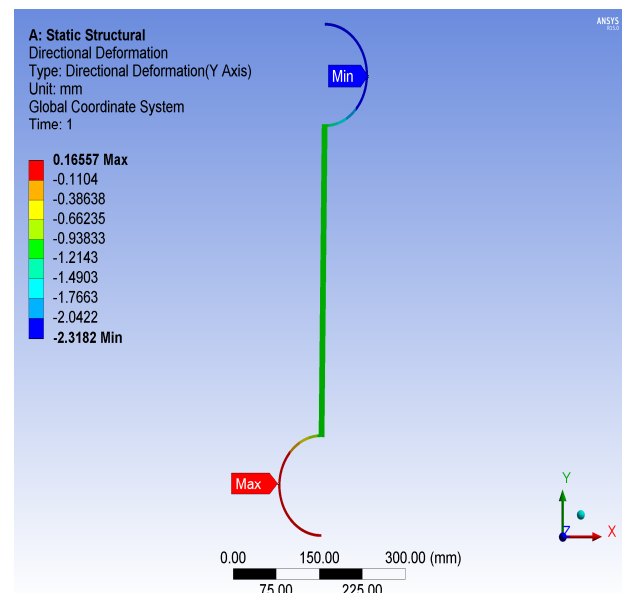


Fig (7): Vertical Deflection for 4N force

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From fig (6) and (7), for 4N force the deflection of top end of beam is 11.17 mm in horizontal direction and 2.15 mm in vertical direction. The type of mesh used is coarse mesh (free mesh), minimum element size is 3 mm, minimum edge length is 4 mm, elements=1518, node=10770.

V.2 SUMMARY OF ALL RESULTS

Table (2): Result Table

| Sr. No. | Weight (gm) | Horizontal Deflection (mm) | | | Vertical Deflection (mm) | | |
|---------|-------------|----------------------------|-------|--------------|--------------------------|-------|--------------|
| | | Analytical | Ansys | Experimental | Analytical | Ansys | Experimental |
| 1 | 100 | 3.01 | 2.79 | 4 | 0.59 | 0.53 | 0.6 |
| 2 | 200 | 6.02 | 5.58 | 5 | 1.18 | 1.15 | 1.3 |
| 3 | 300 | 9.04 | 8.37 | 7 | 1.77 | 1.73 | 1.8 |
| 4 | 400 | 12.05 | 11.17 | 10 | 2.36 | 2.15 | 2.5 |
| 5 | 500 | 15.06 | 13.96 | 13 | 2.95 | 2.69 | 3.1 |

From the table (2) it indicates that, the horizontal deflection is more compared to vertical deflection. The results of analytical solutions are in close match with results obtained from approach of ansys. The error in both results is 7-9% only and the results obtained from experimental setup are also in close match with the analytical and ansys.

V.3 COMPARISON OF ALL RESULTS

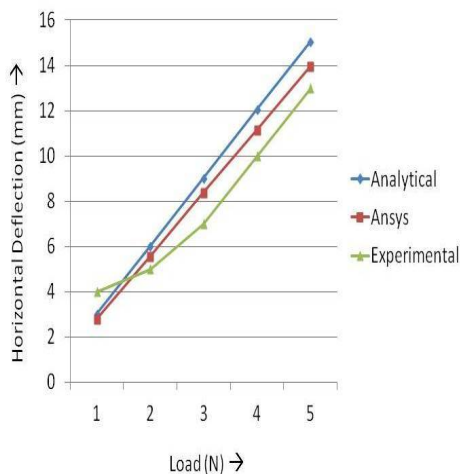


Fig (8): Horizontal Deflection vs. Load

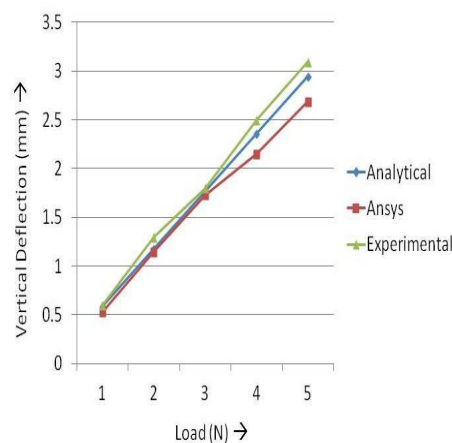


Fig (9): Vertical Deflection vs. Load

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From the fig (8) and (9), it shows that the graph of both horizontal and vertical deflection vs. load is linear. In case of analytical and ansys the graph is nearly coincident but it slightly differs from experimental results due to parallax errors.

VI. CONCLUSIONS

From analytical and experimental values, following conclusions can be drawn.

1. As load increases, correspondingly horizontal and vertical deflection increases upto the maximum value of 3.1 mm for vertical and 18 mm for horizontal.
2. It was observed that experimental values for horizontal and vertical deflection for beam are lower than analytical and ansys.
3. The results of analytical solutions are in close match with results obtained from experimental readings. The error in both results is less than 10%.
4. This error was due to parallax error in experimental setup measurement, while measuring horizontal deflection.

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