

Sensing of Wideband Spectrum Channels by Sub Sampling Rate Subspace Estimator

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ABSTRACT: The work will focus the area of spectrum sensing methodology used in cognitive radio. The main challenge in the spectrum sensing is the need of a very high sampling rate for sensing the wideband signal. Hence a wideband spectrum sensing model will be developed using MATLAB software based programming environment. The algorithm will utilize a sub sampling scheme that can help in reducing the requirement of very high sampling rate during wideband spectrum sensing. For this purpose a finite number of noisy samples will be generated initially and the correlation of these finite samples will be evaluated and to find out the occupied and vacant channels of the spectrum, we use subspace estimation algorithm.

KEYWORDS: Cognitive Radio, Sub Nyquist Sampling, Primary User, Secondary User, Multicoset Sampling.

I. INTRODUCTION

Radio frequency (RF) spectrum is a precious but tightly regulated resource due to its unique and very important role in wireless communications. With the expansion of wireless services, the requirements for the RF spectrum are constantly increasing, leading to scarce spectrum resources. On the other hand, it has been reported that localized temporal and geographic spectrum utilization is extremely low [1]. New spectrum policies are being evolved by the Federal Communications Commission (FCC) that will permit secondary users to opportunistically access a licensed band, when the primary user (PU) is absent. Cognitive radio [2], [3] has become a reliable solution to solve the spectrum scarcity problem in the next generation cellular networks by utilizing opportunities in time, frequency and space domains. Cognitive radio is an advanced software-defined radio that automatically recognizes its surrounding RF stimuli and adapts intelligently its operating parameters to network infrastructure while meeting user demands. Since cognitive radios are also considered as secondary users for using the licensed spectrum, a critical requirement of cognitive radio networks is to efficiently utilize under-utilized spectrum (denoted as spectral opportunities) without causing any harmful interference to the PUs. PUs has no obligation to share and change their operating parameters for sharing spectrum with cognitive radio networks. Hence, cognitive radios should be able to detect spectral opportunities independently without any help from PUs; this ability is called spectrum sensing, which is one of the most important components in cognitive radio networks. Many narrowband spectrum sensing algorithms have been proposed in the literature [4] and references therein, including matched-filtering, energy detection [5], and cyclostationary feature detection. While present narrowband spectrum sensing algorithms have dealt with exploiting spectral opportunities over narrow frequency range, cognitive radio networks will eventually be required to utilize spectral opportunities over wide frequency range from hundreds of megahertz (MHz) to several gigahertz (GHz) for achieving higher opportunistic throughput. This is driven by the famous Shannon's formula that, under certain conditions, the maximum theoretically achievable bit rate is directly proportional to the spectral bandwidth. Hence, unlike narrowband spectrum sensing, wideband spectrum sensing goals to find more spectral opportunities over wide frequency range and achieve higher opportunistic aggregate throughput in cognitive radio networks. However, traditional wideband spectrum sensing techniques based on standard analog-to-digital converter (ADC) could lead to unaffordable high sampling rate or implementation complexity; thus, revolutionary wideband spectrum sensing techniques become increasingly important. The traditional methods for finding out the occupied and vacant channels depend upon knowledge of signal properties, hence it create a large amount of uncertainty in the estimation outcome. Our proposed method will be independent of

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signal since it works on any self generated test noisy signals. Hence the uncertainty in estimation results can be reduced up to a large extent; hence we can enhance the vacant and occupied channels detection accuracy without using a high sensing at high sampling rate.

II. RELATED WORK

Yvan Lamelas Polo, Ying Wang, Ashish Pandharipande, and Geert Leus, (2009) [6], presented a compressive wide-band spectrum sensing scheme for cognitive radios. The received analog signal at the cognitive radio sensing receiver is transformed into a digital signal using an analog-to-information converter. The autocorrelation of this compressed signal is then used to reconstruct an estimate of the signal spectrum. Then evaluate the performance of this scheme in terms of the mean squared error of the power spectrum density estimate and the probability of detecting signal occupancy. It presented a compressive wide-band spectrum sensing scheme wherein an AIC operates on the received analog signal. Performance evaluation using MSE and probability of detection showed that the proposed scheme performs comparably to the scheme based on **Z. Tian and G. B. Giannakis[15]**. The loss in incoherence thus does not substantially affect spectrum estimation and spectrum occupancy detection.

For cognitive radio networks, efficient and robust spectrum sensing is a crucial enabling step for dynamic spectrum access. Cognitive radios need to not only rapidly identify spectrum opportunities over very wide bandwidth, but also make reliable decisions in noise-uncertain environments. Cyclic spectrum sensing techniques work well under noise uncertainty, but require high-rate sampling which is very costly in the wideband regime.

Zhi Tian, et. al. (2012) [7], developed robust and compressive wideband spectrum sensing techniques by exploiting the unique sparsity property of the two-dimensional cyclic spectra of communications signals. For this, a new compressed sensing framework is proposed for extracting useful second-order statistics of wideband random signals from digital samples taken at sub-Nyquist rates. The time-varying cross-correlation functions of these compressive samples are formulated to reveal the cyclic spectrum, which is then used to simultaneously detect multiple signal sources over the entire wide band. Because the proposed wideband cyclic spectrum estimator utilizes all the cross-correlation terms of compressive samples to extract second-order statistics, it is also able to recover the power spectra of stationary signals as a special case, permitting lossless rate compression even for non-sparse signals

Hongjian Sun et. al. (2013) [8] proposed a Multi-rate asynchronous sub-Nyquist sampling (MASS) for wideband spectrum sensing. Corresponding spectral recovery conditions are derived and the probability of successful recovery is given. Compared to previous approaches, MASS offers lower sampling rate, and is an attractive approach for cognitive radio networks.

Hongjian Sun and Arumugam Nallanathan, (2013) [9] Cognitive radio has emerged as one of the most promising candidate solutions to improve spectrum utilization in next generation cellular networks. A crucial requirement for future cognitive radio networks is wideband spectrum sensing: secondary users reliably detect spectral opportunities across a wide frequency range. The author presented various wideband spectrum sensing algorithms but special attention is paid to the use of sub-Nyquist techniques, including compressive sensing and multi-channel sub-Nyquist sampling techniques.

Chia-Pang Yen et. al. (2013) [10], consider the problem of locating multiple active spectrum sub bands in a wide range of frequency bands. A major challenge associated with such wideband spectrum sensing is that it is either infeasible or too expensive to perform Nyquist sampling on the wideband signal. They proposed a sensing scheme based on a sub-Nyquist sampling method called multicorset sampling, which is similar to the polyphase implementation of the Nyquist sampling, but requires less A/D converters. In contrast to the traditional sub-Nyquist approaches, where the wideband signal is first reconstructed from the sub-Nyquist samples, it developed a method that directly estimates the power spectrum of the wideband signal of interest using the sub-Nyquist samples, by exploiting its statistical properties. It also characterizes the statistical distribution of the proposed power spectrum estimator.

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III. METHODOLOGY

We wish to perfectly reconstruct $x(t) \in M$ from its point-wise samples under two constraints. One is blindness, so that the information about the band locations is not used while acquiring the samples and neither can it be used in the reconstruction process. The other is that the sampling rate required to guarantee perfect reconstruction should be minimal. This problem is solved if either of its constraints is removed. Without the rate constraint, the WKS theorem allows perfect blind-reconstruction for every signal $x(t)$ bandlimited to F from its uniform samples at the Nyquist rate $x(t = n/T)$. Alternatively, if the exact number of bands and their locations are known, then the method of [12] allows perfect reconstruction for every multi-band signal at the minimal sampling rate provided by Landau's theorem [11].

We characterized the signal for sampling as

$$y(f) = Ax(f), \forall f \in F_0 \dots\dots\dots (1)$$

Where $A_{ik} = \frac{1}{LT} \exp(j \frac{2\pi}{L} ci * k)$ in matrix form.

MINIMAL SAMPLING RATE:

We begin with Landau's theorem for the minimal sampling rate of an arbitrary sampling method that permits known-spectrum perfect reconstruction. It is then proved that blind perfect reconstruction requires a minimal sampling rate that is twice the Landau rate.

A. Known spectrum support:

Consider the space of band limited functions restricted to a known support $T \subseteq F$:

$$B_T = \{x(t) \in L^2(\mathbb{R}) \mid \text{supp } X(f) \subseteq T\} \dots\dots\dots (2)$$

A classical sampling scheme takes the values of $x(t)$ on a known countable set of locations $R = \{r_n\}$ for $-\infty < n < \infty$. The set R is called a *sampling set* for B_T , if $x(t)$ can be perfectly reconstructed in a stable way from the sequence of samples $x_R[n] = x(t = r_n)$. The stability constraint requires the existence of constants $\alpha > 0$ and $\beta < \infty$ such that:

$$\alpha \|x - y\|^2 \leq \|x_R - y_R\|^2 \leq \beta \|x - y\|^2, \forall x, y \in B_T \dots\dots\dots (3)$$

Landau [11] proved that if R is a sampling set for B_T then it must have a density $D^-(R) \geq \lambda(T)$, where

$$D^-(R) = \lim_{r \rightarrow \infty} \inf_{y \in \mathbb{R}} \frac{|R \cap [y, y+r]|}{r} \dots\dots\dots (4)$$

is the lower Beurling density, and $\lambda(T)$ is the Lebesgue measure of T . The numerator in (4) counts the number of points from R in every interval of width r of the real axis. This result is usually interpreted as a minimal *average* sampling rate requirement for B_T , and $\lambda(T)$ is called the Landau rate.

B. Unknown Spectrum Support:

Consider the set N of signals bandlimited to F with bandwidth occupation no more than $0 < \Omega < 1$, so that

$$\lambda(\text{supp } X(f)) \leq \frac{\Omega}{T}, \forall x(t) \in N_\Omega$$

The Nyquist rate for N is $1/T$. Note that N is not a subspace so that the Landau theorem is not valid here. Nevertheless, it is intuitive to argue that the minimal sampling rate for N cannot be below T as this value is the Landau rate had the spectrum support been known. A blind sampling set R for N is a sampling set whose design does not assume knowledge of $\text{supp } X(f)$.

Similarly to (3) the stability of R requires the existence of $\alpha > 0$ and $\beta < \infty$ such that:

$$\alpha \|x - y\|^2 \leq \|x_R - y_R\|^2 \leq \beta \|x - y\|^2, \forall x, y \in N_\Omega \dots\dots\dots (5)$$

Theorem 1 (Minimal sampling rate): Let R is a blind sampling set for N . Then,

$$D^-(R) \geq \min \left\{ \frac{2\Omega}{T}, \frac{1}{T} \right\} \dots\dots\dots (6)$$

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SPECTRUM-BLIND RECONSTRUCTION

We, firstly, develop the theory needed for SBR. These results are then used in to construct two efficient algorithms for blind signal reconstruction. The theoretical results are explained in the following steps: We first note that when considering blind-reconstruction, we cannot use the prior of [13]. Using this we develop a sufficient condition for blind perfect reconstruction which is

$$x(f) \text{ is } \sigma(A) - \text{ sparse}, \forall f \in F_0 \dots\dots\dots (7)$$

Where $\sigma(A)$ is Krushal Rank of matrix A. Furthermore, we find the value of L, p, C for perfect reconstruction under certain conditions for every signal in subset M.

Conditions for blind perfect reconstruction

For every $f \in F_0$ the system of (1) is undetermined since there are fewer equations than unknowns. The prior assumed in this paper is that for every $f \in F_0$ the vector $x(f)$ is sparse but in contrast to [13] the location of the non-zero values is unknown. In this case

$$x(f) \text{ is } p - \text{ sparse}, \forall f \in F_0 \dots\dots\dots (8)$$

is still necessary for blind perfect reconstruction as found in known-spectrum perfect reconstruction. The following theorem from the CS literature is used to provide a sufficient condition.

Theorem 2 : Suppose x is a solution of $y = Ax$. If $\|x\|_0 \leq \sigma(A)/2$ then x is the unique sparsest solution of the system. Note that the condition of the theorem is not necessary as there are examples that the sparsest solution x of $y = Ax$ is unique while $x > \sigma(A)/2$. Using above Theorem 2, it is evident that perfect reconstruction is possible for every signal satisfying

$$x(f) \text{ is } \frac{\sigma(A)}{2} - \text{ sparse}, \forall f \in F_0 \dots\dots\dots (9)$$

As before, choosing a universal pattern makes the set of signals that conform with (9) the widest possible. Note that a factor of two distinguishes between the sufficient conditions of (7) and of (9), and results from the fact that here we do not know the locations of the non-zero values in $x(f)$. Note that (9) provides a condition under which perfect reconstruction is possible; however, it is still unclear how to find the original signal. Although the problem is similar to that described in the CS literature, here finding the unique sparse vector must be solved for each value f in the continuous interval F_0 , which clearly cannot be implemented.

In practice, conditions (7) and (9) are hard to verify since they require knowledge of $x(t)$ and depend on the parameters of the multicoset sampling. We therefore prefer to develop conditions on the class M which characterizes multi-band signals based on their intrinsic properties: the number of bands and their widths. It is more likely to know the values of N(no. of frequency) and B(bandwidth) in advance than to know if the signals to be sampled satisfy (7) or (9). The following theorem describes how to choose the parameters L, p and C so that the sufficient conditions for perfect reconstruction hold true for every $x(t) \in M$. The theorem is valid for both known and blind reconstruction with a slight difference resulting from the factor of two in the sufficient conditions.

Theorem 3 (Uniqueness): Let $x(t) \in M$ be a multi-band signal. If:

- 1) The value of L is limited by

$$L \leq \frac{1}{BT} \dots\dots\dots (10)$$
- 2) $p \geq N$ for known reconstruction or $p \geq 2N$ for blind,
- 3) C is a universal pattern,

Then, for every $f \in F_0$, the vector $x(f)$ is the unique solution of (1).

Proof: If L is limited by (10) then for the i^{th} band $T_i = [a_i, b_i]$ we have

$$\lambda(T_i) \leq B \leq \frac{1}{LT}, 1 \leq i \leq N$$

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Therefore, $f \in T_i$ implies

$$f + \frac{k}{LT} \notin T_i, \forall k \neq 0$$

For every $f \in F_0$ the vector $x(f)$ takes the values of $X(f)$ on a set of L points spaced by $1/LT$. Consequently, the number of non-zero values in $x(f)$ is no greater than the number of the bands, namely $x(f)$ is N -sparse. Since C is a universal pattern, $\sigma(A) = p$. This implies that conditions (7) and (9) are satisfied.

The Beurling lower density (i.e. the average sampling rate) of a multicoset sampling set is

$$\frac{1}{T_{avg}} = \frac{p}{LT} \dots\dots\dots (11)$$

Note that the condition on the value of p implies the minimal sampling rate requirement. To see this, substitute (10) into (11):

$$\frac{1}{T_{avg}} = \frac{p}{LT} \leq pB \dots\dots\dots (12)$$

If the signals are known to lie in M then the Landau rate is NB , which is implied by $p \geq N$. Theorem 1 requires an average sampling rate of $2NB$, which can be guaranteed if $p \geq 2N$.

IV. RESULT AND DISCUSSION

This section covers the results that are obtained by our channel spectrum sensing by subspace estimation algorithm. Our first step is to generate an input signal containing multiple frequencies.

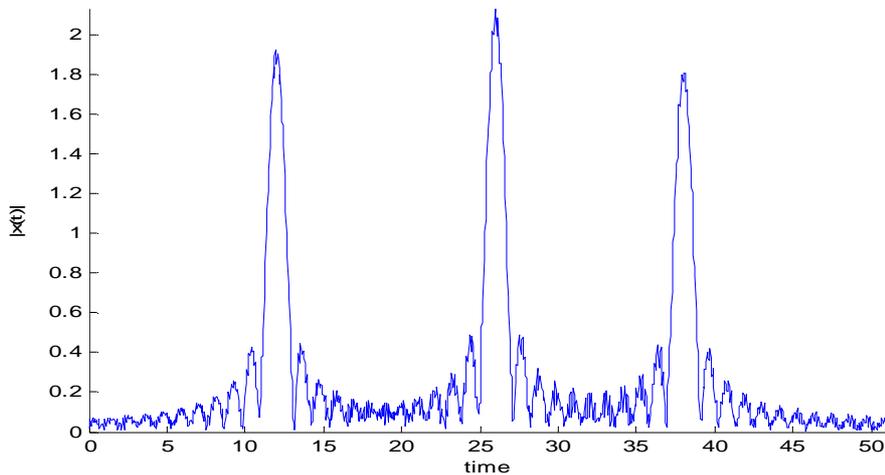


Figure 1: Time domain plot of signal x(t) having multiple frequencies.

The figure 1 shows that a signal is generated as $x(t)$ having summation of three sinc functions at different time delay ‘ t_i ’ and instantaneous frequency f_i with following parameters:

$$x(t) = \sum E_i(n)^{1/2} * B_i(n) * \text{sinc}(B_i(n)*(t-t_i(n))) * \exp(2j*\pi*f_i(n)*t)$$

$f_i = [5.2 \quad 11.4 \quad 16.6]$ Mega hertz;

$t_i = [12 \quad 26 \quad 38]$

$E_i = [4 \quad 4.8 \quad 3.6 \quad 5.2]$

$B_i = 0.9 \quad 0.9 \quad 0.9$;

$F_s = 20$ (Sampling frequency)

$T = 0.05$ sec (Sample time)

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LL = 1024 (Length of signal)
t = 0 to 51.5 sec (Time vector representing)
NFFT = Next power of 2 from length of x

The total no. of active frequency are $N=3$, thus the maximum channel occupancy are $\Omega_{max}=N_{max}/L=0.135$ and maximum number of active channels are $L=Fs/B=22$. The maximum number of active channel cells are $p=7$ for sub Nyquist factor $\alpha =p/L$ and $\alpha > \Omega_{max}$

The multicaset sample set are found as

$$C = [0 \quad 1 \quad 2 \quad 10 \quad 11 \quad 12 \quad 13]$$

When signal is passed through these sample set, we have gotten a down sampling matrix of order 7×1024 . Traditionally, number of sources is being equivalent to the number of eigen value.

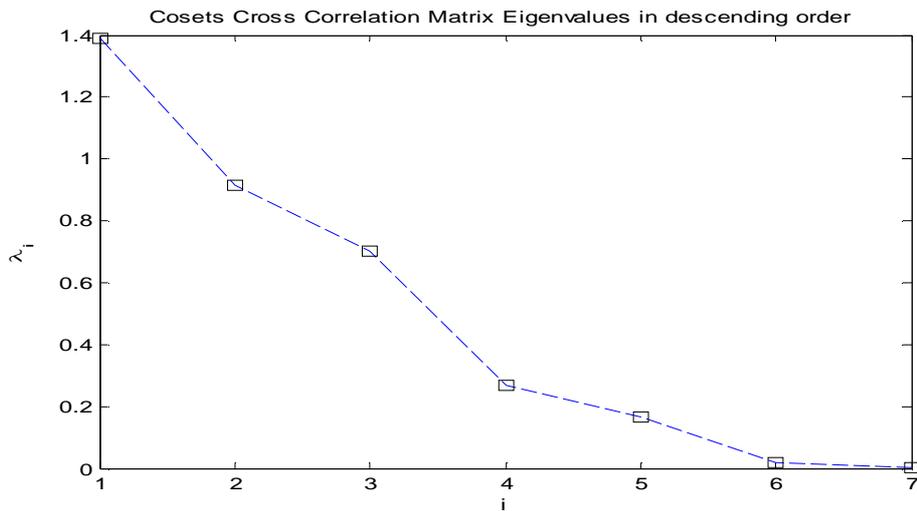


Figure 2: Cosets auto correlation matrix eigen values (λ_i) plot in the descending order

Figure 2 shows the corresponding eigen values of the correlation matrix obtained from down sampled signal in descending order. We have gotten 7 eigen values.

Eigen values of $R_{xx} = \lambda_i = [0.00062478, 0.018077, 0.16656, 0.26725, 0.70339, 0.91458, 1.3895]$.

Several tests have been proposed for determining the number of sources in the presence of statistical fluctuations. The most common of these tests, are the Akaike information criterion (AIC) and Rissanen’s minimum description length (MDL) criterion.

AIC method is to provide the order of a model using information theory. Using the expression

$$AIC(m) = -N(M - m) \log \left(\frac{g(m)}{a(m)} \right) + m(2M - m) \dots \dots \dots (13)$$

The number of sources is integer, which, for $m \in \{0, 1, \dots, M - 1\}$, minimizes the following quantity. Where $g(m)$ and $a(m)$ are the geometric and arithmetic means of the $(M - m)$ smallest eigen values of the covariance matrix of the observation respectively. The first term belongs to the log-likelihood residual error, while the other is a penalty for over-fitting. This criterion does not determine the true number of sources with a probability of one, even with an infinite number of samples. The MDL approach is also based on information theoretic arguments, and the selected model order is the one which minimizes the code length needed to describe the data.

$$MDL(m) = -N(M - m) \log \left(\frac{g(m)}{a(m)} \right) + (1/2)m(2M - m) \log N \dots \dots \dots (14)$$

In this paper we use the form of the MDL given in [14]

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MDL = [1768.7 639.25 330.51 113.42 87.039 72.247]

Number of significant values of MDL represents the non-zero sources are 6. Since there are 7 eigen vectors, the noise signal related vectors are represented as the eigen vectors out off significant vectors given as neglecting significant 6 eigen vectors. The noise eigen vector is found as -

$$U_n = \begin{bmatrix} -0.038371 - 0.10129i \\ 0.063955 + 0.14176i \\ -0.0031033 + 0.072472i \\ 0.42839 - 0.3801i \\ 0.40555 - 0.067345i \\ 0.32247 - 0.21121i \\ 0.55967 \end{bmatrix}$$

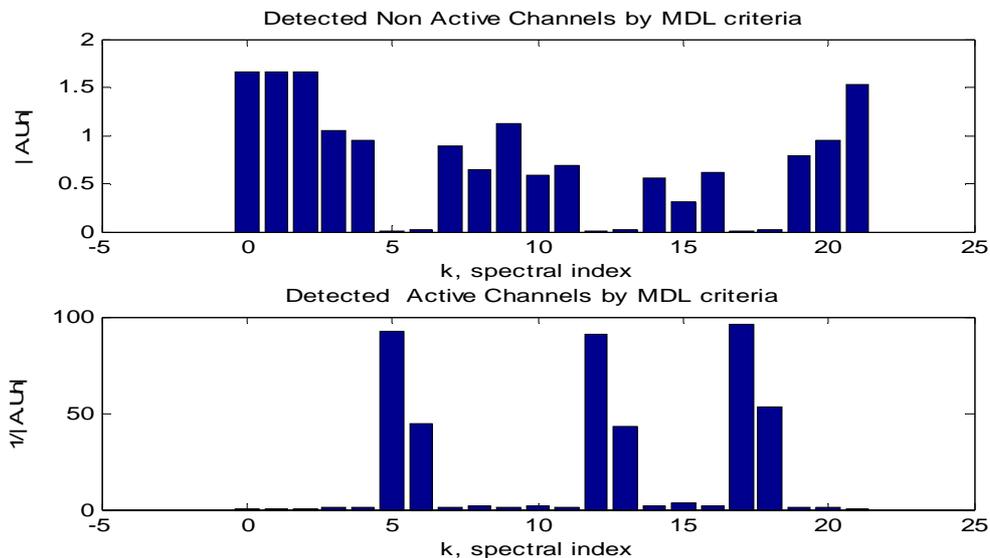


Figure 3: Signal representing noise channels (Top) representation and signal channel (bottom) representation

Figure 3 represents the signal and noise channel representation of signal we taken in figure 1.

Here the signal that represents only the noise can be obtained by matrix multiplication of A (described in eq. 1) with U_n (noise eigen vector associated with signal) given as:

$$A \cdot U_n = [1.6565 \ 1.6665 \ 1.6525 \ 1.0478 \ 0.94349 \ 0.010861 \ 0.022289 \ 0.88596 \ 0.63902 \ 1.125 \\ 0.58591 \ 0.69512 \ 0.011028 \ 0.02322 \ 0.56427 \ 0.3167 \ 0.61895 \ 0.010422 \ 0.018866 \ 0.78628 \ 0.95499 \\ 1.524].$$

Thus the channels representing the signal only are:

$$1/U_n \cdot A = [0.60367 \ 0.60006 \ 0.60516 \ 0.95434 \ 1.0599 \ 92.071 \ 44.864 \ 1.1287 \ 1.5649 \ 0.88887 \ 1.7067 \\ 1.4386 \ 90.678 \ 43.067 \ 1.7722 \ 3.1576 \ 1.6156 \ 95.949 \ 53.004 \ 1.2718 \ 1.0471 \ 0.65618]$$

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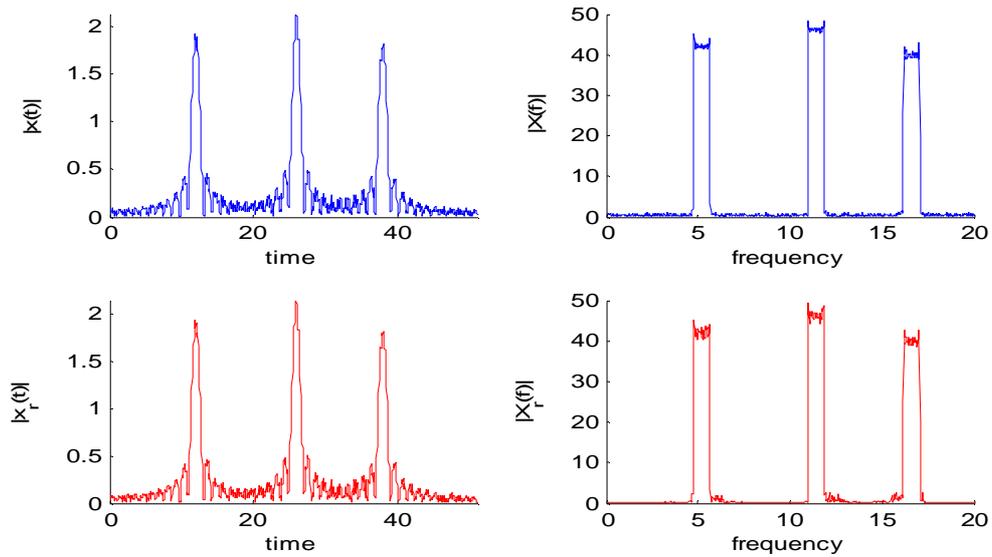


Figure 4: Original Signal top left in time domain and in frequency domain (top right) reconstructed signal using column restriction in time and frequency domain (bottom left and right).

After finding the channels that represents signal, we reconstruct the original signal $x(t)$. The reconstructed signal are denoted as $x_r(t)$. Figure 4 represent the original signal and reconstructed signal in the time domain as well as in frequency domain.

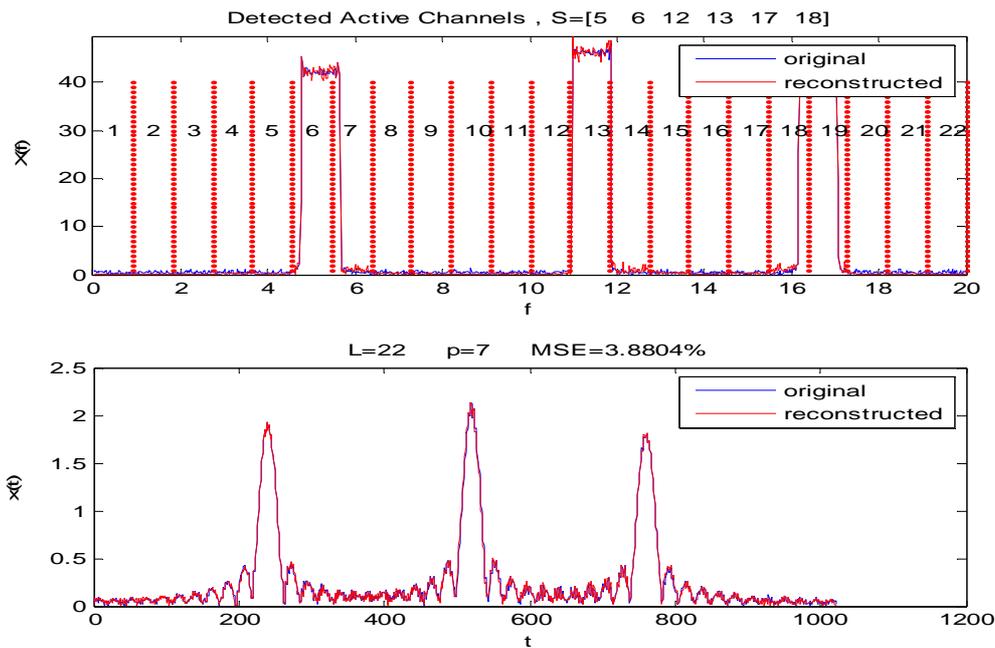


Figure 5: Reconstructed signal obtained using the interpolation in frequency domain (top) and time domain (bottom).

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Figure 5 shows the original noisy signal and the reconstructed signal using column restriction matrix. In this figure the MSE value is also shown having value of 3.88% approx. We can observe that original signal is in blue color with noise and reconstructed signal is in red, both are complete superimposing thus it represents perfect signal reconstruction and the detected active channels are 5th, 6th, 12th, 13th, 17th and 18th.

V. CONCLUSION

This work is related to the development of a signal reconstruction scheme for multi-band signals by assuming the blind multicorset sampling satisfying the minimal sampling rate desirability. Programming based environment using MATLAB is developed for simulating the continuous nature of the reconstruction approach into a finite dimensional problem without any losses occurred due to discretization. We have proved that the solution can be obtained by finding the unique sparsest solution matrix using column restriction matrix. The work composed of a compressive wide-band spectrum sensing method operates on the received analog signal. Spectrum estimation is done based on multicorset approach based reconstruction by the use of autocorrelation function of the resulting compressed signal. The estimated spectrums are applied for detecting the spectrum occupancy of the system. Performance evaluation is performed by MSE and probability of detection proves that the proposed scheme performs comparably to the other past scheme. It proves that loss in incoherence due to lower sampling rate does not significantly distort the spectrum estimation and spectrum occupancy measurements.

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