

# Interesting Properties of strongly $p^*g\alpha$ Connected Spaces and Compact Spaces

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**ABSTRACT:** The aim of this paper is to introduce and study the concept of strongly pre stargeneralized alpha compactness and strongly prestartgeneralized alpha-connectedness which is stronger than the notions of connectedness of topological spaces. Some characterizations of these concepts are discussed.

**KEYWORDS:** Strongly prestar generalizedalpha connected spaces.Strongly pre star generalized compact spaces.

## I. INTRODUCTION

In 1970, Levine (1970) introduced the concepts of generalized closed sets of topological spaces as a generalization of closed sets. Mashhour et al., (1982) studied the concept of preopen sets in topological spaces. The purpose of the present paper is to introduce and investigate the concepts of strongly prestartgeneralized compactness and strongly prestart generalized connectedness which is stronger than the concepts of connectedness of topological spaces.

## II. PRELIMINARIES

Throughout this paper  $(X, \tau), (Y, \sigma)$  will always denote topological spaces on separation axioms are assumed, unless otherwise mentioned when  $A$  is a subset of  $(X, \tau), cl(A)$  and  $int(A)$  denote the closure and interior respectively. We have to recall some known results.

### DEFINITION 2.1:

1. A generalized closed (briefly  $g$ -closed) set [7] if  $cl(A) \subseteq U, A \subseteq U$  and  $U$  is open in  $(X, \tau)$ .
2. A generalized preclosed (briefly  $gp$ -closed) set [16] if  $pcl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is open in  $(X, \tau)$ .
3. A generalized preregular closed (briefly  $gpr$ -closed) set [15] if  $pcl(A) \subseteq U$  whenever  $A \subseteq U$ .
4. A subset  $A$  of  $(X, \tau)$  is said to be  $p^*g\alpha$ -closed if  $pcl(A) \subseteq U$  whenever  $A \subseteq U$  and  $U$  is  $*g\alpha$ -open in  $(X, \tau)$ .

### DEFINITION-2.2:

A mapping  $f(X, \tau) \rightarrow (Y, \sigma)$  is called

- (i) Strongly  $p^*g\alpha$ -continuous – if the inverse image of every closed set of  $(Y, \sigma)$  is strongly  $p^*g\alpha$  closed in  $(X, \tau)$ .
- (ii) A strongly  $p^*g\alpha$  irresolute if  $f^{-1}(V)$  is strongly  $p^*g\alpha$  closed in  $(X, \tau)$  for every strongly  $p^*g\alpha$  closed set  $V$  of  $(Y, \sigma)$ .

### DEFINITION 2.3:

A mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  is called

- i.  $P$ -preopen if  $f(v)$  is a pre-open set of  $(Y, \sigma)$ , for each  $v$  is pre-open set in  $(X, \tau)$ .
- ii. Pre-strongly  $p^*g\alpha$ -closed, if the image of each strongly  $p^*g\alpha$ -closed set of  $(X, \tau)$  is strongly  $p^*g\alpha$ -closed in  $(Y, \sigma)$ .
- iii. Pre-strongly  $p^*g\alpha$ -open if the image of each strongly  $p^*g\alpha$ -open set in  $(X, \tau)$  is strongly  $p^*g\alpha$ -open in  $(Y, \sigma)$ .
- iv. Super –strongly  $p^*g\alpha$ -open if the image of each strongly  $p^*g\alpha$ -open set of  $(X, \tau)$  is open in  $(Y, \sigma)$ .

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## DEFINITION 2.4[10]:

A space  $(X, \tau)$  is called strongly compact [10] if every cover of  $S$  by preopen sets of  $(X, \tau)$  has a finite sub cover.

## LEMMA 2.5:

In a topological space  $(X, \tau)$ . Then :

- (i) Every pre-open set is strongly  $p^*g\alpha$  – open.
- (ii) Every strongly  $p^*g\alpha$ -open set is  $p^*g\alpha$ -open.

## DEFINITION 2.6:

- (iii) A collection  $\{A_i : i \in I\}$  of strongly  $p^*g\alpha$ -open sets in a topological space  $(X, \tau)$  is called a strongly pre-open cover of a subset  $B$  of  $(X, \tau)$  if  $B \subseteq \bigcup \{A_i : i \in I\}$ .
- (iv) A topological space  $(X, \tau)$  is strongly  $p^*g\alpha$ - compact if every strongly  $p^*g\alpha$  open cover of  $X$  has a finite sub cover.
- (v) A subset  $S$  of a space  $(X, \tau)$  is strongly  $p^*g\alpha$ -compact relative to  $X$ , if every cover of  $S$  by strongly  $p^*g\alpha$ -open sets of  $X$  has a finite sub cover.
- (vi) A subset  $B$  of a topological space  $(X, \tau)$  is said to be strongly  $p^*g\alpha$  compact if  $B$  is strongly  $p^*g\alpha$  compact as a subset of  $X$ .

## III. PROPERTIES OF STRONGLY $p^*g\alpha$ COMPACT SPACES

### Theorem 3.1:

For a space  $(X, \tau)$ , the following statements are equivalents:

- (i)  $(X, \tau)$  is strongly  $p^*g\alpha$ -compact.
- (ii) Any family of strongly  $p^*g\alpha$ -closed subsets of  $(X, \tau)$  satisfying finite intersection property.
- (iii) Any family of strongly  $p^*g\alpha$ -closed subset of  $(X, \tau)$  with empty intersection has a finite subfamily with empty intersection.

### Proof:

(i) $\Rightarrow$ (ii)

Let  $\{P_i : i \in I\}$  be a family of strongly  $p^*g\alpha$ -closed subsets of  $(X, \tau)$  which satisfy the finite intersection property. To prove that  $\bigcap_{i \in I} P_i \neq \emptyset$ . Suppose that the converse.  $\bigcap_{i \in I} P_i = \emptyset$ . Then  $X = \bigcup_{i \in I} P_i^c$  and therefore  $\{P_i^c : i \in I\}$  is a strongly  $p^*g\alpha$ -open cover of  $(X, \tau)$ . Thus by hypothesis, there exists a finite subset  $I_0$  of  $I$  such that  $X = \bigcup_{i \in I_0} P_i^c$  and hence  $\bigcap_{i \in I_0} P_i \neq \emptyset$  Which is a contradiction with the assumption. Therefore  $\bigcap_{i \in I} P_i \neq \emptyset$

(ii) $\Rightarrow$ (i) suppose that  $\{G_i : i \in I\}$  is a strongly  $p^*g\alpha$ -open cover of  $(X, \tau)$  and  $(X, \tau)$  is not strongly  $p^*g\alpha$ -compact. Then there exists a strongly  $p^*g\alpha$ -open cover of  $(X, \tau)$  has no finite subcover. Hence for a finite subset  $I_0$  of  $I$ ,  $\bigcup_{i \in I_0} G_i \neq X$  therefore  $\bigcap_{i \in I_0} G_i^c \neq \emptyset$  and thus the family  $\{G_i^c : i \in I\}$  satisfy the finite intersection property which is a contradiction with the assumption and hence  $(X, \tau)$  is strongly  $p^*g\alpha$  compact.

(i) $\Rightarrow$ (iii) let  $\{P_i : i \in I\}$  be a collection of strongly  $p^*g\alpha$ -closed subset of  $(X, \tau)$  with  $\bigcap_{i \in I} P_i = \emptyset$ , hence  $\bigcup_{i \in I} P_i^c = X$ . so that the family  $\{P_i^c : i \in I\}$  is a strongly  $p^*g\alpha$ -open cover of  $(X, \tau)$ . Therefore by hypothesis, there exist a finite subset  $\{P_{i_k} : k = 1, 2, \dots, n\}$  of  $(X, \tau)$  such that  $X = \bigcup_{i \in I} P_{i_k}^c$  hence  $\bigcap_{k=1}^n P_{i_k} = \emptyset$ . Hence, there exists a finite sub family with empty intersection.

(iii) $\Rightarrow$ (i) suppose that  $\{G_i : i \in I\}$  is a collection of strongly  $p^*g\alpha$  open cover of  $(X, \tau)$ , such that  $\bigcup_{i \in I} G_i = X$ . Hence  $\bigcap_{i \in I} G_i^c = \emptyset$ . Thus there exist a collection of strongly  $p^*g\alpha$ -closed sub set  $\{G_i^c : i \in I\}$  of  $(X, \tau)$  such that  $\bigcap_{i \in I} G_i^c = \emptyset$ . Hence by hypothesis a finite sub collection  $\{G_{i_k} : k=1, 2, \dots, n\}$  of  $(X, \tau)$  such that  $\bigcap_{k=1}^n G_{i_k}^c = \emptyset$ . So  $X = \bigcup_{k=1}^n G_{i_k}$  and therefore  $(X, \tau)$  is strongly  $p^*g\alpha$ -compact.

### Theorem 3.2 :

A strongly  $p^*g\alpha$  continuous image of a strongly  $p^*g\alpha$  compact space is compact.

### Proof:

Let  $f : (x, \tau) \rightarrow (Y, \sigma)$  be a strongly  $p^*g\alpha$  continuous mapping from a strongly  $p^*g\alpha$  – compact space  $(x, \tau)$  on to topological space  $(y, \sigma)$ . Suppose that  $\{A_i : i \in I\}$  is open cover of  $(y, \sigma)$ . Then  $\{f^{-1}(A_i) : i \in I\}$  is a strongly  $p^*g\alpha$

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open cover of  $(X, \tau)$ . But  $(X, \tau)$  is strongly  $p^*ga$ -compact, then there exists a finite sub cover  $\{f^{-1}(A_1), f^{-1}(A_2), \dots, f^{-1}(A_n)\}$ , but  $f$  is onto, hence  $f(X) = Y = \{A_1, \dots, A_n\}$ . Hence  $(Y, \sigma)$  is compact.

**Theorem 3.3:**

The image of a strongly  $p^*ga$  compact subset under a strongly  $p^*ga$  irresolute mapping is strongly  $p^*ga$  – compact.

**Proof:**

Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a strongly  $p^*ga$ -irresolute mapping and  $\{V_i: i \in I\}$  be any family strongly  $p^*ga$  open cover of a subset  $f(G)$  of  $Y$ . Since  $f$  is strongly  $p^*ga$  irresolute, then  $\{f^{-1}(V_i): i \in I\}$  is strongly  $p^*ga$  open cover of a subset  $G$  of  $X$ . But  $G$  is strongly  $p^*ga$ -compact, then there exists a finite subset  $I_0$  of  $I$  Such that  $G \subseteq \bigcup_{i \in I_0} \{f^{-1}(V_i): i \in I_0\}$  which implies that  $G \subseteq f^{-1}(\bigcup_{i \in I_0} V_i)$  and therefore  $f(G) \subseteq \bigcup_{i \in I_0} V_i$ . This shows that  $f(G)$  is strongly  $p^*ga$ -compact.

**Corollary 3.4.** If a mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$  is surjection strongly  $p^*ga$  irresolute and  $(X, \tau)$  is a strongly  $p^*ga$  compact space, then  $(Y, \sigma)$  is strongly  $p^*ga$  compact.

**Proof.** Suppose that  $\{G_i: i \in I\}$  is a collection of strongly  $p^*ga$  open cover of  $(Y, \sigma)$ . Then,  $\{f^{-1}(G_i): i \in I\}$  is a strongly  $p^*ga$  open cover of  $(X, \tau)$ . But,  $(X, \tau)$  is strongly  $p^*ga$  compact, hence there exists a finite subset  $I_0$  of  $I$  such that  $X = \bigcup_{i \in I_0} \{f^{-1}(G_i): i \in I_0\}$  and therefore  $Y = \bigcup_{i \in I_0} \{G_i: i \in I_0\}$ . So,  $(Y, \sigma)$  is strongly  $p^*ga$  compact.

**Theorem 3.5.** Let  $f: (X, \tau) \rightarrow (Y, \sigma)$  be a bijective pre-strongly  $p^*ga$  open mapping and  $(Y, \sigma)$  be a strongly  $p^*ga$  compact space. Then  $(X, \tau)$  is strongly  $p^*ga$  compact.

**Proof.** Assume that  $\{U_i: i \in I\}$  is a strongly  $p^*ga$  open cover of  $(X, \tau)$ . and  $f$  is a pre-strongly  $p^*ga$  open mapping, then  $\{f(U_i): i \in I\}$  is a strongly  $p^*ga$  open cover of  $(Y, \sigma)$ . But,  $(Y, \sigma)$  is strongly  $p^*ga$  compact, hence there exists a finite subset  $I_0$  of  $I$  such that  $Y = \bigcup_{i \in I_0} \{f(U_i): i \in I_0\}$ , so by a bijective of,  $f, X = \bigcup_{i \in I_0} \{U_i: i \in I_0\}$  and therefore  $(X, \tau)$  is strongly  $p^*ga$  compact.

**Theorem 3.6.** If  $f: (X, \tau) \rightarrow (Y, \sigma)$  is a bijective  $P$ -preopen mapping and  $(Y, \sigma)$  is a strongly  $p^*ga$  compact space, then  $(X, \tau)$  is strongly compact.

**Proof.** Let  $\{G_i: i \in I\}$  be a preopen cover of  $(X, \tau)$ . Then,  $\{f(G_i): i \in I\}$  is a preopen cover of  $(Y, \sigma)$  hence by **Lemma 2.5**  $\{f(G_i): i \in I\}$  is a strongly  $p^*ga$  open cover of  $(Y, \sigma)$ . Since,  $(Y, \sigma)$  is a strongly  $p^*ga$  compact space, then there exists a finite subset  $I_0$  of  $I$  such that  $Y = \bigcup_{i \in I_0} \{f(G_i): i \in I_0\}$  and thus  $X = \bigcup_{i \in I_0} \{G_i: i \in I_0\}$ . So,  $(X, \tau)$  is strongly compact.

**Theorem 3.7.** Let  $A, B$  be two subsets of a space  $(X, \tau)$ ,  $A$  be strongly  $p^*ga$  compact relative to  $(X, \tau)$  and  $B$  be a strongly  $p^*ga$  closed subset of  $(X, \tau)$ . Then  $A \cap B$  is strongly  $p^*ga$  compact relative to  $(X, \tau)$ .

**Proof.** Let  $\{V_i: i \in I\}$  be a strongly  $p^*ga$  open cover of  $A \cap B$  and  $X - B$  be a strongly  $p^*ga$  open subset of  $(X, \tau)$ . Then  $(X - B) \cup \bigcup_{i \in I} \{V_i: i \in I\}$  is strongly  $p^*ga$  Sopen cover of  $A$  which is strongly  $p^*ga$  compact relative to  $(X, \tau)$ , hence there exists a finite subset  $I_0$  of  $I$  such that  $A \subseteq \bigcup_{i \in I_0} \{V_i: i \in I_0\} \cup (X - B)$ . Therefore  $A \cap B \subseteq \bigcup_{i \in I_0} \{V_i: i \in I_0\}$  and hence  $A \cap B$  is strongly  $p^*ga$  compact relative to  $(X, \tau)$ .

**Theorem 3.8.** If  $\{A_j: j \in J\}$  is a finite family of strongly  $p^*ga$  compact subsets relative to a space  $(X, \tau)$ , then  $\bigcup_{j \in J} \{A_j: j \in J\}$  is a strongly  $p^*ga$  compact relative to  $(X, \tau)$ .

**Proof.** Suppose that  $\{V_i: i \in I\}$  is a strongly  $p^*ga$  open cover of  $\bigcup_{j \in J} \{A_j: j \in J\}$ . Then,  $\{V_i: i \in I\}$  is a strongly  $p^*ga$  open cover of  $A_j$ , for each  $j \in J$ . Hence, there exists a finite subset  $I_0$  of  $I$  such that  $A_j \subseteq \bigcup_{i \in I_0} \{V_i: i \in I_0\}$ , for each  $j \in J$ . Therefore,  $\bigcup_{j \in J} A_j \subseteq \bigcup_{i \in I_0} \{V_i: i \in I_0\}$ . Thus,  $\bigcup_{j \in J} \{A_j: j \in J\}$  is a strongly  $p^*ga$  compact relative to  $(X, \tau)$ .

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## IV. STRONGLY PRESTAR GENERALIZED ALPHA CONNECTED SPACES

**Definition 4.1.** Two non-empty subsets  $A$  and  $B$  in a space  $(X, \tau)$  are called strongly Prestargeneralized Alpha-separated (briefly, strongly  $p^*ga$ separated) if and only if  $A \cap stp^*ga-cl(B) = \emptyset$  and  $stp^*ga-cl(A) \cap B = \emptyset$ .

**Remark 4.2.** For a topological space  $(X, \tau)$ , the following statements are hold:

- (i) Each separation is strongly  $p^*ga$ separation by the fact that  $stp^*ga-cl(A) \subseteq cl(A)$ , for each  $A \subseteq X$ .
- (ii) Any two strongly  $p^*ga$ separated sets are always disjoint but the converse in general is not true as shown by the following example.

**Example 4.3.** If  $X = \{a, b, c, d\}$  with a topology  $\tau = \{X, \emptyset, \{c\}, \{a, b\}, \{a, b, c\}\}$ , then two subsets  $\{a, c\}$  and  $\{b, d\}$  of  $\tau$  are disjoint but not strongly  $p^*ga$ separated.

**Theorem 4.4** Let  $A$  and  $B$  be non-empty subsets of a space  $(X, \tau)$ . Then the following statements are hold:

- (i) If  $A$  and  $B$  are strongly  $p^*ga$ separated sets,  $A_1$  and  $B_1$  are non-empty sets such that  $A_1 \subseteq A$  and  $B_1 \subseteq B$ , then  $A_1$  and  $B_1$  are also strongly  $p^*ga$ separated.
- (ii) If  $A \cap B = \emptyset$  and either both  $A$  and  $B$  are strongly  $p^*ga$ open or strongly  $p^*ga$ closed, then  $A$  and  $B$  are strongly  $p^*ga$ separated.
- (iii) If both  $A$  and  $B$  are either strongly  $p^*ga$ open or strongly  $p^*ga$ closed and if then  $U$  and  $V, U = A \cap (X-B)$  and  $V = B \cap (X-A)$ , are strongly  $p^*ga$ separated.

**Proof.**

(i) obviously.

(ii) Let  $A, B \in Stp^*gaO(X, \tau)$ . Then  $X-A$  and  $X-B$  are strongly  $p^*ga$ closed sets. But,  $A, B$  are disjoint, then  $A \subseteq X-B$  which implies that  $stp^*ga-cl(A) \subseteq stp^*ga-cl(X-B) = X-B$ , and so  $stp^*ga-cl(A) \cap B = \emptyset$ .

Similarly,  $stp^*ga-cl(B) \cap A = \emptyset$ . Hence,  $A$  and  $B$  are strongly  $p^*ga$ separated. For the case of strongly  $p^*ga$ closed sets  $A$  and  $B$  are given.

(iii) Assume that  $A$  and  $B$  are strongly  $p^*ga$ open sets. Then  $X-A$  and  $X-B$  are strongly  $p^*ga$ closed. Since,  $U \subseteq X-B$ , hence  $stp^*ga-cl(U) \subseteq stp^*ga-cl(X-B) = X-B$  and so,  $stp^*ga-cl(U) \cap B = \emptyset$ . Therefore,  $stp^*ga-cl(U) \cap V = \emptyset$ .

Similarly. We can prove that  $stp^*ga-cl(V) \cap U = \emptyset$ . Since,  $A$  and  $B$  are strongly  $p^*ga$ closed sets, then  $A = stp^*ga-cl(A)$  and  $B = stp^*ga-cl(B)$ . But,  $U \subseteq X-B$ , then  $U \cap stp^*ga-cl(B) = \emptyset$  and hence  $stp^*ga-cl(U) \cap V = \emptyset$ . Similarly. We can show that  $stp^*ga-cl(U) \cap V = \emptyset$ . Therefore, in both cases  $U$  and  $V$  are strongly  $p^*ga$ separated.

**Theorem 4.5.** If  $G$  and  $H$  are strongly  $p^*ga$ open sets in  $(X, \tau)$  such that  $A \subseteq G, B \subseteq H$  and  $A \cap H = \emptyset, B \cap G = \emptyset$ , then  $A$  and  $B$  are non-empty strongly  $p^*ga$ separated sets in  $(X, \tau)$ .

**Proof.** Suppose that  $G$  and  $H$  are strongly  $p^*ga$ open sets such that  $A \subseteq G, B \subseteq H$  and  $A \cap H = \emptyset, B \cap G = \emptyset$ . Then,  $X-H$  and  $X-G$  are strongly  $p^*ga$ closed sets and hence

$stp^*ga-cl(A) \subseteq X-H \subseteq X-B$  and  $stp^*ga-cl(B) \subseteq X-G \subseteq X-A$ . Therefore,  $stp^*ga-cl(A) \cap B = \emptyset$  and  $stp^*ga-cl(B) \cap A = \emptyset$ . Thus,  $A$  and  $B$  are non empty strongly  $p^*ga$  separated sets.

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**Definition 4.6.** A space  $(X, \tau)$  is called a strongly Prestar generalized alpha connected (briefly, strongly  $p^*ga$ connected) space if  $X$  cannot be written as a disjoint union of two non-empty strongly  $p^*ga$ open sets.

**Definition 4.7.** A subset  $S$  of a space  $(X, \tau)$  is said to be strongly prestart generalized alpha connected (briefly, strongly  $p^*ga$ connected) relative to  $(X, \tau)$  if there is no subsets  $A$  and  $B$  are strongly  $p^*ga$ separated relative to  $(X, \tau)$  and  $S = A \cup B$ . In other words, a subset  $S$  cannot be expressed as the union of two non-empty strongly  $p^*ga$ separated sets  $A, B$ .

**Remark 4.8.** For a space  $(X, \tau)$ , every strongly  $p^*ga$ connected set is connected (resp. GO-connected), but the converse may not be true as shown by the following examples.

**Example 4.9.** If  $X = \{a, b, c, d\}$  with a topology  $\tau = \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$ , then the subset  $A = \{a, c\}$  is connected but not strongly  $p^*ga$ connected.

**Example 4.10.** If  $X = \{a, b, c, d\}$  with a topology  $\tau = \{X, \emptyset, \{b, d\}, \{a, b, d\}, \{b, c, d\}\}$ , then the subset  $B = \{a, b, d\}$  is GO-connected but not strongly  $p^*ga$ connected.

**Remark 4.11.** Every strongly  $p^*ga$ connected space is connected.

**Example 4.12.** Let  $X = \{a, b, c\}$  with a topology  $\tau = \{X, \emptyset, \{a, b\}\}$ . Then,  $(X, \tau)$  is connected but it is not strongly  $p^*ga$ connected. Since,  $\{b, c\}$  is both strongly  $p^*ga$ open and strongly  $p^*ga$ closed.

**Theorem 4.13.** In a space  $(X, \tau)$  if  $E$  is strongly  $p^*ga$ connected, then  $st.p^*ga-cl(E)$  is strongly  $p^*ga$ connected,

**Proof.** Assume that  $st.p^*ga-cl(E)$  is strongly  $p^*ga$  disconnected, then there exist two non - empty strongly  $p^*ga$  separated sets  $G$  and  $H$  in  $(X, \tau)$

Such that  $st.p^*ga-cl(E) = G \cup H$ . Now  $E = (G \cap E) \cup (H \cap E)$  and

$$st.p^*ga-cl(G \cap E) \subseteq st.p^*ga-cl(G)$$

$$st.p^*ga-cl(H \cap E) \subseteq st.p^*ga-cl(H) \text{ and } G \cap H = \emptyset, \text{ this}$$

implies  $st.p^*ga-cl(G \cap E) \cap st.p^*ga-cl(H \cap E) = \emptyset$ , Thus

$st.p^*ga-cl(G \cap E) \cap (H \cap E) = \emptyset$ . Similarly, we can Prove  $st.p^*ga-cl(H \cap E) \cap (G \cap E) = \emptyset$  and have  $E$  is strongly  $p^*ga$  disconnected.

**Theorem 4.14.**

For a space  $(X, \tau)$ , the following are equivalent:

(i)  $(X, \tau)$  is strongly  $p^*ga$ connected,

(ii) The only subsets of  $(X, \tau)$  which are both strongly  $p^*ga$ open and strongly  $p^*ga$ closed are the empty set  $\emptyset$  and  $X$ .

(iii) Each strongly  $p^*ga$ continuous mapping of  $(X, \tau)$  into a discrete space  $(Y, \sigma)$  with at least two points is a constant mapping.

**Proof.** (i)  $\rightarrow$  (ii). Let  $G$  be strongly  $p^*ga$ open and strongly  $p^*ga$ closed of  $(X, \tau)$ . Then,  $X - G$  is both strongly  $p^*ga$ open and strongly  $p^*ga$ closed. Since,  $X$  is the disjoint union two strongly  $p^*ga$ open sets  $G$  and  $X - G$  one of these must be empty, that is,  $G = \emptyset$  or  $G = X$ .

(ii)  $\rightarrow$  (i). Suppose that  $X = C \cup D$ , where  $C$  and  $D$  are disjoint non-empty strongly  $p^*ga$ open and strongly  $p^*ga$ closed subsets of  $(X, \tau)$ , then  $C$  is both strongly  $p^*ga$ open and strongly  $p^*ga$ closed, by hypothesis,  $C = \emptyset$  or  $X$ .

(ii)  $\rightarrow$  (iii). Assume that  $f: X \rightarrow Y$  be a strongly  $p^*ga$ continuous map. Then,  $X$  is covered by

strongly  $p^*ga$  open and strongly  $p^*ga$ closed covering  $\{f^{-1}(y), y \in Y\}$ , by assumption,  $f^{-1}(y) = \emptyset$  or  $X$ , for each  $y \in Y$ . If  $f^{-1}(y) = \emptyset$  for all  $y \in Y$ , then fails to be map. Then there exists only point  $y \in Y$  such that  $f^{-1}(y) \neq \emptyset$  and hence  $f^{-1}(y) = X$ . Thus, shows that  $f$  is a constant map.

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(iii)  $\rightarrow$  (ii). Let  $G$  be both strongly  $p^*$ gaopen and strongly  $p^*$ gaclosed in  $(X, \tau)$ . Suppose  $G \neq \emptyset$  and  $f: X \rightarrow Y$  be a strongly  $p^*$ gacontinuous map defined by  $f(G) = \{y\}$  and  $f(X - G) = \{w\}$  for some distinct points  $y$  and  $w$  in  $Y$ , by assumption,  $f$  is constant. Therefore,  $G = X$ .

### Theorem 4.15.

For a subjective mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$ , the following statements are hold:

- (i) If  $f$  is strongly  $p^*$ gacontinuous and  $X$  is strongly  $p^*$ gaconnected, then  $Y$  is connected.
- (ii) If  $f$  is strongly  $p^*$ gairresolute and  $X$  is strongly  $p^*$ gaconnected, then  $Y$  is strongly  $p^*$ gaconnected.

**Proof.**(i) Suppose that  $(Y, \sigma)$  is not connected and  $Y = A \cup B$ , where  $A$  and  $B$  are disjoint non-empty open sets in  $(Y, \sigma)$ . Since,  $f$  is surjection strongly  $p^*$ ga continuous, then  $X = f^{-1}(A) \cup f^{-1}(B)$ , where  $f^{-1}(A)$  and  $f^{-1}(B)$  are disjoint non-empty strongly  $p^*$ gaopen sets in  $(X, \tau)$  which is a contradiction with the fact that  $(X, \tau)$  is strongly  $p^*$ gaconnected. Hence,  $(Y, \sigma)$  is connected.

(ii) Let  $(Y, \sigma)$  be not strongly  $p^*$ gaconnected. Then, there exists two non-empty disjoint strongly  $p^*$ gaopen sets  $A$  and  $B$ , where  $Y = A \cup B$ . Since,  $f$  is strongly  $p^*$ gairresolute, then  $X = f^{-1}(A) \cup f^{-1}(B)$ , where  $f^{-1}(A)$  and  $f^{-1}(B)$  are disjoint non-empty strongly  $p^*$ gaopen sets in  $(X, \tau)$  which is contradiction with fact that  $(X, \tau)$  is strongly  $p^*$ gaconnected. Hence,  $(Y, \sigma)$  is strongly  $p^*$ gaconnected.

### Theorem 4.16.

For a bijective mapping  $f: (X, \tau) \rightarrow (Y, \sigma)$ , the following statements are hold:

- (i) If  $f$  is pre-strongly  $p^*$ gaclosed and  $K$  is strongly  $p^*$ gaconnected relative to  $(Y, \sigma)$ , then  $f^{-1}(K)$  is strongly  $p^*$ gaconnected relative to  $(X, \tau)$ .
- (ii) If  $f$  is super strongly  $p^*$ gaopen and  $K$  is connected relative to  $(Y, \sigma)$ , then  $f^{-1}(K)$  is strongly  $p^*$ ga connected relative to  $(X, \tau)$ .

**Proof:**(i) Let  $f^{-1}(K)$  be not strongly  $p^*$ gaconnected relative to  $(X, \tau)$ .

Then there exists disjoint non-empty strongly  $p^*$ gaopen sets  $A$  and  $B$  where  $f^{-1}(K) = A \cup B$ . Since,  $f$  is injective pre-strongly  $p^*$ ga open, then  $K = f(A) \cup f(B)$ , where  $f(A)$  and  $f(B)$  are non-empty disjoint strongly  $p^*$ gaclosed sets in  $(Y, \sigma)$  which is a contradiction with the fact that  $K$  is strongly  $p^*$ gaconnected. Hence,  $f^{-1}(K)$  is strongly  $p^*$ gaconnected relative to  $(X, \tau)$ .

(ii) Suppose that  $f^{-1}(K)$  is not strongly  $p^*$ gaconnected relative to  $(X, \tau)$ . Then, there exist disjoint non-empty strongly  $p^*$ gaopen sets  $A$  and  $B$  where  $f^{-1}(K) = A \cup B$ . Since,  $f$  is bijective super strongly  $p^*$ ga open map and  $K = f(A) \cup f(B)$  which is a contradiction with the fact that  $K$  is connected relative to  $(Y, \sigma)$ . Therefore,  $f^{-1}(K)$  is strongly  $p^*$ ga connected relative to  $(X, \tau)$ .

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