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A Study of Boundary Value Problem in Association with I -Function: A Review

Deepika Bhandari¹, Punit Bansal²

Department of Physics, Seth R L Saharia Government PG College, Kaladera, India¹

Department of Mathematics, Government College, Malpura, India²

ABSTRACT: In this paper, the authors make a model of a boundary value problem and then obtain its solution involving the products of I -function and a general class of polynomials.

KEYWORDS: General Class of Polynomials, I -function, Expansion Formula, Hermite Polynomial.

I. INTRODUCTION

The operational techniques are important tools to compute various problems in various fields of science which are used in the works of Chaurasia [4], Chandel, Agrawal, and Kumar [2], Chandel and Sengar[3], and Kumar [5] to find out several results in various problems in different field of sciences and thus motivated by this work, we construct a model problem for temperature distribution in a rectangular plate under prescribed boundary conditions and then evaluate its solution involving A -function with the product of the general class of polynomials.

The general class of polynomials is defined by Srivastava and Panda [11, 12] as:

$$S_{n_1, \dots, n_r}^{m_1, \dots, m_r}(x_1, \dots, x_r) = \sum_{k_1=0}^{[n_1/m_1]} \dots \sum_{k_r=0}^{[n_r/m_r]} \frac{(-n_1)_{m_1 k_1}}{k_1!} \dots \frac{(-n_r)_{m_r k_r}}{k_r!} F[n_1, k_1; \dots; n_r, k_r] x_1^{k_1} \dots x_r^{k_r} \quad (1.1)$$

Where, m_1, \dots, m_r are arbitrary positive integers and the coefficients $F[n_1, k_1; \dots; n_r, k_r]$ are arbitrary constants real or complex. Finally, we derive some new particular cases and find their applications also.

The I -function introduced by Saxena [6] will be represented and defined as follows:

$$I[Z] = I_{p_i, q_i; r}^{m, n}[Z] = I_{p_i, q_i; r}^{m, n} \left[z \left| \begin{matrix} (a_j, \alpha_j)_{1, n}, (a_{ji}, \alpha_{ji})_{n+1, p_i} \\ (b_j, \beta_j)_{1, m}, (b_{ji}, \beta_{ji})_{m+1, q_i} \end{matrix} \right. \right] = \frac{1}{2\pi\omega} \int_L \chi(\xi) d\xi \quad (1.2)$$

where $\omega = \sqrt{-1}$

$$\chi(\xi) = \frac{\prod_{j=1}^m \Gamma(b_j - \beta_j \xi) \prod_{j=1}^n \Gamma(1 - a_j + \alpha_j \xi)}{\sum_{i=1}^r \left\{ \prod_{j=m+1}^{q_i} \Gamma(1 - b_{ji} - \beta_{ji} \xi) \prod_{j=n+1}^{p_i} \Gamma(a_{ji}, \alpha_{ji} \xi) \right\}} \quad (1.3)$$

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$p_i, q_i (i = 1, \dots, r), m, n$ are integers satisfying $0 \leq n \leq p_i, 0 \leq m \leq q_i, (i = 1, \dots, r), r$ is finite $\alpha_j, \beta_j, \alpha_{ji}, \beta_{ji}$ are real and a_j, b_j, a_{ji}, b_{ji} are complex numbers such that $\alpha_j (b_h + v) \neq \beta_h (a_j - v - k)$ for $v, k = 0, 1, 2, \dots$

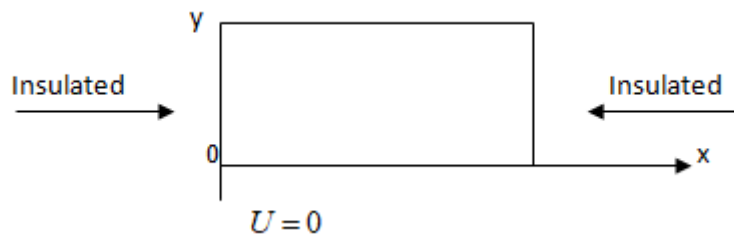
We shall use the following notations:

$$A^* = (a_j, \alpha_j)_{1,n}, (a_{ji}, \alpha_{ji})_{n+1,p_i}; B^* = (b_j, \beta_j)_{1,m}, (b_{ji}, \beta_{ji})_{m+1,q_i}$$

II. A BOUNDARY VALUE PROBLEM

We consider a rectangular plate such that,

$$U = f(x) \quad \left(\frac{a}{2}, \frac{b}{2} \right)$$



Where the boundary value conditions are:

$$\frac{\partial^2 U}{\partial x^2} + \frac{\partial^2 U}{\partial y^2} = a, \quad 0 < x < \frac{a}{2}, \quad 0 < y < \frac{b}{2} \tag{2.1}$$

$$\frac{\partial U}{\partial x} \Big|_{x=0} = \frac{\partial U}{\partial x} \Big|_{x=\frac{a}{2}} = 0, \quad 0 < y < \frac{b}{2} \tag{2.2}$$

$$U(x, 0) = 0, \quad 0 < x < \frac{a}{2} \tag{2.3}$$

$$U \left(x, \frac{b}{2} \right) = f(x) = \left(\cos \frac{\pi x}{a} \right) S_{n_1, \dots, n_r}^{m_1, \dots, m_r} \left[y_1 \left(\cos \frac{\pi x}{a} \right)^{2\rho} \right] \tag{2.4}$$

$$A_{p,q}^{m,n} \left[z \left(\cos \frac{\pi x}{a} \right)^{2\sigma} \Big|_{1(a_j, \alpha_j)_p} \Big|_{1(b_j, \beta_j)_q} \right]$$

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Where, $0 < x < \frac{a}{2}$ provided that $\text{Re}(\eta) > -1, \sigma > 0$.

$U(x, y)$ is the temperature distribution in the rectangular plate at the point (x, y) .

III. MAIN INTEGRAL

In our investigations, we appeal to the modified formula due to Kumar [5] as,

$$\int_0^{a/2} \left(\cos \frac{\pi x}{a} \right)^\eta \cos \frac{2m\pi x}{a} dx = \frac{a\Gamma(\eta+1)}{2^{\eta+1} \left(\frac{\eta}{2} + m + 1 \right) \left(\frac{\eta}{2} - m + 1 \right)} \quad (3.1)$$

Where, m is a positive integer and $\text{Re}(\eta) > -1$, then we evaluate an integral

$$\begin{aligned} & \int_0^{a/2} \left(\cos \frac{\pi x}{a} \right)^\eta \cos \frac{2m\pi x}{a} S_{n_1, \dots, n_r}^{m_1, \dots, m_r} \left[y \left(\cos \frac{\pi x}{a} \right)^{2\rho} \right] \\ & I_{p_i, q_i; r}^{m, n} \left[z \left(\cos \frac{\pi x}{a} \right)^{2\sigma} \left| \begin{matrix} A^* \\ B^* \end{matrix} \right. \right] dx \\ &= \frac{a}{2^{\eta+1}} \sum_{k_1=0}^{[n_1/m_1]} \dots \sum_{k_r=0}^{[n_r/m_r]} \frac{(-n)_{m_1 k_1}}{k_1!} \dots \frac{(-n)_{m_r k_r}}{k_r!} \\ & F[n_1, k_1; \dots, n_r, k_r] I(k) \left(\frac{y}{4\rho} \right)^{k_1} \dots \left(\frac{y}{4\rho} \right)^{k_r} \end{aligned} \quad (3.2)$$

Where,

$$\begin{aligned} I(k) &= I_{p_i+1, q_i+1; r}^{m, n+1} \left[\frac{z}{4^\sigma} \right. \\ & \left. \left(-\eta - 2\rho k_1 - \dots - 2\rho k_r; 2\sigma \right), A^* \right] \\ & \left[B^*, \left(-\frac{\eta}{2} - m - \rho k_1 - \dots - \rho k_r; \sigma \right) \right] \end{aligned} \quad (3.3)$$

Provided that $F[n_1, k_1; \dots, n_r, k_r]$ are arbitrary functions of $n_1, k_1; \dots, n_r, k_r$, real or complex independent of x, y, ρ , the conditions of (2.4) and (3.1) are satisfied and

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$$\operatorname{Re}\left(\eta + \sigma \frac{b_{ji}}{\beta_{ji}}\right) > -1, \quad |\arg z| \leq \frac{1}{2} \pi \Omega,$$

Where

$$\Omega \equiv \sum_{j=1}^n \alpha_j + \sum_{j=1}^m \beta_j - \sum_{j=n+1}^{p_i} \alpha_{ji} - \sum_{j=m+1}^{q_i} \beta_{ji} > 0$$

IV. SOLUTION OF BOUNDARY VALUE PROBLEM

In this section, we obtain the solution to the boundary value problem stated in section (2) using (2.1), (2.2), and (2.3) with the help of the techniques referred to by Zill [13] as:

$$U(x, y) = A_0 y + \sum_{p=1}^{\infty} A_p \sinh \frac{2p\pi y}{a} \cos \frac{2p\pi x}{a}, \quad 0 < x < \frac{a}{2}, \quad 0 < y < \frac{a}{2} \quad (4.1)$$

For $y = \frac{b}{2}$, we find that

$$U\left(x, \frac{b}{2}\right) = f(x) = \frac{A_0 b}{2} + \sum_{p=1}^{\infty} A_p \sinh \frac{p\pi b}{a} \cos \frac{2p\pi x}{a}, \quad 0 < x < \frac{a}{2} \quad (4.2)$$

Now making an appeal to (2.4) and (4.2) and then interchanging both sides with respect to x from 0 to $\frac{a}{2}$, we derive,

$$A_0 = \frac{2}{b\sqrt{\pi}} \sum_{k_1=0}^{[n_1/m_1]} \dots \sum_{k_r=0}^{[n_r/m_r]} (-n_1)_{m_1 k_1} \dots (-n_r)_{m_r k_r}$$

$$F[n_1, k_1; \dots; n_r, k_r] I_1(k) \frac{y^{k_1}}{k_1!} \dots \frac{y^{k_r}}{k_r!} \quad (4.3)$$

Where

$$I_1(k) = I_{p_i+1, q_i+1; r}^{m, n+1} \left[z, \left(-\frac{1}{2} - \frac{\eta}{2} - \rho k_1 - \dots - \rho k_r; \sigma \right), A^* \right. \\ \left. B^*, \left(-\frac{\eta}{2} - \rho k_1 - \dots - \rho k_r; \sigma \right) \right] \quad (4.4)$$

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Where all conditions of (2.4), (3.1) and (3.3) are satisfied.

Again appealing to (2.4) and (4.2) and then multiply by $\cos \frac{2m\pi x}{a}$ both sides and thus integrating that result concerning x from 0 to $\frac{a}{2}$, we find,

$$A_m = \frac{1}{2^{\eta-1} \sinh \frac{p\pi b}{a}} \sum_{k_1=0}^{[n_1/m_1]} \dots \sum_{k_r=0}^{[n_r/m_r]} \frac{(-n_1)_{m_1 k_1}}{k_1!} \dots \frac{(-n_r)_{m_r k_r}}{k_r!} F[n_1, k_1; \dots, n_r, k_r]$$

$$I(k) \left(\frac{y}{4^\rho}\right)^{k_1} \dots \left(\frac{y}{4^\rho}\right)^{k_r} \tag{4.5}$$

Provided that all conditions of (2.4), (3.1), and (3.3) are satisfied.

Finally, appealing to the result (4.1), (4.3), and (4.5), we derive the required solution to the boundary value problem,

$$U(x, y) = \frac{2y}{b\sqrt{\pi}} \sum_{k_1=0}^{[n_1/m_1]} \dots \sum_{k_r=0}^{[n_r/m_r]} \left[\prod_{j=1}^r \left((-n_j)_{m_j k_j} \frac{y^{k_j}}{k_j!} \right) \right] F[n_1, k_1; \dots, n_r, k_r] +$$

$$\sum_{m=1}^{\infty} \frac{\sinh \frac{2m\pi y}{a} \cos \frac{2m\pi x}{a}}{2^{\eta-1} \sinh \frac{m\pi b}{a}} \sum_{k_1=0}^{[n_1/m_1]} \dots \sum_{k_r=0}^{[n_r/m_r]} \left[\prod_{j=1}^r \left((-n_j)_{m_j k_j} \left(\frac{y}{4^\rho}\right)^{k_j} \frac{1}{k_j!} \right) \right]$$

$$F[n_1, k_1; \dots, n_r, k_r] I(k) \tag{4.6}$$

Provided that all conditions of (2.4), (3.1), and (3.3) are satisfied.

V. PARTICULAR CASES

In this section, we do some setting of different parameters of our results and then drive some particular cases as stated here as taking $m_1 = \dots = m_r = \gamma$ and

$$F[n_1, k_1; \dots, n_r, k_r] = \left(\frac{h}{(\nu)^\gamma}\right)^{k_1 + \dots + k_r} \frac{1}{(1 + p - n_1 - \dots - n_r)_{\gamma(k_1 + \dots + k_r)}} \quad \text{in (1.1)}$$

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We get,

$$S_{n_1, \dots, n_r}^{\gamma, \dots, \gamma} [x_1, \dots, x_r] = \frac{(-v)^{-n_1 \dots - n_r}}{(-p)_{n_1 + \dots + n_r}} (x_1)^{n_1/\gamma} \dots (x_r)^{n_r/\gamma} H_{n_1, \dots, n_r}^{(h, \gamma, v, p)} [(x_1)^{-1/\gamma} \dots (x_r)^{-1/\gamma}] \quad (5.1)$$

Thus, we obtain an integral for the product of a class of polynomials of several variables and cosine functions as

$$\int_0^{a/2} \left(\cos \frac{\pi x}{a} \right)^\eta \cos \frac{2m\pi x}{a} \frac{(-v)^{-n_1 \dots - n_r}}{(-p)_{n_1 + \dots + n_r}} \left[y \left(\cos \frac{\pi x}{a} \right)^{2\rho} \right]^{n_1/\gamma} \dots \left[y \left(\cos \frac{\pi x}{a} \right)^{2\rho} \right]^{n_r/\gamma} \\ H_{n_1, \dots, n_r}^{(h, \gamma, v, p)} \left[\left[y \left(\cos \frac{\pi x}{a} \right)^{2\rho} \right]^{r/\gamma} \right] I_{p_i, q_i; r}^{m, n} \left[z \left(\cos \frac{\pi x}{a} \right)^{2\sigma} \left| \begin{matrix} A^* \\ B^* \end{matrix} \right. \right] dx \\ = \frac{a}{2^{\eta+1}} \sum_{k_1=0}^{[n_1/\gamma]} \dots \sum_{k_r=0}^{[n_r/\gamma]} \frac{(-n)_{\gamma k_1}}{k_1!} \dots \frac{(-n)_{\gamma k_r}}{k_r!} \left[\frac{h}{(-v)^\gamma} \right]^{k_1 + \dots + k_r} \\ \frac{1}{(1 + \rho - n_1 - \dots - n_r)_{\gamma(k_1 + \dots + k_r)}} I(k) \left(\frac{y}{4\rho} \right)^{k_1} \dots \left(\frac{y}{4\rho} \right)^{k_r} \quad (5.2)$$

Provided that all conditions of (2.4), (3.1), and (3.2) are satisfied.

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