

Study of an Electrical Circuit with Unknown Resistances and Voltages using Graph Theoretic Approach and Superposition Theorem

Chanchal Boruah¹, Chandra Chutia²

Assistant Professor, Department of Mathematics, Arya Vidyapeeth College, Guwahati-781016, Assam, India¹

Associate Professor, Department of Mathematics, Jorhat Institute of Science and Technology, Jorhat-785010, Assam, India²

ABSTRACT: In Mathematics, Graph Theory is a branch where we study about some points or nodes and the connections between them defined by a particular relation. So there are many application of Graph Theory in our daily life. In Electrical Engineering, electrical elements are connected in a particular way for passing current. So, electrical circuits can be studied with the help of graph theoretic approach also. Electrical circuits can be simplified by using many well known theorems of Electrical Engineering. An electrical circuit can be converted to graphs using their connections. From the graph we can define many matrices using the connectivity of the points. In this paper, we shall try to study branch currents of an electrical circuit with unknown resistances and voltages by using both network equilibrium equations and Superposition theorem.

KEYWORDS: Graph theory, Electrical circuits, Fundamental tie-set matrix, Branch impedance matrix.

2000 .S. subject classification: 05C50

I. INTRODUCTION

In Electrical Engineering, Graph Theory can be used to verify the properties of different electrical circuits. Electrical circuits consist of different electrical elements. Flowing current through the circuits obeys different laws such as Kirchhoff's current laws, Kirchhoff's voltage laws etc. Graph theory comes into account in the electrical circuits in the manner by which the elements or devices are connected in the circuits.

II. DEFINITIONS

A. Graph: A graph is a set of ordered pair $G = (V, E)$ of sets where $E = \{\{x, y\} : x, y \in V\}$ is defined by a particular relation among the elements of V . The elements of V are called vertices (or nodes) of the graph G and the elements of E are called edges. The vertex set V must not be non empty.

B. Electrical Circuits: Different electrical elements such as resistors, capacitors, inductors, diodes, transistors etc are connected to form an electrical circuit. Its behaviour depends upon the characteristic of each of internally connected elements and the rule by which they are connected together. This rule gives a relation between electrical circuits with graph theory.

III. FORMATION OF GRAPH FROM AN ELECTRICAL CIRCUIT

To form a graph from an electrical circuit all the electrical elements are replaced by lines with circles or dots at both ends. A branch is a line representing one network element or combination of elements between two points. Junction of two branches is called a node. The ends of an isolate branch are also a node. The number of branches incident to a node

is called its degree.

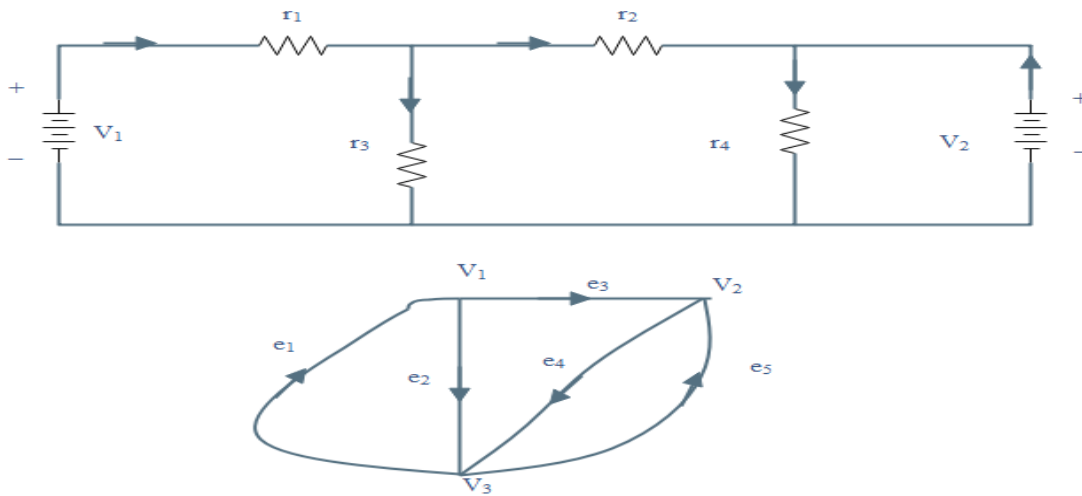


Figure 1: A graph from the circuit

IV. MATRICES ASSOCIATED TO A GRAPH

A. Fundamental Tie-Set matrix (Fundamental Loop Matrix) :

A fundamental loop is a loop formed by only one link associated with other twigs. In fundamental tie set matrix B $[b_{ij}]$, all branches form the rows and all loops obtained from a particular tree forms the columns such that

$$b_{ij} = \begin{cases} 1, & \text{if branch } b_j \text{ in loop } i \text{ are in the same direction} \\ -1, & \text{if branch } b_j \text{ in loop } i \text{ are in the opposite direction} \\ 0, & \text{if branch } b_j \text{ is not in loop } i \end{cases} \quad (1)$$

B. Branch Impedance Matrix $[Z_b]$:

It is a square matrix of order m where m is the no of branches having branch impedance as the diagonal elements and mutual impedance as off diagonal elements. If there is no transformer or mutual sharing then off diagonal elements is zero.

C. Application of Graph Theory in Network Equilibrium Equations:

If V_{sk} be the voltage source in a branch k having impedance z_k and carrying current i_k , then the branch voltage $v_k = z_k i_k + V_{sk}$

For the whole circuit, in matrix form this can be written as $[V_b] = [Z_b][I_b] + [V_s]$ where $[Z_b]$ is the branch impedance matrix, $[I_b]$ is the column vector of branch currents and $[V_s]$ is the column vector of source voltage. Now Kirchhoff's Voltage law states that the algebraic sum of voltages in any closed path of network traversed in a single direction is zero.

In matrix form it is given by

$$\begin{aligned} [B][V_b] &= 0 \\ \Rightarrow [B]\{[Z_b][I_b] + [V_s]\} &= 0 \\ \Rightarrow [B][Z_b][I_b] &= -[B][V_s] \end{aligned} \quad (2)$$

Also the branch current matrix equation is $[I_b] = [B^T][I_L]$ where $[I_L]$ is the loop current and $[B^T]$ is the transpose of $[B]$.

$$\text{So we get } [B][Z_b][B^T][I_L] = -[B][V_s] \quad (3)$$

We will use this equation to analyze the following circuits and also in the equivalent circuits obtained by Superposition theorem.

IV. SUPERPOSITION THEOREM OF ELECTRICAL CIRCUIT

If a number of voltage or current sources are acting simultaneously in a linear network then the resultant current in any branch is the algebraic sum of the currents that would be produced in it, when each source acts alone replacing all other independent sources by their internal resistances.

V. FINDING BRANCH CURRENT USING GRAPH THEORY AND MATRIX

Let us consider the circuit with two unknown resistances r_1 and r_2 and two unknown voltages v_1 and v_2 such that $r_i, v_i \neq 0$ as shown in the figure below

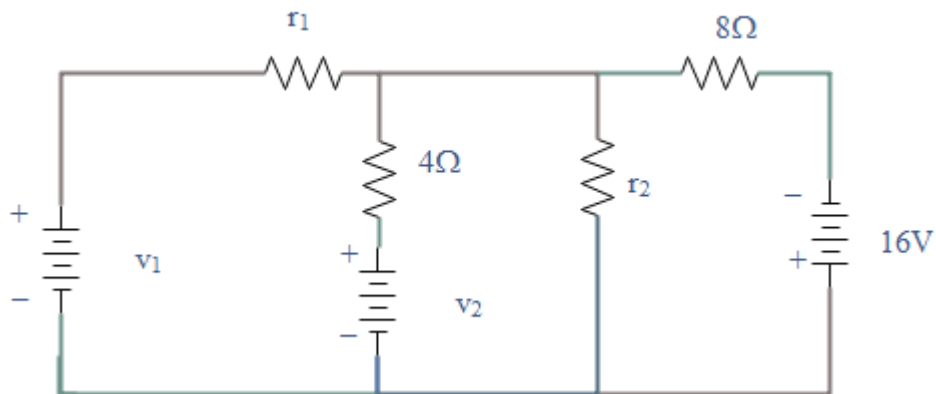


Figure2: Circuit with two unknown resistances and voltages

We will find the branch currents using both graph theoretic approach and Superposition theorem.

A. Graph Theoretic Approach:

The graph obtained from the circuit in Figure 2 is

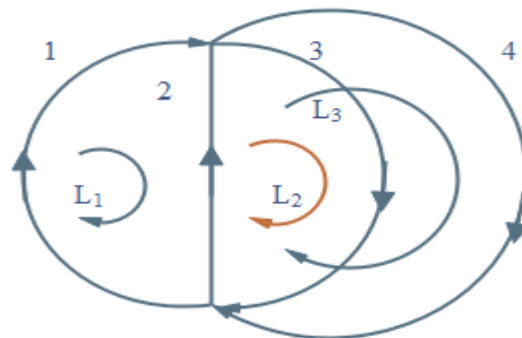


Figure 3: Graph from circuit

Here

$$B = \begin{bmatrix} 1 & -1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix}, \quad Z_b = \begin{bmatrix} r_1 & 0 & 0 & 0 \\ 0 & 4 & 0 & 0 \\ 0 & 0 & r_2 & 0 \\ 0 & 0 & 0 & 8 \end{bmatrix},$$

$$B^T = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad I_L = \begin{bmatrix} I_{L_1} \\ I_{L_2} \\ I_{L_3} \end{bmatrix}, \quad V_s = \begin{bmatrix} v_1 \\ v_2 \\ 0 \\ 16 \end{bmatrix}$$

So putting in

$$[B][Z_b][B^T][I_L] = -[B][V_s]$$

$$\Rightarrow \begin{bmatrix} r_1 + 4 & -4 & -4 \\ -4 & 4 + r_2 & 4 \\ -4 & 4 & 12 \end{bmatrix} \begin{bmatrix} I_{L_1} \\ I_{L_2} \\ I_{L_3} \end{bmatrix} = \begin{bmatrix} v_2 - v_1 \\ -v_2 \\ -(v_2 + 16) \end{bmatrix}$$

$$\Rightarrow XI_L = Y \text{ where } X = [B][Z_b][B^T], Y = -[B][V_s]$$

So there exist a unique circuit for unique values of 'r₁' and 'v₁'. Solving we have

$$I_{L_1} = \frac{(-16 - 3v_1)r_2 + 2r_2v_2 - 8v_1}{8r_1 + 3r_1r_2 + 8r_2} A (\text{Ampere}), I_{L_2} = \frac{16r_1 - 2r_1v_2 - 8v_1}{8r_1 + 3r_1r_2 + 8r_2} A$$

$$I_{L_3} = \frac{-64r_1 - 16r_1r_2 - (64 + 4v_1)r_2 - r_1r_2v_2}{32r_1 + 12r_1r_2 + 32r_2} A, \quad 8r_1 + 3r_1r_2 + 8r_2 \neq 0$$

Again to find the individual branch currents we have

$$[I_b] = [B^T][I_L]$$

$$\Rightarrow \begin{bmatrix} i_1 \\ i_2 \\ i_3 \\ i_4 \end{bmatrix} = \begin{bmatrix} \frac{(-16 - 3v_1)r_2 + 2r_2v_2 - 8v_1}{8r_1 + 3r_1r_2 + 8r_2} \\ \frac{-16r_1r_2 + 8v_1r_2 - 8r_1v_2 - r_1r_2v_2 - 8r_2v_2}{32r_1 + 12r_1r_2 + 32r_2} \\ \frac{16r_1 - 2r_1v_2 - 8v_1}{8r_1 + 3r_1r_2 + 8r_2} \\ \frac{-64r_1 - 16r_1r_2 - (64 + 4v_1)r_2 - r_1r_2v_2}{32r_1 + 12r_1r_2 + 32r_2} \end{bmatrix}$$

Here i_k denotes the currents through respective branches $k=1,2,3,4$

B. Verification of the Result by Superposition Theorem:

Now we will verify the above result with the help of Superposition Theorem. We will find the current through resistor 'r₂'.

From the given circuit, at first we consider the voltage source 'v₁' only. Other two voltage sources are removed by short circuit. Then the circuit becomes

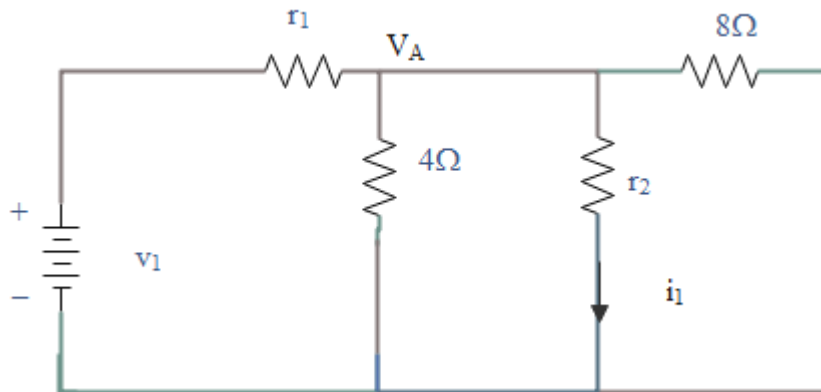


Figure 4: Circuit with voltage source 'v₁' only

Let us consider the node voltage at A be 'V_A'. Then from Kirchhoff's Current Law we have

$$\frac{V_A - v_1}{r_1} + \frac{V_A}{4} + \frac{V_A}{r_2} + \frac{V_A}{8} = 0$$

$$\Rightarrow V_A = \frac{8v_1 r_2}{8r_1 + 8r_2 + 3r_1 r_2} V$$

So current through resistance 'r₂' in this case $i_1 = \frac{V_A}{r_2} = \frac{8v_1 r_2}{8r_1 + 8r_2 + 3r_1 r_2} A = \frac{8v_1}{8r_1 + 8r_2 + 3r_1 r_2} A$

Now considering the voltage source 'v₂' only and short circuiting the other two sources we have

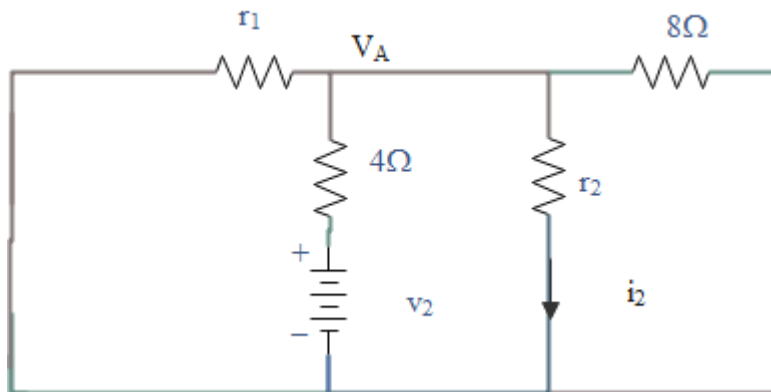


Figure 5: Circuit with voltage source 'v₂' only

Let us consider the node voltage at A be 'V_A'. Then from Kirchhoff's Current Law we have

$$\frac{V_A}{r_1} + \frac{V_A - v_2}{4} + \frac{V_A}{r_2} + \frac{V_A}{8} = 0$$

$$\Rightarrow V_A = \frac{2r_1 r_2 v_2}{8r_1 + 8r_2 + 3r_1 r_2} V$$

So current through resistance 'r₂' in this case $i_2 = \frac{V_A}{r_2} = \frac{2r_1 r_2 v_2}{8r_1 + 8r_2 + 3r_1 r_2} A = \frac{2r_1 v_2}{8r_1 + 8r_2 + 3r_1 r_2} A$

Now considering the voltage source 16 V only and short circuiting the other two sources we have

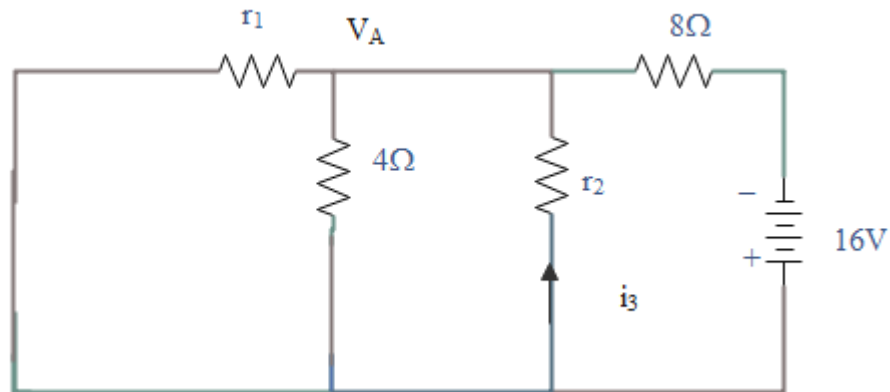


Figure 6: Circuit with 16V only

Let us consider the node voltage at A be ' V_A '. Then from Kirchhoff's Current Law we have

$$\frac{V_A}{r_1} + \frac{V_A}{4} + \frac{V_A}{r_2} + \frac{V_A + 16}{8} = 0$$

$$\Rightarrow V_A = \frac{-16r_1r_2}{8r_1 + 8r_2 + 3r_1r_2} V$$

So current through resistance ' r_2 ' $i_3 = \frac{V_A}{r_2} = \frac{-16r_1r_2}{8r_1 + 8r_2 + 3r_1r_2} A = \frac{-16r_1}{8r_1 + 8r_2 + 3r_1r_2} A$

So by Superposition Theorem total current through resistor ' r_2 '

$$i_{r_2} = i_1 + i_2 + i_3 = \frac{8v_1 + 2r_1v_2 - 16r_1}{8r_1 + 3r_1r_2 + 8r_2} A, \quad 8r_1 + 3r_1r_2 + 8r_2 \neq 0, \text{ which is same as the current obtained in matrix form}$$

through resistance ' r_2 ' (i_3 in case of matrix method).

In case of matrix method negative sign is just showing that current is in opposite direction.

VI. RESULTS

In $X I_L = Y$ we see that

- 1) The matrix X is symmetric.
- 2) The diagonal elements of X
 - $a_{11} = r_1 + 4 = \text{sum of resistances in } L_1$
 - $a_{22} = 4 + r_2 = \text{sum of resistances in } L_2$
 - $a_{33} = 12 = 4 + 8 = \text{sum of resistances in } L_3$

So the a_{ii} element (diagonal) of matrix X is the sum of the resistances of loop L_i of I_L .

- 3) Without Using nodal analysis and mesh analysis we can find the loop current and branch current by using graph theory only.

VII. CONCLUSION

From the above study we observed that the obtained current is same in both process i.e. by graph theoretic approach using network equilibrium equation and by Superposition theorem in a circuit with unknown resistances and voltages.

The values of currents can be obtained for different values of ' r_i ' and ' v_i ' by using network equilibrium equation without knowing the nodal and mesh analysis of electrical circuits.

REFERENCES

- [1] Banerjee Anirban, Jost Jurgen (2008). On the spectrum of the normalized graph Laplacian, Linear Algebra and its Applications, Elsevier, 428 (2008) 3015–3022.
- [2] Boruah Chanchal, Gogoi Krishna, Chutia Chandra. Analysis of Some Electrical Circuits with the Help of Graph Theory Using Network Equilibrium Equations, International Journal of Innovative Research in Science, Engineering and Technology, Vol. 6, Issue 1, January 2017.
- [3] Bryant Randal E. Symbolic Simulation—Techniques and Applications. Computer Science Department, Paper 240. (1990).
- [4] Chakrabarti Abhijit. Circuit Theory Analysis and Synthesis, Dhanpat Rai & Co.
- [5] Chang Ting-Jung, Tam Bit-Shun (2010). Graphs with maximal signless Laplacian spectral radius, Linear Algebra and its Applications 432 (2010) 1708–1733.
- [6] Cvetkovic Dragos, Simic Slobodan K. (2010). Towards a spectral theory of graphs based on the signless Laplacian, II, Linear Algebra and its Applications, Elsevier, 432 (2010) 2257–2272.
- [7] Deo Narsingh. Graph Theory with Applications to Engineering and Computer Science, Phi Learning Private Limited, New Delhi.
- [8] Heydari Abbas, Taeri Bijan (2008). On the characteristic polynomial of a special class of graphs and spectra of balanced trees, Linear Algebra and its Applications 429 (2008) 1744–1757.
- [9] Van Dam E.R., Haemers W.H., Koolen J.H., Spence E. (2006). Characterizing distance-regularity of graphs by the spectrum, Journal of Combinatorial Theory, Series A 113 (2006) 1805–1820.
- [10] Wang Wei, Xu Cheng-Xian (2010). On the asymptotic behavior of graphs determined by their generalized spectra, Discrete Mathematics 310 (2010) 70-76.