



e-ISSN: 2319-8753 | p-ISSN: 2320-6710

# IJIRSET

International Journal of Innovative Research in  
**SCIENCE | ENGINEERING | TECHNOLOGY**

# INTERNATIONAL JOURNAL OF INNOVATIVE RESEARCH

IN SCIENCE | ENGINEERING | TECHNOLOGY

Volume 9, Issue 12, December 2020

**ISSN** INTERNATIONAL  
STANDARD  
SERIAL  
NUMBER  
INDIA

**Impact Factor: 7.512**

9940 572 462

6381 907 438

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# Properties of $*R$ -closed sets and $*A$ -closed sets in Topology

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**ABSTRACT:** The purpose of this paper is to define and study a new class of sets, called the class of  $*R$ -closed sets, which lies between the class of  $(gsp)^*$ -closed sets and the class of  $gsp$ -closed sets. Also, we define and study of a new class of generalized closed sets, namely,  $*A$ -closed sets, which lies between the class of  $gs$ -closed sets and  $gsp$ -closed sets. The basic properties of these  $*R$ -closed sets,  $*R$ -open sets,  $*rT_{1/2}$ -spaces,  $*r$ -continuous functions,  $*r$ -irresolute functions,  $*r$ -regular spaces and  $*r$ -normal spaces and  $*A$ -closed sets,  $*A$ -open sets,  $*A_c$ -spaces,  $*A_s$ -spaces,  $\#A$ -spaces,  $*a$ -continuous functions,  $*a$ -irresolute functions and  $*a$ -connected spaces are studied in this paper.

**Mathematics Subject Classification :** 54A05, 54B05, 54C08, 54D10

**Keywords and Phrases :** semiopen sets,  $\alpha$ -open sets, semipreclosed sets, semi- $\alpha$ -open sets,  $gsp$ -closed sets and  $(gsp)^*$ -closed sets,  $gs$ -closed sets, semi- $\alpha$ -closed sets, semi- $\alpha$ -open sets, semicontinuity and semiprecontinuity.

## I. INTRODUCTION

After, introduction of  $g$ -closed sets in topology for the first time by Levine [15], it is noticed that many of the weaker forms of closed sets have been generalized in topological spaces. In 1963, 1965, 1982 and 1986, respectively, N.Levine [14], O.Njastad [28], A.S.Mashhour et al [17] and D.Andrijevic [1] have defined and studied the concepts of semiopen sets,  $\alpha$ -open sets, preopen sets and semipreopen sets in topology. In 1994, 1995, 1998, 2000, 2002, 2006, respectively, H.Maki et al [16], Dontchev [8], Noiri et al [29], M.K.R.S.Veera Kumar [40&42], P.Rajajeswari et al [34], have defined and studied the concepts of  $\alpha g$ -closed sets,  $gsp$ -closed sets,  $gp$ -closed sets,  $g^*$ -closed sets,  $\psi$ -closed sets and strongly  $\alpha g$ -closed sets. The purpose of this paper is to define and study a new class of sets, called the class of  $*R$ -closed sets, which lies between the class of  $(gsp)^*$ -closed sets and the class of  $gsp$ -closed sets. Also, we define and study of a new class of generalized closed sets, namely,  $*A$ -closed sets, which lies between the class of  $gs$ -closed sets and  $gsp$ -closed sets. The basic properties of these  $*R$ -closed sets,  $*R$ -open sets, Also, the basic properties of these  $*R$ -closed sets,  $*R$ -open sets,  $*rT_{1/2}$ -spaces,  $*r$ -continuous functions,  $*r$ -irresolute functions,  $*r$ -regular spaces and  $*r$ -normal spaces and  $*A$ -closed sets,  $*A$ -open sets,  $*A_c$ -spaces,  $*A_s$ -spaces,  $\#A$ -spaces,  $*a$ -Hausdorff spaces,  $*a$ -continuous functions,  $*a$ -irresolute functions and  $*a$ -connected spaces are studied in this paper.

## II. PRELIMINARIES

Throughout this paper  $(X, \tau)$  and  $(Y, \sigma)$  (or simply  $X$  and  $Y$ ) always means topological spaces on which no separation axioms are assumed unless explicitly stated. Let  $A$  be a subset of space  $X$ . We denote the closure of  $A$  and the interior of  $A$  by  $Cl(A)$  and  $Int(A)$  respectively. A subset  $A$  of a space  $X$  is called regular open (in brief,  $r$ -open) if  $A = Int Cl(A)$  and regular closed (in brief,  $r$ -closed) if  $A = Cl Int(A)$ .

The following definitions and results are useful in the sequel :

**Definition 2.1 :** A subset  $A$  of a space  $X$  is said to be:

- (i) preopen [17] if  $A \subset Int Cl(A)$ .
- (ii) semiopen [14] if  $A \subset Cl Int(A)$ .
- (iii) semipreopen [1] if  $A \subset Cl Int Cl(A)$ .
- (iv)  $\alpha$ -open [28] if  $A \subset Int Cl Int(A)$ .
- (v) semi- $\alpha$ -open [20] (in short,  $s\alpha$ -open) if  $A \subset Cl Int Cl Int(A)$

The complement of a preopen (resp. semiopen, semipreopen,  $\alpha$ -open, semi- $\alpha$ -open) set of a space  $X$  is called preclosed [9] (resp. semiclosed [4], semipreclosed [1],  $\alpha$ -closed [19],  $s\alpha$ -closed [30])

**Definition 2.2 :** The union of all semi- $\alpha$ -open sets contained in A is called the semi- $\alpha$ -interior of A and is denoted by  $s\alpha Int(A)$  [27].

$spInt(A)$  [ 1],  $sInt(A)$ [ 6] and  $\alpha Int(A)$  [19],  $pInt(A)$ [ 18] can be similarly defined .

**Definition 2.3 :** The intersection of all semi- $\alpha$ -closed sets containing A is called the semi- $\alpha$ -closure of A and is denoted by  $s\alpha Cl(A)$  [27].

$spCl(A)$  [ 1],  $sCl(A)$ [5] and  $\alpha Cl(A)$  [19],  $pCl(A)$  [ 9] can be similarly defined .

**Definition 2.4 [22]:** The union of all gsp-open sets contained in A is called the gsp-interior of A and is denoted by  $gspInt(A)$ .

$gsInt(A)$  [ 21],  $sgInt(A)$ [ 21] and  $\alpha gInt(A)$  [ 26],  $gpInt(A)$ [ 26] can be similarly defined .

**Definition 2.5 [22]:** The intersection of all gsp-closed sets containing A is called the gsp-closure of A and is denoted by  $gspCl(A)$ .

$gsCl(A)$  [ 21],  $sgCl(A)$ [21] and  $\alpha gCl(A)$  [26],  $gpCl(A)$  [ 26] can be similarly defined .

**Lemma 2.6 [22] :** For every subset  $A \subset X$ , the following hold.

- (i)  $gspCl(X-A) = X-gspInt(A)$  .
- (ii)  $gspInt(X-A) = X-gspCl(A)$ .

**DEFINITION 2.7 :** A subset A of a space X is called :

- (i) g-closed set [15] if  $Cl(A) \subset U$  whenever  $A \subset U$  and U is open in X .
- (ii) gs-closed set [2] if  $sCl(A) \subset U$  whenever  $A \subset U$  and U is open in X .
- (iii) sg-closed set [3] if  $sCl(A) \subset U$  whenever  $A \subset U$  and U is semiopen in X .
- (iv) gsp-closed set [8] if  $spCl(A) \subset U$  whenever  $A \subset U$  and U is open in X .
- (v) (gsp)\*-closed set [12] if  $Cl(A) \subset U$  whenever  $A \subset U$  and U is gsp-open in X .
- (vi)  $\alpha^*$ -closed set [46] if  $Cl(A) \subset U$  whenever  $A \subset U$  and U is  $\alpha$ -open in X .
- (vii)  $\alpha g$ -closed set [16] if  $\alpha Cl(A) \subset U$  whenever  $A \subset U$  and U is open in X .
- (viii)  $G^*$ -closed set [ 23] if  $gspCl(A) \subset U$  whenever  $A \subset U$  and U is g-open in X .
- (ix) \*G-closed set [25] if  $gsCl(A) \subset U$  whenever  $A \subset U$  and U is g-open in X .
- (x)  $N^*$ -closed set [26] if  $\alpha gCl(A) \subset U$  whenever  $A \subset U$  and U is g-open in X .
- (xi) \*N-closed set [26] if  $gpCl(A) \subset U$  whenever  $A \subset U$  and U is g-open in X .
- (xii)  $\alpha g^*$ -closed set [37] if  $\alpha Cl(A) \subset U$  whenever  $A \subset U$  and U is  $\alpha g$ -open in X .
- (xiii)  $g^\#$ -closed set [43] if  $Cl(A) \subset U$  whenever  $A \subset U$  and U is  $\alpha g$ -open in X .
- (xiv)  $g^\#s$ -closed set [44] if  $sCl(A) \subset U$  whenever  $A \subset U$  and U is  $\alpha g$ -open in X .
- (xv) gp-closed set [29] if  $pCl(A) \subset U$  whenever  $A \subset U$  and U is open in X .
- (xvi)  $g^*$ -closed set [40] if  $Cl(A) \subset U$  whenever  $A \subset U$  and U is g-open in X .
- (xvii)  $g^*sp$ -closed set [24] if  $spCl(A) \subset U$  whenever  $A \subset U$  and U is g-open in X .
- (xviii) (gs)\*-closed set [11] if  $Cl(A) \subset U$  whenever  $A \subset U$  and U is gs-open in X .
- (xix)  $g^*s$ -closed set [20] if  $sCl(A) \subset U$  whenever  $A \subset U$  and U is gs-open set in X .
- (xx) strongly  $\alpha g$ -closed set [ 34] if  $\alpha Cl(A) \subset U$  whenever  $A \subset U$  and U is g-open set in X .
- (xxi)  $gp^*$ -closed set [ 37] if  $Cl(A) \subset U$  whenever  $A \subset U$  and U is gp-open set in X .
- (xxii)  $g^\wedge$ -closed set [45] if  $Cl(A) \subset U$  whenever  $A \subset U$  and U is semiopen set in X .
- (xxiii)  $g^\wedge$ -closed set [41] if  $Cl(A) \subset U$  whenever  $A \subset U$  and U is  $g^\wedge$ -open set in X .

- (xxiv)  $gsg$ -closed [39] if  $Cl(A) \subset U$ , whenever  $A \subset U$  and  $U$  is  $sg$ -open set in  $X$ .
- (xxv)  $\alpha gs$ -closed set [33] if  $\alpha Cl(A) \subset U$  whenever  $A \subset U$  and  $U$  is semiopen set in  $X$ .
- (xxvi)  $\psi$ -closed set [42] if  $sCl(A) \subset U$  whenever  $A \subset U$  and  $U$  is  $sg$ -open set in  $X$ .
- (xxvii)  $\psi^*$ -closed set [47] if  $sCl(A) \subset U$  whenever  $A \subset U$  and  $U$  is  $\psi$ -open set in  $X$ .
- (xxviii) Strongly  $gs$ -closed [10] if  $sCl(A) \subset U$  whenever  $A \subset U$  and  $U$  is  $g$ -open set in  $X$ .

The complement of a  $m$ -closed set of a space  $X$  is called, respectively,  $m$ -open set in  $X$  where  $m = g, gs, sg, gsp, (gsp)^*, \dots, st, gs$ -closed set.

**Definition 2.8** [35 &36]: The union of all  $g$ -open sets contained in  $A$  is called the  $g$ -interior of  $A$  and is denoted by  $gInt(A)$ .

**Definition 2.9** [35 &36]: The intersection of all  $g$ -closed sets containing  $A$  is called the  $g$ -closure of  $A$  and is denoted by  $gCl(A)$ .

**Definition 2.10** [36]: A subset  $A$  of a space  $X$  is called semi $^*$ - $\alpha$ -open if  $A \subset gCl(\alpha Int(A))$ .

**Definition 2.11** [35]: A subset  $A$  of a space  $X$  is called semi $^*$ - $\alpha$ -closed if  $gInt(\alpha Cl(A)) \subset A$ .

**Lemma 2.12** [27]: For  $s\alpha Cl(A)$  and  $s\alpha Int(A)$ , the following hold.

- (i)  $s\alpha Cl(X-A) = X - s\alpha Int(A)$ .
- (ii)  $s\alpha Int(X-A) = X - s\alpha Cl(A)$ .

**Definition 2.13:** A function  $f : X \rightarrow Y$  is called :

- (i) semicontinuous [14] if the inverse image of each open set of  $Y$  is semiopen in  $X$ .
- (ii)  $\alpha$ -continuous [19] if the inverse image of each open set of  $Y$  is  $\alpha$ -open in  $X$ .
- (iii)  $s\alpha$ -continuous [30] if the inverse image of each open set of  $Y$  is  $s\alpha$ -open in  $X$ .

**Definition 2.19** : A function  $f : X \rightarrow Y$  is called :

- (i)  $sg$ -continuous [38] if the inverse image of each closed set of  $Y$  is  $sg$ -closed in  $X$ .
- (ii)  $gs$ -continuous [7] if the inverse image of each closed set of  $Y$  is  $gs$ -closed in  $X$ .
- (iii)  $gsp$ -continuous [8] if the inverse image of each closed set of  $Y$  is  $gsp$ -closed in  $X$ .
- (iv)  $\alpha g$ -continuous [16] if the inverse image of each closed set of  $Y$  is  $\alpha g$ -closed in  $X$ .

**Definition 2.20** [31]: A space  $X$  is said to be semi-connected if  $X$  cannot be written as the disjoint union of two nonempty semi-open sets in  $X$ .

**Definition 2.21** [32]: A space  $X$  is said to be  $\beta$ -connected (=semipre-connected) if  $X$  cannot be written as the disjoint union of two nonempty  $\beta$ -open (=semipre-open) sets in  $X$ .

### III.PROPERTIES OF \*R-CLOSED SETS

We introduce the following .

**Definition 3.1:** A subset  $A$  of a space  $X$  is said to be  $^*R$ -closed set if  $s\alpha Cl(A) \subset U$  whenever  $A \subset U$  and  $U$  is  $g$ -open in  $X$ .

The complement of a  $^*R$ -closed set is called  $^*R$ -open set.

The family of all  $^*R$ -open sets of  $X$  is denoted by  $^*RO(X)$  and that of all  $^*R$ -closed sets of  $X$  is denoted by  $^*RF(X)$ . Also, the family of all  $^*R$ -open sets of  $X$  containing a point  $x$  is denoted by  $^*RO(x)$ .

**Lemma 3.2** : Every closed set (and hence semiclosed set,  $\alpha$ -closed set and  $s\alpha$ -closed set) is  $^*R$ -closed set.

**Lemma 3.3** : Every,  $(gsp)^*$ -closed (and hence  $g^*$ -closed,  $g^\#$ -closed,  $gp^*$ -closed,  $\alpha g^*$ -closed,  $g^*s$ -closed,  $g^\#s$ -closed,  $(gs)^*$ -closed,  $st, \alpha g$ -closed,  $st, gs$ -closed) set is  $^*R$ -closed set.

**Lemma 3.4:**  $*R$ -closed set is  $g$ - $*sp$ -closed set and hence is  $gsp$ -closed set &  $I^*$ -closed set.

**Lemma 3.5 :** Every  $g$ -closed set ( and hence  $G^*$ -closed set ,  $*G$ -closed set ,  $N^*$ -closed set ,  $*N$ -closed set ) and  $*R$ -closed set are independent each other..

We , define the following.

**Definition 3.6:** A subset  $A$  of space  $X$  is called  $*R$ -open set if  $F \subset \text{Int}(A)$  whenever  $F \subset A$  and  $F$  is  $g$ -closed set in  $X$ .

**Definition 3.7:** Let  $x \in X$  .A subset  $U \subset X$  is called  $*r$ -neighbourhood (in brief ,  $*r$ -nbd.) of  $x$  in  $X$  if there exists  $A \in *RO(x)$  such that  $A \subset U$  (i.e.  $x \in A \subset U$ ) .

**Definition 3.8 :** The union of all  $*R$ -open sets which contained in  $A$  is called the  $*R$ -interior of  $A$  and is denoted by  $*rInt(A)$  .

**Definition 3.9 :** The intersection of all  $*R$ -closed sets containing set  $A$  is called the  $*R$ -closure of  $A$  and is denoted  $*rCl(A)$ .

**Lemma 3. 10 :** Let  $x \in X$ , then  $x \in *rCl(A)$  if and only if  $V \cap A \neq \emptyset$  for every  $*R$ -open set  $V$  containing  $x$ .

**Remark 3.11 :** The  $*r$ -closure of a set  $A$  is  $*R$ -closed set.

We , state the following.

**Lemma 3.12:** Let  $A$  and  $B$  be subsets of  $X$ . Then , the following statements hold for the  $*R$ -closure operator :

- (i)  $*rCl(\emptyset) = \emptyset$  and  $*rCl(X) = X$  .
- (ii)  $A$  is  $*R$ -closed set iff  $A = *rCl(A)$  .

(iii) If  $A \subset B$  ,  $*rCl(A) \subset *rCl(B)$  .

(iv)  $*rCl(*rCl(A)) = *rCl(A)$  .

(v)  $*rCl(A)$  is  $*R$ -closed set in  $X$  containing point  $x$ .

**Lemma 3.13 :** For every subset  $W \subset X$  , the following are hold :

- (i)  $*rCl(X-W) = X - *rInt(A)$ .
- (ii)  $*rInt(X-W) = X - *rCl(A)$ .

**Lemma 3.14:** Let  $A$  and  $B$  be subsets of  $X$ . Then,

- (i)  $*rCl(A) \cup *rCl(B) \subset *rCl(A \cup B)$
- (ii)  $*rCl(A \cap B) \subset *rCl(A) \cap *rCl(B)$

**Theorem 3.15:** If  $\text{Int}(A) \subset B \subset A$  and  $A$  is  $*R$ -open, then  $B$  is  $*R$ -open.

**Proof:** Obvious.

**Theorem 3.16 :** Let  $A \subset X$  be an  $*R$ -open set ,then  $A$  is  $*r$ -nbd. of each of its points.

**Proof :** Since for every point  $x$  in a  $*R$ -open set  $A$  ,  $x \in A \subset A$ .

Next , we give some properties of  $*R$ -interior operator.

**Lemma 3.17 :**  $A$  is  $*R$ -open iff  $A = *rInt(A)$ .

**Lemma 3.18 :** Let  $A$  and  $B$  be subsets of a spa $X$  .Then,

- (i) If  $A \subset B$  then  $*rInt(A) \subset *rInt(B)$  .
- (ii)  $*rInt(A) \cup *rInt(B) \subset *rInt(A \cup B)$ .
- (iii)  $*rInt(A \cap B) \subset *rInt(A) \cap *rInt(B)$ .

We , define the following .

**Definition 3.19 :**  $*rfr(A)$  is called the  $*R$ -frontier of  $A \subset X$  and is defined by  $*rfr(A) = *rCl(A) - *rInt(A)$ .

Clearly,  $*rfr(A) \subset \text{fr}(A)$  , frontier of  $A \subset X$ .

We. state the following.

**Lemma 3.20:** For aubset  $A$  of a space  $X$  , the following statements hold :

- (i)  $*rCl(A) = *rInt(A) \cup *rfr(A)$  .
- (ii)  $*rInt(A) \cap *rfr(A) = \emptyset$ .
- (iii)  $*rfr(A) = *rCl(A) \cap *rCl(X-A)$ .

**Proof :**

We , state the following .

**Lemma 3.21 :**  $*rfr(A)$  is  $*R$ -closed set in  $X$ .

We, define the following.

**Definition 3.22:** A subset  $A \subset X$  is called  $*R$ -clopen if it is both  $*R$ -open set and  $*R$ -closed set.

The family of all  $*R$ -clopen sets of  $X$  is denoted by  $*RCO(X)$ .



Clearly, every clopen set is  $*R$ -clopen set.

We give the following.

**Lemma 3.23 :** In a space  $X$ , if  $U$  is  $*R$ -open set then  $*rCl(U)$  is  $*R$ -clopen set.

**Lemma 3.24 :**  $*rfr(A) = \emptyset$  iff  $A \in *RCO(X)$ .

**Lemma 3.25 :** Let  $A \subset X$ , then the following hold true.

(i)  $*rfr(A) = *rfr(X-A)$ .

(ii)  $A \in *RO(X)$  iff  $*rfr(A) \subset (X-A)$  i.e.,  $A \cap *rfr(A) = \emptyset$ .

(iii)  $A \in *RF(X)$  iff  $*rfr(A) \subset A$ .

(iv)  $*rfr(*rfr(*rfr(A))) = *rfr(*rfr(A))$ .

We, define the following.

**Definition 3.26:** A function  $f: X \rightarrow Y$  is called  $*r$ -continuous if  $f^{-1}(V)$  is  $*R$ -closed in  $X$  for every closed subset  $V$  of  $Y$ .

Clearly, every continuous function is  $*r$ -continuous function.

In view of Lemma -3.5, we give the following.

**Lemma 3.27 :** Let  $f: X \rightarrow Y$  be a function, then the following hold.

(i) If  $f$  is semicontinuous, then it is  $*r$ -continuous.

(ii) If  $f$  is  $\alpha$ -continuous, then it is  $*r$ -continuous.

(iii) If  $f$  is  $s\alpha$ -continuous, then it is  $*r$ -continuous.

We, define the following.

**Definition 3.28 :** A function  $f: X \rightarrow Y$  is called  $*r$ -irresolute continuous if  $f^{-1}(V)$  is  $*R$ -closed in  $X$  for every  $*R$ -closed subset  $V$  of  $Y$ .

**Definition 3.29 :** A function  $f: X \rightarrow Y$  is called strongly  $*r$ -continuous if the inverse image of each  $*R$ -open set of  $Y$  is open in  $X$ .

**Definition 3.30 :** A function  $f: X \rightarrow Y$  is called  $\alpha$   $*r$ -continuous if the inverse image of each  $\alpha$ -open set of  $Y$  is  $*R$ -open in  $X$ .

**Definition 3.31 :** A function  $f: X \rightarrow Y$  is called  $s$   $*r$ -continuous if the inverse image of each semiopen set of  $Y$  is  $*R$ -open in  $X$ .

**Definition 3.32:** A function  $f: X \rightarrow Y$  is called pre- $*r$ -continuous if the inverse image of each  $s\alpha$ -open set of  $Y$  is  $*R$ -open in  $X$ .

We, state the following

**Theorem 3.33 :** Let  $f: X \rightarrow Y$  be a function, then the following are equivalent :

(i)  $f$  is  $*r$ -continuous.

(ii) The inverse image of each open set  $A$  in  $Y$  is  $*R$ -open in  $X$ .

**Theorem 3.34 :** Let  $f: X \rightarrow Y$  be  $*r$ -continuous and  $g: Y \rightarrow Z$  be continuous, then  $g \circ f: X \rightarrow Z$  is  $*r$ -continuous.

**Theorem 3.35 :** A function  $f: X \rightarrow Y$  is  $*r$ -irresolute if for each  $*R$ -open set  $A$  of  $Y$ ,  $f^{-1}(A)$  is  $*R$ -open set in  $X$ .

**Theorem 3.36 :** If  $f: X \rightarrow Y$  is  $*r$ -irresolute function, then for each  $B \subset X$ ,  $f(*rCl(A)) \subset *rCl(f(A))$ .

We, state the following.

**Theorem 3.37 :** Let  $f: X \rightarrow Y$  be  $*r$ -irresolute and  $g: Y \rightarrow Z$  be  $\alpha$   $*r$ -continuous, then  $g \circ f: X \rightarrow Z$  is  $\alpha$   $*r$ -continuous.

**Theorem 3.38:** Let  $f: X \rightarrow Y$  be  $*r$ -irresolute and  $g: Y \rightarrow Z$  be  $s$   $*r$ -continuous, then  $g \circ f: X \rightarrow Z$  is  $s$   $*r$ -continuous.

**Theorem 3.39 :** Let  $f: X \rightarrow Y$  be  $*r$ -irresolute and  $g: Y \rightarrow Z$  be  $s\alpha$   $*r$ -continuous, then  $g \circ f: X \rightarrow Z$  is  $s\alpha$   $*r$ -continuous.

We, define the following

**Definition 3.40 :** A bijective function  $f: X \rightarrow Y$  is called  $*r$ -homeomorphism if both  $f$  and  $f^{-1}$  are  $*r$ -irresolute functions.

We, define the following.

**Definition 3.41 :** A space  $X$  is called  $*r$ -regular if for each  $*R$ -closed set  $F$  and each point  $x \in X-F$ , there exist disjoint  $*R$ -open sets  $U$  and  $V$  in  $X$  such that  $x \in U$  and  $F \subset V$ .

**Definition 3.42 :** A space  $X$  is called  $**r$ -regular if for each closed set  $F$  and each point  $x \in X-F$ , there exist disjoint  $*R$ -open sets  $U$  and  $V$  in  $X$  such that  $x \in U$  and  $F \subset V$ .

Clearly, every  $*r$ -regular space is  $**r$ -regular space.



**Definition 3.43 :** A space  $X$  is called  $*r$ -normal if for each pair of disjoint  $*R$ -closed sets  $A$  and  $B$  of  $X$ , there exist disjoint  $*R$ -open sets  $U$  and  $V$  in  $X$  such that  $A \subset U$  and  $B \subset V$ .

**Definition 3.44 :** A space  $X$  is called  $**r$ -normal if for each pair of disjoint closed sets  $A$  and  $B$  of  $X$ , there exist disjoint  $*R$ -open sets  $U$  and  $V$  in  $X$  such that  $A \subset U$  and  $B \subset V$ .

Clearly, every  $*r$ -normal space is  $**r$ -normal space.

**Definition 3.45 :** A space  $X$  is called  $*r-T_1$  if for each pair of distinct points  $x, y \in X$ , there exists a  $*R$ -open set  $A$  containing  $x$  but not  $y$  and a  $*R$ -open set  $B$  containing  $y$  but not  $x$ .

Clearly, the following is stated.

**Lemma 3.46 :** A space  $X$  is  $*r-T_1$  iff every singleton is  $*R$ -closed set.

We, prove the following.

**Theorem 3.47 :** Every  $*r-T_1$  space and  $*r$ -normal space  $X$  is  $*r$ -regular space.

**Proof :**

**Theorem 3.48 :**  $*r$ -homeomorphic image of  $*r$ -normal space is  $*r$ -normal, i.e.,  $*r$ -normality is  $*r$ -topological property.

**Proof :** Obvious.

#### IV. PROPERTIES OF $*A$ -CLOSED SETS

We, define the following.

**Definition 4.1 :** A subset  $A$  of a space  $X$  is said to be  $*A$ -closed set if  $s\alpha Cl(A) \subset U$  whenever  $A \subset U$  and  $U$  is open in  $X$ .

The complement of a  $*A$ -closed set is called  $*A$ -open set.

**Definition 4.2 :** A subset  $A$  is called  $\#A$ -closed if  $s^*\alpha Cl(A) \subset U$ , whenever  $A \subset U$  and  $U$  is open set in  $X$ .

In view of Definitions-2.4 and 3.2- 3.8, we have the following implications hold true :

**Lemma 4.3:** Every closed (and hence  $\alpha$ -closed, semiclosed, semi- $\alpha$ -closed) set is  $*A$ -closed set.

**Lemma 4.4 :** Every semipreclosed set and  $*A$ -closed set are independent.

**Lemma 4.5 :** Every  $*A$ -closed set is  $gsp$ -closed set.

**Lemma 4.6:** Every  $g$ -closed (and hence  $sg$ -closed,  $gs$ -closed,  $\alpha^*$ -closed,  $\alpha g$ -closed,  $\alpha g^*$ -closed,  $(gsp)^*$ -closed,  $g^\#$ -closed,  $g^\#s$ -closed,  $\#A$ -closed,  $\alpha g s$ -closed,  $\psi$ -closed,  $gsg$ -closed,  $*R$ -closed,  $\psi^*$ -closed,  $st.\alpha g$ -closed,  $g^\wedge$ -closed) set is  $*A$ -closed set.

**Lemma 4.7 :** A subset  $A$  of space  $X$  is called  $*A$ -open set if  $F \subset s\alpha Int(A)$  whenever  $F \subset A$  and  $F$  is closed set in  $X$ .

We, define the following

**Definition 4.8 :** The union of all  $*A$ -open sets which contained in  $A$  is called the  $*A$ -interior of  $A$  and is denoted by  $*aInt(A)$ .

**Definition 4.9 :** The intersection of all  $*A$ -closed sets containing set  $A$  is called the  $*A$ -closure of  $A$  and is denoted  $*aCl(A)$ .

**Lemma 4.10 :** Let  $x \in X$ , then  $x \in *aCl(A)$  if and only if  $V \cap A \neq \emptyset$  for every  $*A$ -open set  $V$  containing  $x$ .

**Remark 4.11 :** The  $*A$ -closure of a set  $A$  is  $*A$ -closed set.

We, state the following.

**Lemma 4.12:** Let  $A$  and  $B$  be subsets of  $X$ . Then

- (i)  $*aCl(\emptyset) = \emptyset$  and  $*aCl(X) = X$
- (ii) If  $A \subset B$ ,  $*aCl(A) \subset *aCl(B)$ .
- (iii)  $A \subset *aCl(A)$

**Lemma 4.13:** Let  $A$  and  $B$  be subsets of  $X$ . Then

- (i)  $*aCl(A) = *aCl(*aCl(A))$
- (ii)  $*aCl(A) \cup *aCl(B) \subset *aCl(A \cup B)$
- (iii)  $*aCl(A \cap B) \subset *aCl(A) \cap *aCl(B)$

**Theorem 4.14:** If  $s\alpha Int(A) \subset B \subset A$  and  $A$  is  $*A$ -open, then  $B$  is  $*A$ -open.

**Proof:** Obvious.

**Lemma 4.15 :** For any  $A \subset X$ , then  $s\alpha Int(s\alpha Cl(A) - A) = \emptyset$ .

**Proof :** Obvious.

**Lemma 4.16:** If  $A \subset X$  is  $*A$ -closed, then  $s\alpha Cl(A) - A$  is  $*A$ -open.

**Proof :** Obvious.

We, define the following.

**Definition 4.17 :** A function  $f: X \rightarrow Y$  is called  $*a$ -continuous if  $f^{-1}(V)$  is  $*A$ -closed in  $X$  for every closed subset  $V$  of  $Y$ .



Clearly, every  $*a$ -continuous function is  $gsp$ -continuous.

**Lemma 4.18:** Let  $f : X \rightarrow Y$  be a function. Then,

- (i) If  $f$  is continuous (and hence  $\alpha$ -continuous, semicontinuous) function, then it is  $*a$ -continuous.
- (ii) If  $f$  is  $a^*$ -continuous function, then it is  $*a$ -continuous
- (iii) If  $f$  is  $sg$ -continuous function, then it is  $*a$ -continuous.
- (iv) If  $f$  is  $gs$ -continuous function, then it is  $*a$ -continuous.
- (v) If  $f$  is  $\alpha g$ -continuous function, then it is  $*a$ -continuous.

**Definition 4.29:** A function  $f : X \rightarrow Y$  is called  $*a$ -irresolute continuous if  $f^{-1}(V)$  is  $*A$ -closed in  $X$  for every  $*A$ -closed subset  $V$  of  $Y$ .

**Lemma 3.20:** Every  $*a$ -irresolute function is  $*a$ -continuous

**Proof:** Suppose  $f : X \rightarrow Y$  is  $*a$ -irresolute. Let  $V$  be any closed subset of  $Y$ . Then  $V$  is  $*A$ -closed set in  $Y$ , lemma - 3.2. Since  $f$  is  $*a$ -irresolute,  $f^{-1}(V)$  is  $*A$ -closed in  $X$ . This proves the lemma.

We, define the following.

**Definition 4.21:** A function  $f : X \rightarrow Y$  is called strongly  $*a$ -continuous if the inverse image of each  $*A$ -open set of  $Y$  is open in  $X$ .

**Definition 4.22:** A function  $f : X \rightarrow Y$  is called  $\alpha^*$ -continuous if the inverse image of each  $\alpha$ -open set of  $Y$  is  $*A$ -open in  $X$ .

**Definition 4.23:** A function  $f : X \rightarrow Y$  is called  $s^*$ -continuous if the inverse image of each semiopen set of  $Y$  is  $*A$ -open in  $X$ .

**Definition 4.24:** A function  $f : X \rightarrow Y$  is called  $sp^*$ -continuous if the inverse image of each semipreopen set of  $Y$  is  $*A$ -open in  $X$ .

**Definition 4.25:** A function  $f : X \rightarrow Y$  is called  $g^*$ -continuous if the inverse image of each  $g$ -open set of  $Y$  is  $*A$ -open in  $X$ .

We, state the following

**Theorem 4.26:** Let  $f : X \rightarrow Y$  be a function, then the following are equivalent :

- (iii)  $f$  is  $*a$ -continuous.
- (iv) The inverse image of each open set  $A$  in  $Y$  is  $*A$ -open in  $X$ .

**Theorem 4.27:** Let  $f : X \rightarrow Y$  be  $*a$ -continuous and  $g : Y \rightarrow Z$  be continuous, then  $g \circ f : X \rightarrow Z$  is  $*a$ -continuous.

**Theorem 4.28:** A function  $f : X \rightarrow Y$  is  $*a$ -irresolute if for each  $*A$ -open set  $A$  of  $Y$ ,  $f^{-1}(A)$  is  $*A$ -open set in  $X$ .

**Theorem 4.29:** Let  $f : X \rightarrow Y$  and  $g : Y \rightarrow Z$  be two  $*a$ -irresolute functions, then  $g \circ f$  is also  $*a$ -irresolute function.

**Theorem 4.30:** Let  $f : X \rightarrow Y$  be a function. Then the following are equivalent.

- (i)  $f$  is  $*a$ -continuous
- (ii) The inverse image of each open set  $Y$  is  $*A$ -open in  $X$
- (iii) The inverse image of each closed set in  $Y$  is  $*A$ -closed in  $X$

In view of Definition -3.10 and Lemma-3.11, we prove the following.

**Theorem 4.31:** If a function  $f : X \rightarrow Y$  is  $*a$ -continuous then  $f(*aCl(A)) \subseteq Cl(f(A))$  for every subset  $A$  of  $X$ .

**Theorem 4.32:** If a function  $f : X \rightarrow Y$  is  $*a$ -irresolute then  $f(*aCl(A)) \subseteq *aCl(f(A))$  for every subset  $A$  of  $X$ .

**Proof:** Similar to Th.3.23.

**Theorem 4.33:** Let  $f : X \rightarrow Y$  be  $*a$ -continuous and  $g : Y \rightarrow Z$  be continuous, then  $g \circ f : X \rightarrow Z$  be  $*a$ -continuous.

**Proof:** Let  $V$  be any open subset of  $Z$ . Then  $g^{-1}(V)$  is open in  $Y$ , since  $g$  is continuous function. Again,  $f$  is  $*a$ -continuous and  $g^{-1}(V)$  is open set in  $Y$  then  $f^{-1}(g^{-1}(V)) = (g \circ f)^{-1}(V)$  is  $*A$ -open in  $X$ . This shows that  $g \circ f$  is  $*a$ -continuous.

**Theorem 4.34:** Let  $f : X \rightarrow Y$  be  $*a$ -continuous and  $g : Y \rightarrow Z$  be strongly  $*a$ -continuous, then  $g \circ f : X \rightarrow Z$  be  $*a$ -irresolute.

We, define the following

**Definition 4.35:** A function  $f : X \rightarrow Y$  is called contra  $*a$ -continuous if  $f^{-1}(V)$  is  $*A$ -closed in  $X$  for each open set  $V$  in  $Y$

**Theorem 4.36:** Let  $f : X \rightarrow Y$  be  $*a$ -continuous and  $g : Y \rightarrow Z$  be contra-continuous, then  $g \circ f : X \rightarrow Z$  be contra  $*a$ -continuous.

**Proof:** Obvious.

We, define the following.

**Definition 4.37:** A function  $f : X \rightarrow Y$  is called pre- $*a$ -continuous if the inverse image of each semi- $\alpha$ -open set of  $Y$  is  $*A$ -open in  $X$ .





Clearly, every  $s^*a$ -continuous function is pre- $s^*a$ -continuous function, since every semiopen set is semi- $\alpha$ -open set.

**Theorem 4.38:** Let  $f : X \rightarrow Y$  be  $s^*a$ -irresolute function and  $g : Y \rightarrow Z$  be  $s^*a$ -continuous then  $g \circ f : X \rightarrow Z$  is  $s^*a$ -continuous function.

**Proof :** Obvious.

**Theorem 4.39 :** Let  $f : X \rightarrow Y$  be  $s^*a$ -continuous function and  $g : Y \rightarrow Z$  be irresolute then  $g \circ f : X \rightarrow Z$  is  $s^*a$ -continuous function.

**Proof :** Obvious.

**Theorem 4.40:** Let  $f : X \rightarrow Y$  be  $s^*a$ -irresolute function and  $g : Y \rightarrow Z$  be  $sp^*a$ -continuous then  $g \circ f : X \rightarrow Z$  is  $sp^*a$ -continuous function.

**Proof :** Obvious.

**Theorem 4.41 :** Let  $f : X \rightarrow Y$  be  $sp^*a$ -continuous function and  $g : Y \rightarrow Z$  be semipre-irresolute then  $g \circ f : X \rightarrow Z$  is  $sp^*a$ -continuous function.

**Proof :** Obvious.

**Theorem 4.42:** Let  $f : X \rightarrow Y$  be  $s^*a$ -irresolute function and  $g : Y \rightarrow Z$  be  $\alpha^*a$ -continuous then  $g \circ f : X \rightarrow Z$  is  $\alpha^*a$ -continuous function.

**Proof :** Obvious.

**Theorem 4.43:** Let  $f : X \rightarrow Y$  be  $\alpha^*a$ -continuous function and  $g : Y \rightarrow Z$  be  $\alpha$ -irresolute then  $g \circ f : X \rightarrow Z$  is  $\alpha^*a$ -continuous function.

**Proof :** Obvious.

We, define the following.

**Definition 4.44:** A function  $f : X \rightarrow Y$  is called semi- $\alpha$ -continuous if the inverse image of each open set of  $Y$  is semi- $\alpha$ -open in  $X$ .

Clearly, every semi- $\alpha$ -continuous function is semiprecontinuous and every semicontinuous function is semi- $\alpha$ -continuous.

The routine proofs of the following are omitted.

**Theorem 4.45:** Let  $f : X \rightarrow Y$  be  $\alpha^*a$ -continuous function and  $g : Y \rightarrow Z$  be  $\alpha$ -continuous then  $g \circ f : X \rightarrow Z$  is  $s^*a$ -continuous function.

**Theorem 4.46:** Let  $f : X \rightarrow Y$  be  $s^*a$ -continuous function and  $g : Y \rightarrow Z$  be semicontinuous then  $g \circ f : X \rightarrow Z$  is  $s^*a$ -continuous function.

**Theorem 4.47:** Let  $f : X \rightarrow Y$  be  $sp^*a$ -continuous function and  $g : Y \rightarrow Z$  be semipre-continuous then  $g \circ f : X \rightarrow Z$  is  $s^*a$ -continuous function.

**Theorem 4.48:** Let  $f : X \rightarrow Y$  be pre- $s^*a$ -continuous function and  $g : Y \rightarrow Z$  be semi- $\alpha$ -continuous function then  $g \circ f : X \rightarrow Z$  is  $s^*a$ -continuous function.

Next, we introduce the following.

**Definition 4.49 :** A space  $X$  is said to be a :

- (i)  $*A_c$ -space if every  $*A$ -closed set in it is closed.
- (ii)  $*A_s$ -space if every  $*A$ -closed set in it is semiclosed.
- (iii)  $\#A$ -space if every  $*A$ -closed set in it is  $\alpha$ -closed.
- (iv)  $gT_a$ -space if every  $*A$ -closed set in it is  $g$ -closed.
- (v)  $gspT_a$ -space if every  $gsp$ -closed set in it is  $*A$ -closed.
- (vi)  $gsT_a$ -space if every  $*A$ -closed set in it is  $gs$ -closed.
- (vii)  $sgT_a$ -space if every  $*A$ -closed set in it is  $sg$ -closed.
- (viii)  $\alpha gT_a$ -space if every  $*A$ -closed set in it is  $\alpha g$ -closed.

In view of Definition-3.8, we have the following.

**Lemma 4.50 :** Let  $f : X \rightarrow Y$  be a  $s^*a$ -continuous function. Then,

- (i) If  $X$  is a  $*A_c$ -space, then  $f$  is continuous function.
- (ii) If  $X$  is a  $*A_s$ -space, then  $f$  is semicontinuous function.
- (iii) If  $X$  is a  $\#A$ -space, then  $f$  is  $\alpha$ -continuous function.



- (iv) If  $X$  is a  $g_s T_a$ -space, then  $f$  is  $g_s$ -continuous function
- (v) If  $X$  is a  $sg T_a$ -space, then  $f$  is  $sg$ -continuous function
- (vi) If  $X$  is a  $\alpha_g T_a$ -space, then  $f$  is  $\alpha_g$ -continuous function.

**Lemma 4.51 :** Let  $f : X \rightarrow Y$  be a  $gsp$ -continuous function with  $X$  is  $gsp T_a$ -space, then  $f$  is  $*a$ -continuous function.

We, define the following.

**Definition 4.52 :** A space  $X$  is said to be  $*a$ -connected if  $X$  cannot be written as the disjoint union of two nonempty  $*A$ -open sets in  $X$ .

We, give the following.

**Lemma 4.53:** For a space  $X$ , the following are equivalent :

- (i)  $X$  is  $*a$ -connected.
- (ii)  $X$  and  $\emptyset$  are the only subsets of  $X$  which are both  $*A$ -open and  $*A$ -closed.
- (iii) Each  $*a$ -continuous function of  $X$  into some discrete space  $Y$  with at least two points is a constant function.

Proof : Obvious.

**Theorem 4.54:** Let  $f : X \rightarrow Y$  be a function

- (i) If  $X$  is  $*a$ -connected and if  $f$  is  $*a$ -continuous, surjective, then  $Y$  is connected
- (ii) If  $X$  is  $*a$ -connected and if  $f$  is  $*a$ -irresolute, surjective, then  $Y$  is  $*a$ -connected.

**Theorem 4.55:** Let  $f : X \rightarrow Y$  be a function

- (i) If  $X$  is  $*a$ -connected and  $f$  is  $s*a$ -continuous, surjective, then  $Y$  is semi-connected.
- (ii) If  $X$  is  $*a$ -connected and  $f$  is  $sp*a$ -continuous, surjective, then  $Y$  is semipre-connected.
- (iii) If  $X$  is  $*a$ -connected and  $f$  is  $\alpha*a$ -continuous, surjective, then  $Y$  is connected.

Proof : Obvious.

## V. ON $*A$ -HAUSDORFF SPACES

We, define the following.

**Definition 5.1:** A space  $X$  is said to be  $*a$ -Hausdorff if whenever  $x$  and  $y$  are distinct points of  $X$ , there exist disjoint  $G^*$ -open sets  $U$  and  $V$  such that  $x \in U$  and  $y \in V$ .

**Definition 5.2:** A space  $X$  is said to be semi- $\alpha$ -Hausdorff if whenever  $x$  and  $y$  are distinct points of  $X$ , there exist disjoint semi- $\alpha$ -open sets  $U$  and  $V$  such that  $x \in U$  and  $y \in V$ .

**Definition 5.3:** A space  $X$  is said to be  $s$ -Hausdorff if whenever  $x$  and  $y$  are distinct points of  $X$ , there exist disjoint semiopen sets  $U$  and  $V$  such that  $x \in U$  and  $y \in V$ .

**Definition 5.4:** A space  $X$  is said to be  $\alpha$ -Hausdorff if whenever  $x$  and  $y$  are distinct points of  $X$ , there exist disjoint  $\alpha$ -open sets  $U$  and  $V$  such that  $x \in U$  and  $y \in V$ .

**Definition 5.5:** A space  $X$  is said to be  $sp$ -Hausdorff if whenever  $x$  and  $y$  are distinct points of  $X$ , there exist disjoint semipreopen sets  $U$  and  $V$  such that  $x \in U$  and  $y \in V$ .

Next, we prove the following.

**Theorem 5.6 :** Let  $X$  be a space and  $Y$  be Hausdorff. If  $f : X \rightarrow Y$  be  $*a$ -continuous injective, then  $X$  is  $*a$ -Hausdorff.

**Proof :** Let  $x$  and  $y$  be any two distinct points of  $X$ . Then  $f(x)$  and  $f(y)$  are distinct points of  $Y$ , because  $f$  is injective. Since  $Y$  is Hausdorff, there exist two disjoint open sets  $U$  and  $V$  such that  $f(x) \in U$  and  $f(y) \in V$ . Since  $f$  is  $*a$ -continuous and  $U \cap V = \emptyset$ , we have  $f^{-1}(U)$  and  $f^{-1}(V)$  are disjoint  $*A$ -open sets in  $X$  such that  $x \in f^{-1}(U)$  and  $y \in f^{-1}(V)$ . Hence  $X$  is  $*a$ -Hausdorff space.

We state the following.

**Theorem 5.7:** Let  $X$  be a space and  $Y$  be  $*a$ -Hausdorff. If  $f : X \rightarrow Y$  be  $*a$ -irresolute injective, then  $X$  is  $*a$ -Hausdorff.

**Theorem 5.8:** Let  $X$  be a space and  $Y$  be  $s$ -Hausdorff. If  $f : X \rightarrow Y$  be  $s*a$ -continuous injective, then  $X$  is  $*a$ -Hausdorff.

**Theorem 5.9:** Let  $X$  be a space and  $Y$  be  $\alpha$ -Hausdorff. If  $f : X \rightarrow Y$  be  $\alpha*a$ -continuous injective, then  $X$  is  $*a$ -Hausdorff.

**Theorem 5.10:** Let  $X$  be a space and  $Y$  be  $sp$ -Hausdorff. If  $f : X \rightarrow Y$  be  $sp*a$ -continuous injective, then  $X$  is  $*a$ -Hausdorff.

**Theorem 5.11 :** Let  $X$  be a space and  $Y$  be Hausdorff. If  $f : X \rightarrow Y$  be semi- $\alpha$ -continuous injective, then  $X$  is semi- $\alpha$ -Hausdorff.

**Theorem 5.12 :** Let  $X$  be a space and  $Y$  be semi- $\alpha$ -Hausdorff. If  $f: X \rightarrow Y$  be pre- $\ast$ -continuous injective, then  $X$  is  $\ast$ -Hausdorff.

**Theorem 5.13:** Let  $X$  be a space and  $Y$  be  $\ast$ -Hausdorff. If  $f: X \rightarrow Y$  be strongly  $\ast$ -continuous injective, then  $X$  is Hausdorff.

We define the following.

**Definition 5.14 :** A space  $X$  is said to be  $\ast$ -normal if for any pair of disjoint  $\ast$ -closed sets  $A$  and  $B$  of  $X$  there exist disjoint open sets  $U$  and  $V$  in  $X$  such that  $A \subset U$  and  $B \subset V$ .

Clearly, every  $\ast$ -normal space is normal, since every closed set is  $\ast$ -closed.

**Definition 5.15 :** A space  $X$  is said to be  $\ast\ast$ -normal if for any pair of disjoint closed sets  $A$  and  $B$  of  $X$  there exist disjoint  $\ast$ -open sets  $U$  and  $V$  in  $X$  such that  $A \subset U$  and  $B \subset V$ .

Clearly, every normal space is  $\ast\ast$ -normal, since every open set is  $\ast$ -open.

**Definition 5.16 :** A function  $f: X \rightarrow Y$  is said to be  $\ast$ -open if the image of each open set of  $X$  is  $\ast$ -open in  $Y$ .

Clearly, open function is  $\ast$ -open.

**Definition 5.17 :** A function  $f: X \rightarrow Y$  is said to be always  $\ast$ -open if the image of each  $\ast$ -open set of  $X$  is  $\ast$ -open in  $Y$ .

Clearly, every always  $\ast$ -open function is  $\ast$ -open.

**Definition 5.18 :** A function  $f: X \rightarrow Y$  is said to be strongly  $\ast$ -open if the image of each  $\ast$ -open set of  $X$  is open in  $Y$ .

In view of a  $gsp$   $T_a$ -space (Definition- 3.51(v)), we state the following.

**Theorem 5.19 :** The following are equivalent for a  $gsp$   $T_a$ -space  $X$  :

- (i)  $X$  is  $\ast$ -normal,
- (ii) For each pair of disjoint  $gsp$ -closed sets  $A$  and  $B$  of  $X$ , there exist disjoint open sets  $U$  and  $V$  in  $X$  such that  $A \subset U$  and  $B \subset V$ .

We, state the following.

**Theorem 5.20 :** If  $f: X \rightarrow Y$  is an open  $\ast$ -irresolute bijective and  $X$  is  $\ast$ -normal, then  $Y$  is  $\ast$ -normal.

We, prove the normality via  $\ast$ -open sets in the following.

**Theorem 5.21 :** The following are equivalent for a space  $X$  :

- (i)  $X$  is normal;
- (ii) for any disjoint closed sets  $A$  and  $B$  of  $X$ , there exist disjoint  $\alpha$ -open sets  $K$  and  $H$  in  $X$  such that  $A \subset K$  and  $B \subset H$ .
- (iii) for any disjoint closed sets  $A$  and  $B$  of  $X$ , there exist disjoint semi-open sets  $K$  and  $H$  in  $X$  such that  $A \subset K$  and  $B \subset H$ .
- (iv) for any disjoint closed sets  $A$  and  $B$  of  $X$ , there exist disjoint  $s\alpha$ -open sets  $K$  and  $H$  in  $X$  such that  $A \subset K$  and  $B \subset H$ .
- (v) for any disjoint closed sets  $A$  and  $B$  of  $X$ , there exist disjoint  $\ast$ -open sets  $K$  and  $H$  in  $X$  such that  $A \subset K$  and  $B \subset H$ .
- (vi) for any disjoint closed sets  $A$  and  $B$  of  $X$ , there exist disjoint  $gsp$ -open sets  $K$  and  $H$  in  $X$  such that  $A \subset K$  and  $B \subset H$ .
- (vii) for any closed set  $A$  and any open set  $G$  containing  $A$ , there exists a  $gsp$ -open set  $K$  in  $X$  such that  $A \subset K \subset spCl(K) \subset G$ .

Proof : (i)  $\Rightarrow$  (ii). It is obvious since every open set is  $\alpha$ -open set.

(ii)  $\Rightarrow$  (iii). It is obvious since every  $\alpha$ -open set is semi-open set.

(iii)  $\Rightarrow$  (iv). It is obvious since every semiopen set is  $s\alpha$ -open set.

(iv)  $\Rightarrow$  (v). It is obvious since every  $s\alpha$ -open set is  $\ast$ -open set.

(v)  $\Rightarrow$  (vi). It is obvious since every  $\ast$ -open set is  $gsp$ -open set.

(vi)  $\Rightarrow$  (vii). Let  $A$  be a closed set and  $G$  be an open set containing  $A$ . Then  $A$  and  $(X-G)$  are disjoint closed sets of  $X$ . By hypothesis there exist disjoint  $gsp$ -open sets  $V$  and  $H$  such that  $A \subset V$  and  $(X-G) \subset H$ . Since  $(X-G)$  is closed set; we obtain  $(X-G) \subset spInt(H)$  and  $V \cap spInt(H) = \emptyset$ . Therefore, we obtain  $spCl(V) \cap spInt(H) = \emptyset$  and hence  $A \subset V \subset spCl(V) \subset (X-spInt(H)) \subset G$ . This implies the required result.

(vii)  $\Rightarrow$  (i): Let  $A$  and  $B$  be disjoint closed sets of  $X$ . Then  $A \subset (X-B)$  and  $(X-B)$  is open set. By hypothesis, there exists a  $gsp$ -open set  $G$  of  $X$  such that  $A \subset G \subset spCl(G) \subset (X-B)$ . Since  $A$  is closed set, we have  $A \subset spInt(G)$ . Put  $K =$



$\text{IntCl Int (spInt}(G))$  and  $H = \text{Int Cl Int (X - spCl}(G))$ . Then,  $K$  and  $H$  are disjoint open set of  $X$  such that  $A \subset K$  and  $B \subset H$ . Therefore,  $X$  is normal.

We state the following.

**Theorem 5.22 :** If  $f : X \rightarrow Y$  is continuous,  $*a$ -open bijection and  $X$  is  $**a$ -normal space, then  $Y$  is  $**a$ -normal.

**Theorem 5.23 :** If  $f : X \rightarrow Y$  is continuous, always  $*a$ -open bijection and  $X$  is  $**a$ -normal space, then  $Y$  is  $**a$ -normal.

**Theorem 5.24 :** If  $f : X \rightarrow Y$  is continuous, strongly  $*a$ -open bijection and  $X$  is  $**a$ -normal space, then  $Y$  is normal.

## VI. CONCLUSION

In this paper, we defined and studied two new generalized closed sets, namely,  $*R$ -closed sets and  $*A$ -closed sets using  $\alpha$ -closure operator,  $g$ -closed set and open set, respectively. On  $*R$ -closed sets,  $*R$ -open sets,  $*rT_1$ -spaces,  $*r$ -continuous functions,  $*r$ -irresolute functions,  $*r$ -regular spaces and  $*r$ -normal spaces and  $*A$ -closed sets,  $*A$ -open sets,  $*A_c$ -spaces,  $*A_s$ -spaces,  $\#A$ -spaces,  $*a$ -continuous functions,  $*a$ -irresolute functions and  $*a$ -connected spaces are also studied in this paper. On  $*R$ -closed sets, one can establish some more properties.

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