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Strongly α^*_{AS} -Continuous Maps and Perfectly α^*_{AS} -Continuous Maps in Topological Spaces

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ABSTRACT: P. Anbarasi Rodrigo and I. Sahaya Dani introduced properties of α^*_{AS} -closed sets in Topological space. Also, investigated α^*_{AS} -Continuous, Contra α^*_{AS} continuous, α^*_{AS} Homeomorphism and studied its properties. In this paper, we introduce strongly α^*_{AS} -continuous maps, and perfectly α^*_{AS} -continuous maps.

KEYWORDS: - α^*_{AS} -closed sets, strongly α^*_{AS} -continuous maps, perfectly α^*_{AS} -continuous maps.

I. INTRODUCTION

Levine introduced and investigated the concept of strong continuity in topological spaces. Sundaram introduced strongly g-continuous maps and perfectly g-continuous maps in topological spaces. P. Anbarasi Rodrigo and I. Sahaya Dani introduced α^*_{AS} -closed sets in topological spaces. In this paper, we introduce the notion of strongly α^*_{AS} -continuous maps, perfectly α^*_{AS} -continuous maps and obtain some of its basic properties.

II. PRELIMINARIES

The following are the necessary concepts and definitions that are used in this work.

Definition 2.1 A subset A of a topological space X is called α^*_{AS} closed sets if $\alpha cl(A) \subseteq int(U)$ whenever $A \subseteq U$ and U is open in X.

Definition 2.2 Let X and Y be two topological spaces. Then the function $f : X \rightarrow Y$ is said to be continuous if the inverse image of every open(closed) set in Y is open(closed) in X.

Definition 2.3 A function $f: X \rightarrow Y$ is called α^*_{AS} -continuous if $f^{-1}(S)$ is α^*_{AS} -closed in X for every closed set S in Y. That is, if the inverse image of every closed set in Y is α^*_{AS} -closed in X.

Definition 2.4 Let X and Y be two topological Spaces. Let $f: X \rightarrow Y$ be a mapping. Then f is said to be:

Strongly continuous if $f^{-1}(S)$ is both open and closed in X for each subset S in Y.

Perfectly continuous if $f^{-1}(S)$ is both open and closed in X for each open set S in Y.

Strongly g-continuous if $f^{-1}(S)$ is open in X for each g-open set S in Y.

Perfectly g-continuous if $f^{-1}(S)$ is both open and closed in X for each g-open set S in Y.

III. STRONGLY α^*_{AS} -CONTINUOUS MAPS IN TOPOLOGICAL SPACES

In this section, we introduce the concept of strongly α^*_{AS} -continuous maps in topological spaces and study some of its properties.

Definition 3.1 A map $f: X \rightarrow Y$ is said to be strongly α^*_{AS} -continuous if the inverse image of every α^*_{AS} -closed set in Y is closed in X.

Example 3.2 Let $X=Y= \{a, b, c\}$ with $\tau= \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$. Then closed sets are $= \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$. Let $\sigma = \{Y, \emptyset, \{c\}\}$. Then α^*_{AS} - closed sets are $= \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$ Let $f: X \rightarrow Y$ be an identity map. Hence the inverse image of every α^*_{AS} - closed set in Y is closed in X. Thus, f is strongly α^*_{AS} - continuous.

Theorem 3.3 If a map $f: X \rightarrow Y$ is strongly α^*_{AS} -continuous then it is continuous but not conversely.

Proof: Assume that f is strongly α^*_{AS} -continuous. Let S be any closed set in Y . Since every closed set is α^*_{AS} -closed, S is α^*_{AS} -closed in Y . Since f is strongly α^*_{AS} -continuous $f^{-1}(S)$ is closed in X . Therefore, f is continuous.

Remark 3.4 The converse need not be true as seen from the following example.

Example 3.5 Let $X=Y= \{a, b, c, d\}$ and $\tau= \{X, \emptyset, \{a\}, \{b, d\}, \{a, b, d\}\}$, $\sigma = \{Y, \emptyset, \{c\}, \{a, d\}, \{a, c, d\}\}$. Let $f: X \rightarrow Y$ defined by $f(a) = c, f(b) = a, f(c) = b, f(d) = d$ then $f^{-1}(a) = b, f^{-1}(b) = c, f^{-1}(c) = a, f^{-1}(d) = d$. Then f is continuous but not strongly α^*_{AS} -continuous since $\{a, b\}$ is α^*_{AS} -closed in Y but $f^{-1}(\{a, b\}) = \{b, c\}$ is not a closed set of X .

Theorem 3.6 A map $f: X \rightarrow Y$ is strongly α^*_{AS} -continuous if and only if the inverse image of every α^*_{AS} -open set in Y is open in X .

Proof: Assume that f is strongly α^*_{AS} -continuous. Let S be any α^*_{AS} -open set in Y . Then S^c is α^*_{AS} -closed set in Y . Since f is strongly α^*_{AS} -continuous, $f^{-1}(S^c)$ is closed in X . But $f^{-1}(S^c) = X - f^{-1}S$ and so $f^{-1}S$ is open in X . Conversely assume that the inverse image of every α^*_{AS} -open set in Y is open in X . Let S be any α^*_{AS} -closed set in Y . Then S^c is α^*_{AS} -open set in Y . By assumption, $f^{-1}(S^c)$ is open in X . But $f^{-1}(S^c) = X - f^{-1}S$ and so $f^{-1}S$ is closed in X . Therefore, f is strongly α^*_{AS} -continuous.

Theorem 3.7 If a map $f: X \rightarrow Y$ is strongly continuous then it is strongly α^*_{AS} -continuous, but not conversely.

Proof: Assume that f is strongly continuous. Let S be any α^*_{AS} -closed set in Y . Since f is α^*_{AS} -continuous, $f^{-1}(S)$ is closed in X . Therefore, f is strongly α^*_{AS} -continuous.

Remark 3.8 The converse need not be true as seen from the following example.

Example 3.9 Let $X= \{a, b, c, d\}$ and $Y= \{a, b, c\}$. Let $\tau= \{X, \emptyset, \{c\}, \{a, d\}, \{a, c, d\}\}$, $\sigma = \{Y, \emptyset, \{a, c\}\}$. Let $f: X \rightarrow Y$ defined by $f(a) = f(d) = c, f(b) = b, f(c) = a$ then $f^{-1}(a) = c, f^{-1}(b) = b, f^{-1}(c) = ad$. Hence the inverse image of every α^*_{AS} -closed set in Y is closed in X . Thus, f is strongly α^*_{AS} -continuous but not strongly continuous since $f^{-1}(b) = b$ is closed in X but not open in X .

Theorem 3.10 If a map $f: X \rightarrow Y$ is strongly α^*_{AS} -continuous and a map $g: Y \rightarrow Z$ is α^*_{AS} -continuous, then the composition $g \circ f: X \rightarrow Z$ is continuous.

Proof: Let S be any closed set in Z . Since g is α^*_{AS} -continuous $g^{-1}(S)$ is α^*_{AS} -closed in Y . Since f is strongly α^*_{AS} -continuous, $f^{-1}(g^{-1}(S))$ is closed in X . But $(g \circ f)^{-1}(S) = f^{-1}(g^{-1}(S))$. Therefore, $g \circ f$ is continuous.

Theorem 3.11 If a map $f: X \rightarrow Y$ is strongly α^*_{AS} -continuous and a map $g: Y \rightarrow Z$ is α^*_{AS} -irresolute, then $g \circ f: X \rightarrow Z$ is strongly α^*_{AS} -continuous.

Proof: Let S be any α^*_{AS} -closed set in Z . Since g is α^*_{AS} -irresolute, $g^{-1}(S)$ is α^*_{AS} -closed in Y . Also, f is strongly α^*_{AS} -continuous $f^{-1}(g^{-1}(S))$ is closed in X . But $(g \circ f)^{-1}(S) = f^{-1}(g^{-1}(S))$ is closed in X . Hence, $g \circ f: X \rightarrow Z$ is strongly α^*_{AS} -continuous.

Theorem 3.12 If a map $f: X \rightarrow Y$ is α^*_{AS} -continuous and a map $g: Y \rightarrow Z$ is strongly α^*_{AS} -continuous then $g \circ f: X \rightarrow Z$ is α^*_{AS} -irresolute.

Proof: Let S be any α^*_{AS} -closed set in Z . Since g is strongly α^*_{AS} -continuous, $f^{-1}(S)$ is closed in Y . Also, f is α^*_{AS} -continuous, $f^{-1}(g^{-1}(S))$ is α^*_{AS} -closed in X . But $(g \circ f)^{-1}(S) = f^{-1}(g^{-1}(S))$. Hence $g \circ f: X \rightarrow Z$ is α^*_{AS} -irresolute.

Theorem 3.13 The composition of two strongly α^*_{AS} -continuous maps is strongly α^*_{AS} -continuous.

Proof: Let $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be two strongly α^*_{AS} -continuous maps. Let S be a α^*_{AS} -closed set in Z . Since, g is strongly α^*_{AS} -continuous, we get $g^{-1}(S)$ is closed in Y . Then $g^{-1}(S)$ is α^*_{AS} -closed in Y . As f is also strongly α^*_{AS} -continuous, $f^{-1}(g^{-1}(S)) = (g \circ f)^{-1}(S)$ is closed in X . Hence, $(g \circ f)$ is strongly α^*_{AS} -continuous.

Theorem 3.14 If $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be any two maps. Then their composition $g \circ f: X \rightarrow Z$ is strongly α^*_{AS} -continuous if g is strongly α^*_{AS} -continuous and f is continuous.

Proof: Let S be a α^*_{AS} -closed in Z . Since, g is strongly α^*_{AS} -continuous, $g^{-1}(S)$ is closed in Y . Since f is continuous, $f^{-1}(g^{-1}(S)) = (g \circ f)^{-1}(S)$ is closed in X . Hence, $(g \circ f)$ is strongly α^*_{AS} -continuous.

IV. PERFECTLY α^*_{AS} -CONTINUOUS MAPS IN TOPOLOGICAL SPACES

Definition 4.1 A map $f: X \rightarrow Y$ is said to be perfectly α^*_{AS} -continuous if the inverse image of every α^*_{AS} -closed set in Y is both open and closed in X .

Example 4.2 Let $X=Y= \{a, b, c\}$ with $\tau= \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\}$. Then closed sets are $= \{X, \emptyset, \{a\}, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{a, c\}\}$. Let $\sigma = \{Y, \emptyset, \{c\}\}$. Then α^*_{AS} -closed sets are $= \{X, \emptyset, \{a\}, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\}$. Let $f: X \rightarrow Y$ be an identity map. Hence the inverse image of every α^*_{AS} -closed set in Y is both open and closed in X . Thus, f is perfectly α^*_{AS} -continuous.

Theorem 4.3 If a map $f: X \rightarrow Y$ is perfectly α^*_{AS} -continuous then it is continuous but not conversely.

Proof: Assume that f is perfectly α^*_{AS} -continuous. Let S be any closed set in Y . Since every closed set is α^*_{AS} -closed, S is α^*_{AS} -closed in Y . Since f is strongly α^*_{AS} -continuous $f^{-1}(S)$ is both open and closed in X . Therefore, f is continuous.

Remark 4.4 The converse of the above theorem need not be true as seen from the following example.

Example 4.5 Let $X=Y= \{a, b, c, d\}$ and $\tau= \{X, \emptyset, \{a\}, \{b, d\}, \{a, b, d\}\}$, $\sigma = \{Y, \emptyset, \{c\}, \{a, d\}, \{a, c, d\}\}$. Let $f: X \rightarrow Y$ defined by $f(a) = c, f(b) = a, f(c) = b, f(d) = d$ then $f^{-1}(a) = b, f^{-1}(b) = c, f^{-1}(c) = a, f^{-1}(d) = d$. Then f is continuous but not perfectly α^*_{AS} -continuous since $\{a, b\}$ is α^*_{AS} -closed in Y but $f^{-1}(\{a, b\}) = \{b, c\}$ is not an open or closed set of X .

Theorem 4.6 If a map $f: X \rightarrow Y$ from a topological space (X, τ) into a topological space (Y, σ) is perfectly α^*_{AS} -continuous then it is strongly α^*_{AS} -continuous.

Proof: Assume that f is perfectly α^*_{AS} -continuous. Let S be any α^*_{AS} -closed set in Y . Since, f is perfectly α^*_{AS} -continuous, $f^{-1}(S)$ is closed in X . Therefore, f is strongly α^*_{AS} -continuous.

Remark 4.7 The converse of the above theorem need not be true.

Example 4.8 Let $X= \{a, b, c, d\}$ and $Y= \{a, b, c\}$. Let $\tau= \{X, \emptyset, \{c\}, \{a, d\}, \{a, c, d\}\}$, $\sigma = \{Y, \emptyset, \{a, c\}\}$. Let $f: X \rightarrow Y$ defined by $f(a) = f(d) = c, f(b) = b, f(c) = a$ then $f^{-1}(a) = c, f^{-1}(b) = b, f^{-1}(c) = \{a, d\}$. Hence the inverse image of every α^*_{AS} -closed set in Y is closed in X . Thus, f is strongly α^*_{AS} -continuous but not perfectly α^*_{AS} -continuous since $f^{-1}(b) = b, f^{-1}(a, b) = \{b, c\}$ and $f^{-1}(b, c) = \{a, b, d\}$ are closed in X but not open in X .

Theorem 4.9 If a map $f: X \rightarrow Y$ from a topological space (X, τ) into a topological space (Y, σ) is perfectly α^*_{AS} -continuous then it is perfectly continuous.

Proof: Assume that f is perfectly α^*_{AS} -continuous. Let S be any closed set in Y . Since every closed set is α^*_{AS} -closed, S is α^*_{AS} -closed set in Y . Since, f is perfectly α^*_{AS} -continuous, $f^{-1}(S)$ is both open and closed in X . Therefore, f is perfectly continuous.

Remark 4.10 The converse of the above theorem need not be true.

Example 4.11 Let $X= Y= \{a, b, c, d\}$. Let $\tau= \{X, \emptyset, \{b\}, \{c\}, \{a, d\}, \{b, c\}, \{a, b, c\}, \{a, c, d\}, \{a, b, d\}, \tau^c = \{X, \emptyset, \{b\}, \{c\}, \{d\}, \{a, d\}, \{b, c\}, \{a, b, d\}, \{a, c, d\}\}$, $\sigma = \{Y, \emptyset, \{c\}, \{a, d\}, \{a, c, d\}\}$, $\sigma^c = \{Y, \emptyset, \{b\}, \{b, c\}, \{a, b, d\}\}$. Also, α^*_{AS} -closed set in $Y = \{Y, \emptyset, \{b\}, \{a, b\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{a, b, d\}, \{b, c, d\}$. Let $f: X \rightarrow Y$ defined be identity map. Hence the inverse image of every closed set in Y is both closed and open in X . Thus, f is perfectly



continuous but not perfectly α^*_{AS} - continuous since $f^{-1}(a, b, c) = \{a, b, c\}$ is open but not closed in X. Hence, f is not perfectly α^*_{AS} - continuous

Theorem 4.12 A map $f: X \rightarrow Y$ from a topological space (X, τ) into a topological space (Y, σ) is perfectly α^*_{AS} -continuous if and only if $f^{-1}(S)$ is both open and closed in (X, τ) for every α^*_{AS} - open set S in (Y, σ) .

Proof: Let S be any α^*_{AS} - open set in Y. Then S^c is α^*_{AS} - closed in Y. Since, f is perfectly α^*_{AS} - continuous, $f^{-1}(S^c)$ is both open and closed in X. But $f^{-1}(S^c) = X - f^{-1}(S)$ and so $f^{-1}(S)$ is both open and closed in X. Conversely, assume that the inverse image of every α^*_{AS} - closed set in Y is both open and closed in X. Let S be any α^*_{AS} - open in Y. Then S^c is α^*_{AS} - closed in Y. By assumption $f^{-1}(S^c)$ is both open and closed in X. But $f^{-1}(S^c) = X - f^{-1}(S)$ and so $f^{-1}(S)$ is both open and closed in X. Therefore, f is perfectly α^*_{AS} - continuous.

Theorem 4.13 Let (X, τ) be a discrete topological space and (Y, σ) be any topological space. Let $f: X \rightarrow Y$ be a map, then the following statements are true.

- 1) f is strongly α^*_{AS} - continuous
- 2) f is perfectly α^*_{AS} - continuous

Proof: (1) \Rightarrow (2) Let S be any α^*_{AS} - closed set in Y. By hypothesis, $f^{-1}(S)$ is closed in X. Since X is a discrete space, $f^{-1}(S)$ is open in X. $f^{-1}(S)$ is both open and closed in X. Thus, f is perfectly α^*_{AS} - continuous.

(2) \Rightarrow (1) Let S be any α^*_{AS} - closed set in Y. Then, $f^{-1}(S)$ is both open and closed in X. Thus, f is strongly α^*_{AS} - continuous.

Theorem 4.14 If $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ are perfectly α^*_{AS} - continuous, then their composition $g \circ f: X \rightarrow Z$ is also perfectly α^*_{AS} - continuous.

Proof: Let S be a α^*_{AS} - closed set in Z. Since, g is perfectly α^*_{AS} - continuous, we get that $g^{-1}(S)$ is both open and closed in Y. Then $g^{-1}(S)$ is α^*_{AS} - closed in Y. Since f is perfectly α^*_{AS} -continuous, $f^{-1}(g^{-1}(S)) = (g \circ f)^{-1}(S)$ is both open and closed in X. Hence, $g \circ f$ is perfectly α^*_{AS} - continuous.

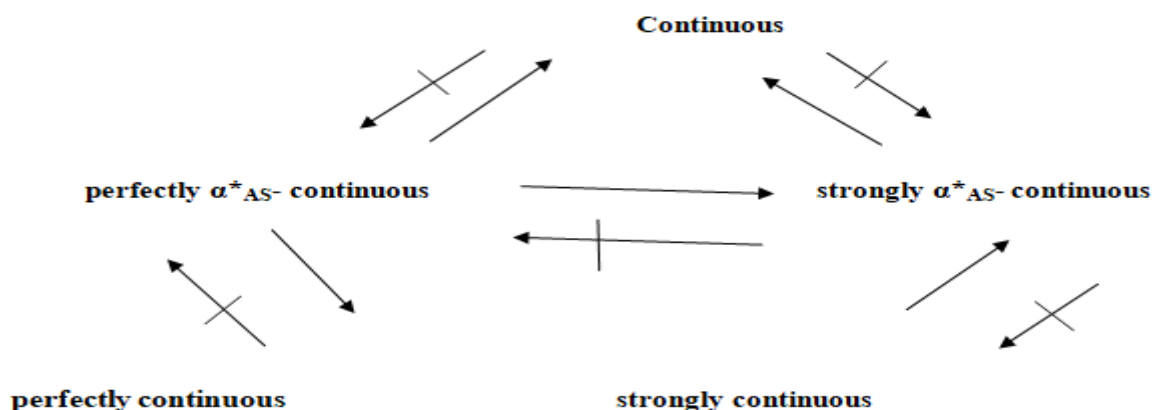
Theorem 4.15 If $f: X \rightarrow Y$ and $g: Y \rightarrow Z$ be any two maps. Then their composition is strongly α^*_{AS} - continuous if g is perfectly α^*_{AS} - continuous and f is continuous.

Proof: Let S be any α^*_{AS} - closed set in Z. Then, $g^{-1}(S)$ is both open and closed in Y. Since, f is continuous. $f^{-1}(g^{-1}(S)) = (g \circ f)^{-1}(S)$ is closed in X. Hence, $g \circ f$ is strongly α^*_{AS} - continuous.

Theorem 4.16 If a map $f: X \rightarrow Y$ is perfectly α^*_{AS} - continuous and a map $g: Y \rightarrow Z$ is strongly α^*_{AS} - continuous then the composition $g \circ f: X \rightarrow Z$ is perfectly α^*_{AS} - continuous.

Proof: Let S be any α^*_{AS} - closed set in Z. Then, $g^{-1}(S)$ is closed in Y. Then $g^{-1}(S)$ is α^*_{AS} - closed in Y. By hypothesis, $f^{-1}(g^{-1}(S)) = (g \circ f)^{-1}(S)$ is both open and closed in X. Therefore, $g \circ f$ is perfectly α^*_{AS} - continuous.

Remark 4.17 From the above observations we have the following implications:





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