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# Analysis and Comparatives Study on Numerical Methods to Solve First Order Ordinary Differential Equations

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**ABSTRACT:** Subsequently Mathematics is a science of statement between us and the logical science, in specific it familiarizes all the instructions and problems as formulas and searches for a solution. We have measured ordinary differential equation of first order with boundary condition. The impartial of this paper is to signify the concepts of numerical analysis and differential equations more precisely the ordinary differential equations (ODE) and also to introduce some numerical methods of solving first order ordinary differential equations for introduction so that reader get accustomed with these concepts before the start of the process. These equations have been solved by Euler's method, Runge-Kutta method, and Adams-Bashforth method with derivation and algorithm. Those methods of solving first order ordinary differential equations have been discussed along with the FORTRAN and MATLAB programming languages also the behaviour of the solutions have been represented graphically for better understanding.

**KEYWORDS:** Numerical Analysis, Differential Equation, IVP, BVP, FORTRAN, MATLAB.

## I. INTRODUCTION

A portion of the mathematics that is generally used in all sciences is the differential equation that will be calculated in this paper. Numerical methods are usually used for resolution mathematical problems that are expressed in science and engineering where it is problematic or even incredible to attain exact solutions. Differential equations are frequently used for mathematical modelling in science and engineering. Various problems of mathematical physics can be started in the form of differential equations. These equations also happen as reformulations of other mathematical problems such as ordinary differential equations and partial differential equations. First a inadequate number of differential equations can be solved methodically. There are many analytical methods for finding the solution of ordinary differential equations. Even then there exist a large number of ordinary differential equations whose solutions cannot be obtained in closed form by using well-known analytical methods, where we have to use initial value problems. The Euler method is traditionally the first numerical technique [1]. The outcome from numerical analysis is an approximation, which can be made closely as wanted. Analysis of errors in numerical methods is an essential part of the learning of numerical analysis [2].

Many authors have attempted to solve initial value problems (IVP) to obtain high accuracy rapidly by using numerous methods, such as Euler method and Runge-Kutta method, and also some other methods. In the author discussed accuracy analysis of numerical solutions of initial value problems (IVP) for ordinary differential equations (ODE) [3], and also in the author discussed accurate solutions of initial value problems for ordinary differential equations with fourth-order Runge-Kutta method [4] and studied on some numerical methods for solving initial value problems in ordinary differential equations [5].

There are many types of practical numerical methods for solving initial value problems for ordinary differential equations. In this paper we present two standard numerical methods Euler improved, Euler modified method and Runge-Kutta for solving initial value problems of ordinary differential equations [6]. Differential equation is one of the crucial, significant parts of mathematics, which has substantial applications, the equation which contains an indefinite function and its derivatives of that unknown function identified as differential equation. In this paper we present two standard numerical methods Euler improved, Euler modified method and Runge-Kutta for solving initial value problems of ordinary differential equations. Numerical analysis is the area of mathematics and computer science that creates analysis and implements algorithms for solving numerically the problems of continuous mathematics [7]. Many mathematicians defined numerical analysis in different ways, By Conte & De Boor. The study of quantitative approximations to the solutions of mathematical problems including considering of the errors and bounds to the errors involved [8]. The study of approximate solutions to mathematical problems, taking into account the extent possible errors [9].



Equations have been solved by Euler's method, Runge-Kutta method, Adams-Bashforth method with derivation and algorithm. Those methods of solving first order ordinary differential equations have been conferred along with the FORTRAN and MATLAB programming languages also the conduct of the solutions have been represented graphically for improved consideration.

### 1.1 Numerical Methods:

Numerical methods for ordinary differential equations are methods used to find numerical approximations to the solutions of ordinary differential equations. Their practice is also identified as numerical combination, although this term is sometimes occupied to mean the calculation of the integrals.

### 1.2 Concept of Numerical Analysis:

#### 1.2.1 Convergence:

A numerical method is said to be convergent if the numerical solution approaches to the exact solution as the steps increase.

#### 1.2.2 Accuracy:

Every method of numerical computing introduces errors. These errors affect the accuracy of the results. The results we obtain must be sufficiently accurate to serve the purpose for which the mathematical model was built.

#### 1.2.3 Consistency and order:

The order shows how well it approximates the solution of numerical method.

#### 1.2.4 Stability:

The stability shows that the error which is occurred solving numerical method is whether damped out or not.

### 1.3 Ordinary differential equation (ODE):

Ordinary differential equation which is an calculation that is unfamiliar function dependent only on one independent variable.

### 1.4 Partial differential equation (PDE):

If the indefinite function depends on two or further independent variables, then the differential equation is a partial differential equation.

### 1.5 Initial Value Problem (IVP):

A differential equation taking given conditions consent us to find the specific function that contents the given differential equation rather than a family of a function, These type of problems are called the initial value problem. The solution of an initial value problem is called a specific solution.

Example:

Consider an initial value problem,

$$\frac{dy}{dx} = 2x + 1; y(0) = 3$$

Here,  $y(0) = 3$  is an initial condition.

### 1.6 Boundary Value Problem (BVP)

If the specified conditions are certain at more than one point and the differential equation are of order two or better than it is called a boundary value problem.

Example:

Consider a boundary value problem,

$$\frac{d^2y}{dx^2} = y + x; y(0) = 1, y(1) = 1$$

Here  $y(0) = 1$  and  $y(1) = 1$  are the boundary conditions of the given problem.



**1.7 Application/Procedure of Differential Equation:**

Differential equation happens in joining with frequent problems that are come upon in the various branches of science and engineering. We specify insufficient such complications in the following list, which could simply be extended to fill many pages.

1. The delinquent of the transmission of temperature in a rod or a slab.
2. The tricky of influential the atmospheres of a wire or a skin.
3. The problematic of influential the motion of a missile, sky rocket, satellite or planet.
4. The tricky of defining the responsibility or present in an electric circuit.
5. The education of the rate of fragmentation of a radioactive constituent or the rate of progress of population.
6. The training of the response of chemicals.
7. The problem of the resolve of curves that have certain geometrical properties.

The scientific invention of such problem's resistance rises to differential equations. But how does this occur? In the conditions below deliberation in each of the overhead problems the objects elaborate obey certain scientific laws. These laws involve numerous rates of alteration of one or more measures with respect to other quantities.

**II. FIRST ORDER ORDINARY DIFFERENTIAL EQUATION**

**2.1 Euler's Method:**

It is the simplest method for solving the initial value problem. It is also one of the oldest and crudest technique. Euler's method offers us with an estimate for the solution of a differential equation. The impression over due Euler's method is to use the thought of limited linearity to join numerous small link sections so that they type up an calculation of the authentic curve.

**2.1.1 Derivation of Euler's Method:**

We consider the differential equation

$$\frac{dy}{dx} = f(x, y) \dots\dots\dots (1)$$

With

$$y(x_0) = y_0 \dots\dots\dots (2)$$

Suppose that we want to solve the equation (1) for y at

$$x = x_r = x_0 + rh, \quad r = 1, 2, 3, \dots$$

Integrating (1) we obtain

$$\begin{aligned} \int_{y_0}^{y_1} dy &= \int_{x_0}^{x_1} f(x, y) dx \\ \Rightarrow [y]_{y_0}^{y_1} &= \int_{x_0}^{x_1} f(x, y) dx \\ \therefore y_1 &= y_0 + \int_{x_0}^{x_1} f(x, y) dx \dots\dots\dots (3) \end{aligned}$$

Assuming that  $f(x, y) = f(x_0, y_0)$  in  $x_0 \leq x \leq x_1$  we get

$$\begin{aligned} y_1 &= y_0 + (x_1 - x_0) f(x_0, y_0) \\ \therefore y_1 &= y_0 + hf(x_0, y_0) \dots\dots\dots (4) \end{aligned}$$

Similarly, for range  $x_1 \leq x \leq x_2$ , we have

$$y_2 = y_1 + \int_{x_0}^{x_1} f(x, y) dx$$

Assuming that we obtain

$$y_2 = y_1 + hf(x_1, y_1)$$



Proceeding in this way, we get the general formula

$$y_{n+1} = y_n + hf(x_n, y_n), n = 0, 1, 2, 3, \dots \dots \dots (5)$$

This method is called the Euler method or Euler-Cauchy method.

**2.1.3 Problem of Euler’s Method:**

Use Euler’s method to approximate the solution of the following initial value problem.

$$y' = e^x - y, 0 \leq x \leq 1, y(0) = -1 \text{ with } h = 0.1$$

And compare with the exact solution.

**Solution and Result:**

Given that,

$$y' = f(x, y) = e^x - y \dots \dots \dots (1)$$

And  $a = 0, b = 1, x_0 = 0, y_0 = -1, h = 0.1$

$$\text{now, } n = \frac{1-0}{h} = \frac{1}{0.1} = 10$$

By Euler’s method we have,

$$y_{n+1} = y_n + hf(x_n, y_n) \dots \dots \dots (2)$$

$$x_{n+1} = x_n + h \dots \dots \dots (3)$$

Where  $n = 0, 1, 2, \dots$

Exact solution of the given equation is,

$$y(x) = \frac{1}{2}e^{2x} - \frac{3}{2}e^{-x}$$

n	x <sub>n</sub>	y <sub>n</sub> (Euler)	y <sub>n</sub> (Exact)	Error
0	0	-1	-1	0
1	0.1	-0.8000	-0.8047	0.0047
2	0.2	-0.6095	-0.6174	0.0079
3	0.3	-0.4264	-0.4363	0.0099
4	0.4	-0.2488	-0.2596	0.0108
5	0.5	-0.0747	-0.0854	0.0107
6	0.6	0.0976	0.0878	0.0098
7	0.7	0.2701	0.2620	0.0081
8	0.8	0.4444	0.4388	0.0057
9	0.9	0.6226	0.6199	0.0026
10	1.0	0.8063	0.8073	0.0011

**Table 1: Exact solution of the given equation**



2.1.4 Graph of the solution:

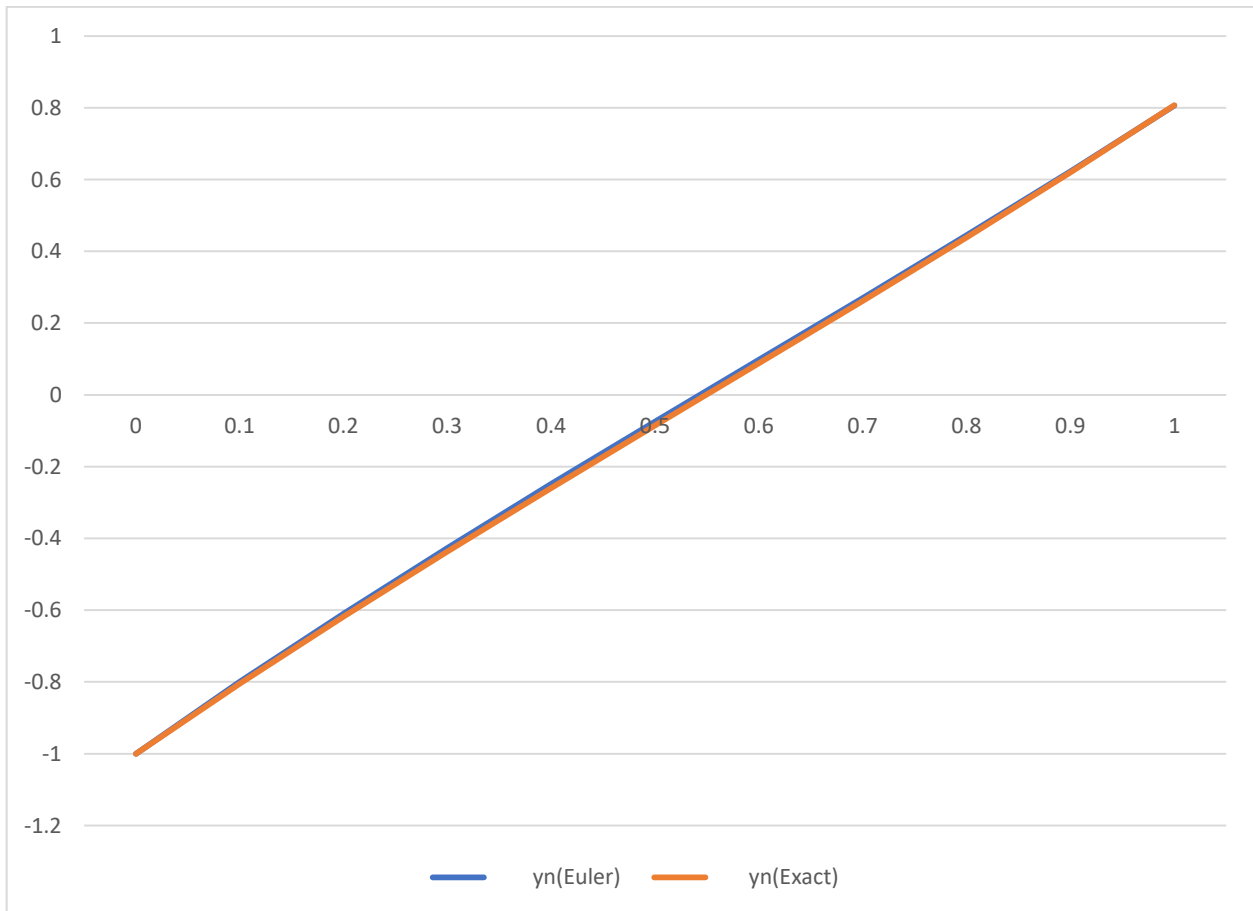


Figure 1: Euler's method comparing with exact solution

III. RUNGE-KUTTA METHOD

Computationally, most efficient methods in terms of accuracy were developed by two German mathematicians, Runge and Kutta. These methods are well known as Runge-Kutta methods or RK method. It is one of the most extensively used technique for result the mathematical solutions of linear and non-linear ordinary differential equations, the expansion of which is complex algebraically.

3.1 Derivation of Runge-Kutta method:

Consider  $\frac{dy}{dx} = f(x, y)$  with the initial condition  $y(x_0) = y_0$

Let h be the interval between equidistant values of x.

Let  $y = y(x)$  has  $(n + 1)$  continuous derivatives  $x_{n+1} = x_n + h$

$$y_{n+1} = y_n + [ak_1 + bk_2]$$

Where,

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf(x_n + \alpha h, y_n + \beta k_1)$$

Now we have to determine the values for  $a, b, \alpha, \beta$

By Taylor's series expansion, we have



$$\frac{dy}{dx} = f(x, y) = f$$

$$\Rightarrow f' = \frac{df}{dx} = \frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} \cdot \frac{dy}{dx} = f_x + f_y f$$

$$\therefore y_{n+1} = y_n + hf(x_n, y_n) + \frac{h^2}{2} f'(x_n, y_n) \dots \dots \dots (1)$$

$$= y_n + hf_n + h^2 \left( \frac{1}{2} f_x + \frac{1}{2} f_y f_n \right) \dots \dots \dots (2)$$

Where  $\left( \frac{1}{2} f_x + \frac{1}{2} f_y f_n \right)$  means value evaluated at  $(x_n, y_n)$

Putting the values of  $k_1$  and  $k_2$  in (1) we get,

$$y_{n+1} = y_n + ahf(x_n, y_n) + bhf(x_n + \alpha h, y_n + \beta hf(x_n, y_n)) \dots \dots \dots (3)$$

We expand the last part of (3) by Taylor's series expansion

$$f(x_n + \alpha h, y_n + \beta hf(x_n, y_n)) = f_n + [\alpha hf_x + \beta hf_y f] \text{ by neglecting the higher order terms.}$$

$$\text{Thus (3)} \Rightarrow y_{n+1} = y_n + ahf_n + bh[f + f_x \alpha h + f_y \beta hf]_n$$

Rearranging we get,

$$y_{n+1} = y_n + (a+b)hf_x + h^2[\alpha hf_x + f_y \beta hf]_n + \dots \dots \dots (4)$$

The equation will be identical to equation (2) if we choose

$$a + b = 1$$

$$\alpha b = \frac{1}{2}$$

$$\beta b = \frac{1}{2}$$

So we have 3 equations to find 4 parameters  $a, b, \alpha, \beta$ .

So we can choose one variable arbitrarily, for example we take  $a = 2/3$  then  $b = 1/3, \alpha = 1, \beta = 1$  we get a special case of Runge-Kutta method of order 2 which is actually modified Euler's method,

$$i, e \ x_{n+1} = x_n + h$$

$$y_{n+1} = y_n + \frac{1}{2} h [f(x_n, y_n) + f(x_n + h, y_n + hf(x_n, y_n))]$$

Now for fourth order Runge-Kutta formula,

$$k_1 = hf(x_n, y_n)$$

$$k_2 = hf \left( x_n + \frac{h}{2}, y_n + \frac{k_1}{2} \right)$$

$$k_3 = hf \left( x_n + \frac{h}{2}, y_n + \frac{k_2}{2} \right)$$

$$k_4 = hf(x_n + h, y_n + k_3), \text{ Then we have, } y_{n+1} = y_n + \frac{1}{6} (k_1 + 2k_2 + 2k_3 + k_4)$$

**3.2 Problem of Runge-Kutta Method:**

Use Runge-Kutta method of 4<sup>th</sup> order to approximate the solution of the following initial value problem.

$$y' = y - x^2 + 1, 0 \leq x \leq 2, y(0) = 0.5 \text{ and } h = 0.2 \text{ in } [0, 2].$$

And compare with actual solution.



**Solution:**

Given that,

$$y' = f(x, y) = y - x^2 + 1 \dots\dots\dots(1)$$

Now, [

$$x_0 = 0, y_0 = 0.5, h = 0.2$$

$$n = \frac{b-a}{h} = \frac{2-0}{0.2} = 10$$

By Runge-Kutta method of order 4 we have,

$$y_{n+1} = y_n + \frac{1}{6}(k_1 + 2k_2 + 2k_3 + k_4) \dots\dots\dots(2)$$

Where

$$k_1 = hf(x, y) \dots\dots\dots(3)$$

$$k_2 = hf\left(x + \frac{h}{2}, y + \frac{k_1}{2}\right) \dots\dots\dots(4)$$

$$k_3 = hf\left(x + \frac{h}{2}, y + \frac{k_2}{2}\right) \dots\dots\dots(5)$$

$$k_4 = hf(x+h, y+k_3) \dots\dots\dots(6)$$

$$x_{n+1} = x_n + h \dots\dots\dots(7)$$

Actual solution of the given problem is:

$$y(x) = (x+1)^2 - 0.5e^x \dots\dots\dots(8)$$

Now the iteration table for the RK-4 method comparing with actual solution is as follows:

n	$x_n$	$Y_n(\text{RK-4})$	$Y_n(\text{Exact})$	Error
0	0.0	0.5000	0.5000	0.0000
1	0.2	0.8293	0.8293	0.0000
2	0.4	1.2140	1.2140	0.0000
3	0.6	1.6489	1.6489	0.0000
4	0.8	2.1272	2.1272	0.0000
5	1.0	2.6408	2.6408	0.0000
6	1.2	3.1798	3.1799	0.0001
7	1.4	3.7323	3.7324	0.0001
8	1.6	4.2834	4.2835	0.0001
9	1.8	4.8150	4.8151	0.0001
10	2.0	5.3053	5.3054	0.0001

**Table 2: iteration table for the RK-4 method comparing with actual solution**





3.3 Graph of The Solution:

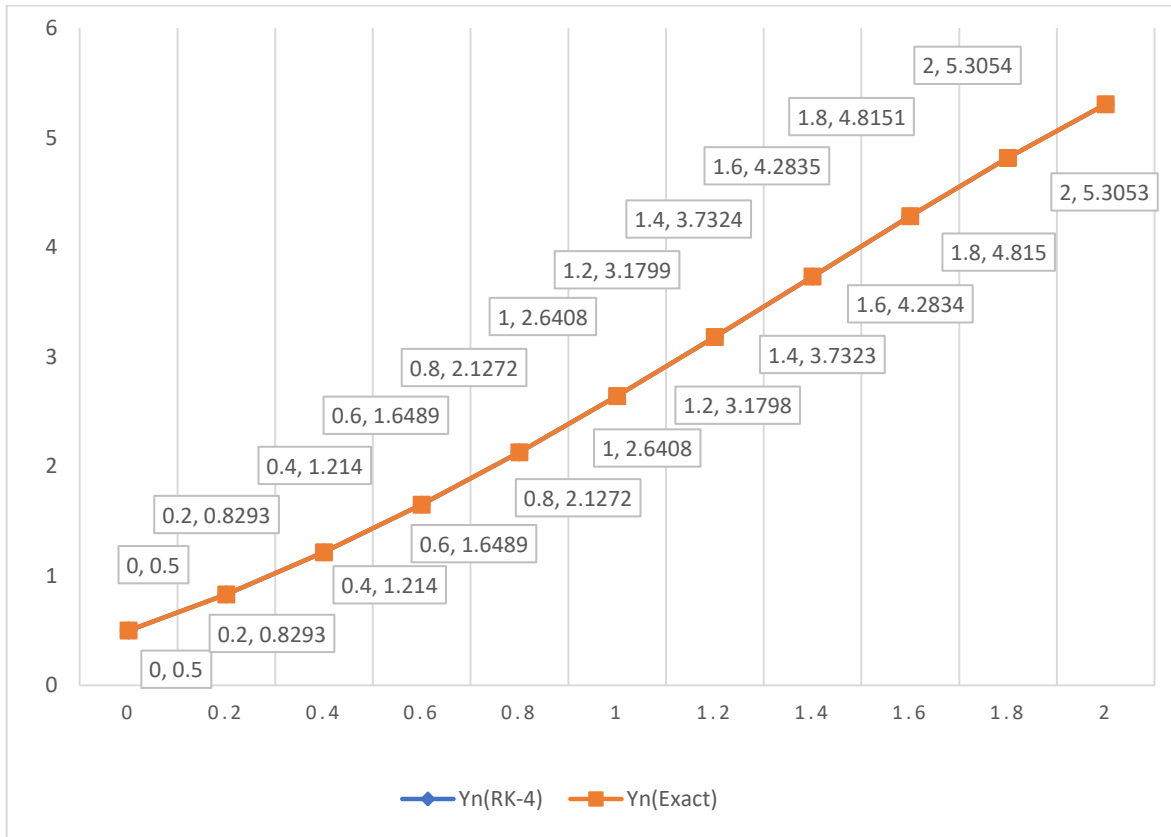


Figure 2: RK-4 method comparing with exact solution

IV. THE ADAMS-BASHFORTH METHOD(THREE-STEP EXPLICIT)

Adams-Bashforth method is a multistep method because it uses  $y_{k-2}, y_{k-1}$  and  $y_k$  in the calculation of  $y_{k+1}$ . This method is not self-starting, three initial points  $(t_0, y_0), (t_1, y_1), (t_2, y_2)$  must be given in order to generate the points

$$\{(t_k, y_k)\}_{k=3}^{k=m}$$

4.1 Derivation of Adams-Bashforth Three-Step Method:

To begin the derivation of a multi-step method, the solution of the initial value problem  $y' = f(t, y), a \leq t \leq b, y(a) = \alpha$  .....(1)

If integrated over the interval  $[t_i, t_{i+1}]$  has the property that

$$y(t_{i+1}) - y(t_i) = \int_{t_i}^{t_{i+1}} f(t, y(t)) dt$$

Consequently

$$y(t_{i+1}) = y(t_i) + \int_{t_i}^{t_{i+1}} f(t, y(t)) dt$$
 .....(2)

To derive an Adams-Bashforth explicit m-step technique, we form the backward difference polynomial  $P_{m-1}(t)$  through  $(t_i, f(t_i, y(t_i))), (t_{i-1}, f(t_{i-1}, y(t_{i-1}))), \dots, (t_{i+1-m}, f(t_{i+1-m}, y(t_{i+1-m})))$ .



Since  $P_{m-1}(t)$  is an interpolator polynomial of degree  $m-1$ , some number  $\xi_i$  in  $(t_{i+1-m}, t_i)$  exists with

$$f(t, y(t)) = P_{m-1}(t) + \frac{f^{(m)}(\xi_i, y(\xi_i))}{m!} (t - t_i)(t - t_{i-1}) \cdots (t - t_{i+1-m}).$$

Introducing the variable substitution  $t = t_i + sh$  with  $dt = hds$  into  $P_{m-1}(t)$  and the error term implies that

$$\begin{aligned} \int_{t_i}^{t_{i+1}} f(t, y(t))dt &= \int_{t_i}^{t_{i+1}} \sum_{k=0}^{m-1} (-1)^k \binom{-s}{k} \nabla^k f(t_i, y(t_i))dt + \int_{t_i}^{t_{i+1}} \frac{f^{(m)}(\xi_i, y(\xi_i))}{m!} (t - t_i)(t - t_{i-1}) \cdots (t - t_{i+1-m})dt \\ &= \sum_{k=0}^{m-1} \nabla^k f(t_i, y(t_i))h(-1)^k \int_0^1 \binom{-s}{k} ds + \frac{h^{m+1}}{m!} \int_0^1 s(s+1)\cdots(s+m-1)f^{(m)}(\xi_i, y(\xi_i))ds \end{aligned}$$

The integrals  $(-1)^k \int_0^1 \binom{-s}{k} ds$  for various values of  $k$  are easily evaluated and are listed the table below as for example when  $k=3$ ,

$$\begin{aligned} (-1)^3 \int_0^1 3ds &= \int_0^1 \frac{(-s)(-s-1)(-s-2)}{1.2.3} ds \\ &= \frac{1}{6} \left[ \frac{s^4}{4} + s^3 + s^2 \right]_0^1 = \frac{1}{6} \left( \frac{9}{4} \right) = \frac{3}{8} \end{aligned}$$

k	0	1	2	3	4	5
$(-1)^k \int_0^1 \binom{-s}{k} ds$	1	1/2	5/12	3/8	251/720	95/288

As a sequence,

$$\begin{aligned} \int_{t_i}^{t_{i+1}} f(t, y(t))dt &= h \left[ f(t_i, y(t_i)) + \frac{1}{2} \nabla f(t_i, y(t_i)) + \frac{5}{12} \nabla^2 f(t_i, y(t_i)) + \cdots \right] \\ &+ \frac{h^{m+1}}{m!} \int_0^1 s(s+1)\cdots(s+m-1)f^{(m)}(\xi_i, y(\xi_i))ds \dots\dots\dots(3) \end{aligned}$$

Since  $s(s+1)\cdots(s+m-1)$  does not change sign on  $[0,1]$  the weighted mean value theorem for integrals can be used to deduce that for some number  $\mu_i$ , where  $t_{i+1-m} < \mu_i < t_{i+1}$ , the error term becomes,

$$\frac{h^{m+1}}{m!} \int_0^1 s(s+1)\cdots(s+m-1)f^{(m)}(\xi_i, y(\xi_i))ds = \frac{h^{m+1}f^{(m)}(\mu_i, y(\mu_i))}{m!} \int_0^1 s(s+1)\cdots(s+m-1)ds$$

Hence the error in (3) simplifies to,

$$h^{m+1}f^{(m)}(\mu_i, y(\mu_i))(-1)^m \int_0^1 \binom{-s}{m} ds \dots\dots\dots(4)$$

Since  $y(t_{i+1}) - y(t_i) = \int_{t_i}^{t_{i+1}} f(t, y(t))dt$  then equation (1) can be written as,

$$\begin{aligned} y(t_{i+1}) &= y(t_i) + h \left[ f(t_i, y(t_i)) + \frac{1}{2} \nabla f(t_i, y(t_i)) + \frac{5}{12} \nabla^2 f(t_i, y(t_i)) + \cdots \right] \\ &+ h^{m+1}f^{(m)}(\mu_i, y(\mu_i))(-1)^m \int_0^1 \binom{-s}{m} ds \dots\dots\dots(5) \end{aligned}$$

To derive the three step Adams-Bashforth method considering equation (5) with  $m=3$  we get,

$$y(t_{i+1}) \approx y(t_i) + h \left[ f(t_i, y(t_i)) + \frac{1}{2} \nabla f(t_i, y(t_i)) + \frac{5}{12} \nabla^2 f(t_i, y(t_i)) + \cdots \right]$$



$$\begin{aligned}
 &= y(t_i) + h\{f(t_i, y(t_i)) + \frac{1}{2}[f(t_i, y(t_i)) - f(t_{i-1}, y(t_{i-1}))]\} + \\
 &\frac{5}{12}[f(t_i, y(t_i)) - 2f(t_{i-1}, y(t_{i-1})) + f(t_{i-2}, y(t_{i-2}))] \\
 &= y(t_i) + \frac{h}{12}[23f(t_i, y(t_i)) - 16f(t_{i-1}, y(t_{i-1})) + 5f(t_{i-2}, y(t_{i-2}))]
 \end{aligned}$$

The three step Adams-Bashforth method is consequently

$$w_0 = \alpha, w_1 = \alpha_1, w_2 = \alpha_2$$

$$w_{i+1} = w_i + \frac{h}{12}[23f(t_i, w_i) - 16f(t_{i-1}, w_{i-1}) + 5f(t_{i-2}, w_{i-2})] \text{ for } i = 2, 3, \dots, N-1$$

**4.2 Problem of Adams-Bashforth Method:**

Solve the following initial value problem using Adams-Bashforth 3 step explicit method.

$$y' = -5y + 5x^2 + 2x, 0 \leq x \leq 1, y(0) = \frac{1}{3} \text{ with } h = 0.1 \text{ and compare with the exact solution.}$$

**Solution:**

Given that

$$y' = -5y + 5x^2 + 2x \dots (1)$$

$$a = 0, b = 1, x_0 = 0, y_0 = \frac{1}{3}, h = 0.1$$

$$N = \frac{b-a}{h} = \frac{1-0}{0.1} = 10$$

By Adams-Bashforth 3 step explicit method we know that,

$$y_{n+1} = y_n + \frac{h}{12}[23f(x_n, y_n) - 16f(x_{n-1}, y_{n-1}) + 5f(x_{n-2}, y_{n-2})] \dots (2)$$

Where  $n = 2, 3, \dots, N-1$

Exact solution of the given problem is,

$$y(x) = x^2 + \frac{1}{3}e^{-5x} \dots (3)$$

At first, we find  $y_1$  and  $y_2$  using RK-4 method.

We know by RK-4 method:

$$y_{n+1} = y_n + \left(\frac{k_1 + 2k_2 + 2k_3 + k_4}{6}\right) \dots (4)$$

Where

$$k_1 = hf(x, y) \dots (5)$$

$$k_2 = hf\left(x + \frac{h}{2}, y + \frac{k_1}{2}\right) \dots (6)$$

$$k_3 = hf\left(x + \frac{h}{2}, y + \frac{k_2}{2}\right) \dots (7)$$

$$k_4 = hf(x+h, y+k_3) \dots (8)$$

$$x_{n+1} = x_n + h \dots (9), n = 0, 1$$

$$\therefore y_1 = 0.2123$$



and  $y_2 = 0.1628$

Now the solution table for the Adams-Bashforth method is as follows:

n	$X_n$	$Y_n(\text{Adam})$	$Y_n(\text{Exact})$	Error
3	0.3	0.1605	0.1644	0.0038
4	0.4	0.1853	0.2051	0.0198
5	0.5	0.2526	0.2774	0.0274
6	0.6	0.3513	0.3766	0.0253
7	0.7	0.4757	0.5001	0.0244
8	0.8	0.6228	0.6461	0.0233
9	0.9	0.7914	0.8137	0.0223
10	1.0	0.9807	1.0022	0.0215

Table 3: solution table for the Adams-Bashforth method

### 4.3 Graph of The Solution:

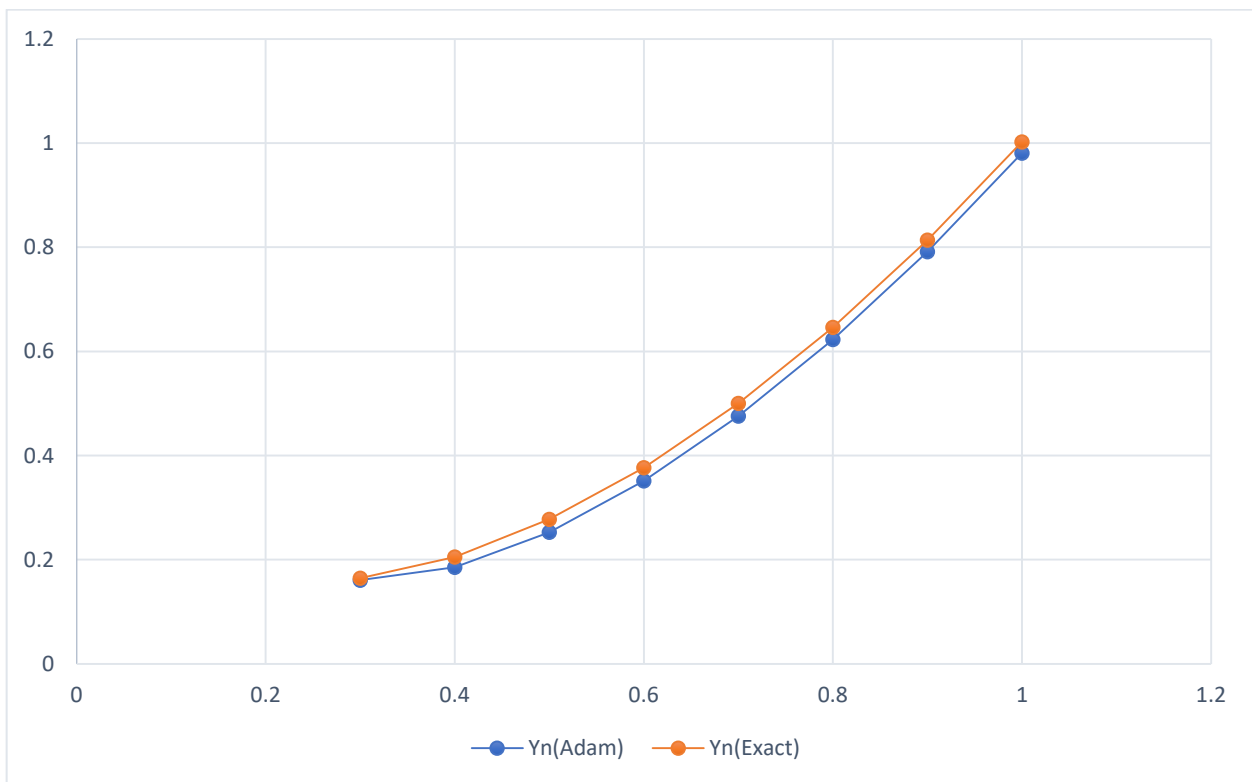


Figure 3: Adams-Bashforth method comparing with the exact solution.

## V. CONCLUSIONS

In this paper, Researcher has studied and observed Euler method, Runge Kutta method, and Adams-Bashforth method are used numerical methods to solve first order ordinary differential equations (ODE) in initial value problems (IVP). Finding more accurate results needs the step size smaller for all methods. From the figures we can see the accuracy of the methods. The numerical solutions attained by the proposed methods are in good arrangement with exact solutions. Comparative study of these methods has been done using different step size with use of MATLAB and FORTRAN programming languages. Various table and graphs the more convergent methods among all off these mentioned

methods. These methods are critically studied and differentiated. It may be concluded that the Runge Kutta method is influential and more efficient in finding numerical solutions of initial value problems (IVP). Euler method stretches least precise results also the behavior of the solutions has been represented graphically for better understanding.

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