

Rocket Engine Nozzle Optimization

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ABSTRACT: Convergent-Divergent nozzle is a device which usually being used in rocket engine which converts chemical energy into kinetic energy. Commonly the C-D nozzle has two major types of contours namely conical and parabolic. The present work consists of analytical and numerical analysis with geometrical optimization of parabolic nozzle. Phase 1 of the study includes CFD based comparison of conical and parabolic nozzle contour using ANSYS Fluent conforming that the performance of parabolic nozzle contour is better. Phase 2 deals with geometrical optimization of parabolic nozzle using Taguchi Methodology which focuses on quality improvement and making products and processes insensitive to manufacturing and environmental variations. Software tool for optimization used here is Minitab. Optimization resulted in maximization of thrust and Mach number considering the area ratio, angle ratio and length ratio as constraints.

KEYWORDS: Convergent-Divergent Nozzle, Taguchi Robust Design, specific impulse, etc.

I. INTRODUCTION

- Optimization, Analytical and Numerical Analysis of Rocket Engine Nozzle. To establish that Parabolic Nozzle geometry is better in performance than Conical Nozzle through CFD based comparison using ANSYS Fluent. To establish a relationship between the input factors in the study – Area Ratio, Angle Ratio, Length Ratio and response factor –Thrust, Mach number. To find out the optimum nozzle parameters in order to achieve better Thrust and an improvised Exit Mach Number using Conventional Taguchi Robust Design Method. Reverification of the flow properties of Thrust Optimized Parabolic Nozzle geometry using CFD Analysis

II. RELATED WORK

Venkatesh.V, C Jaya Pal Reddy worked on the topic of “Modelling and simulation of supersonic nozzle using computational fluid dynamics” and their findings are the work outlined in this report is to design a supersonic convergent-divergent (CD nozzle). A design procedure which can determine the configuration of C-D nozzle is shown by arranging the experimental results using CFD (FLUENT).

Shailendra Kumar Bohidar, Kumar Gaurav, Prakash Kumar Sen worked on the topic of “A brief study about advancement in rocket nozzle used in aircraft science” and their findings are the realization of SSTO vehicles strongly depends on the performance of the engines, which should deliver high performance with low system complexity. Humphrey Iortyer, Nongo Sesugh, Kwaghger Aondonaworked on the topic of “Modelling of chamber pressure for rocket nozzle altitude compensation” and their findings are the optimum thrust of rocket engine is being delivered when the nozzle exit and ambient pressures are balanced.

K.P.S. Surya Narayana, K. Sadashiv Reddy worked on the topic of “Simulation of Convergent Divergent Rocket Nozzle using CFD Analysis” and their findings are in this research paper author has analysis over a C-D rocket nozzle which is performed by varying the number of divisions in mesh (mesh 1 to mesh 5). The various contours of nozzle like Cell Equiangular skew, Cell Reynolds number, Pressure, Velocity, Mach number, and above are calculate.

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Arjun Kundu; Devyanshu Prasad; Sarfraj Ahmed worked on the topic of “Effect of the exit diameter on the performance of the C-D nozzle by using CFD” and their findings are in this research paper the pressure, temperature and velocity are kept same in both the cases at inlet. by changing exit diameter of the nozzle to get the supersonic flow.

III. RESEARCH METHODOLOGY

Based on the equation, the analytically calculated properties of the flow are shown in table:

Parameters	Values	Parameters	Values
R	287 J/Kg.K	P ₀	21 bar
C _p	1005 J/Kg.K	T ₀	400 K
I _{sp}	47588.99 Sec	A _t	3.532 * 10 ⁻³ m ²
m	15 Kg/Sec	P _e	1 bar
T _e	167.61 K	γ	1.4
V _e	683.44 m/Sec	M	28.97 kg/kg.mol
T	10.25 KN	A _i	7.539 * 10 ⁻³ m ²
A _e	12.14 m ²	A _e	0.04918 m ²
M _e	2.633		
P _t	11.09 bar		
T _t	333.33 K		
V _t	365.96 m/Sec		

Table 3.1 Calculated Properties

The ‘Rao nozzle’ starts with Rao’s preferred throat geometry.

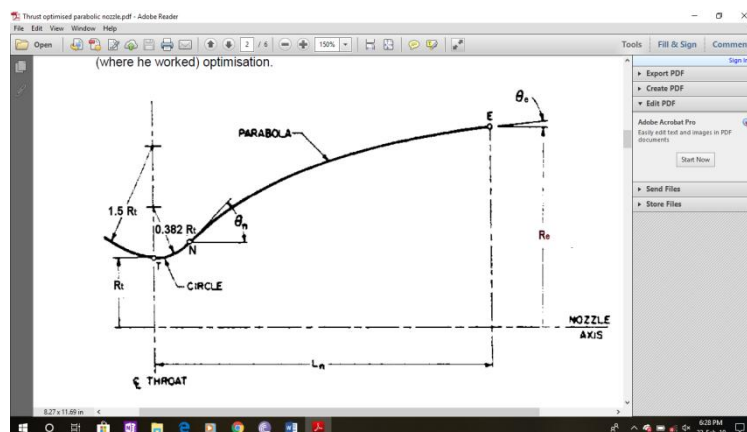


Figure 3.1 Contour of Parabolic Nozzle

The first curve, of radius 1.5R_t, is drawn from an angle of say, -135 degrees, to the throat at -90 degrees (angles measured from the arc’s origin). Then the second curve of radius 0.382R_t is drawn from this angle of -90 degrees to an angle of (θ_n- 90) at inflection point N. (R_t is the throat radius, R_e is the exit radius.) Then a skewed parabola is drawn

from point N to nozzle exit point E, tangent to the throat curve, and starting at an angle of θ_n and ending at an angle of θ_e .

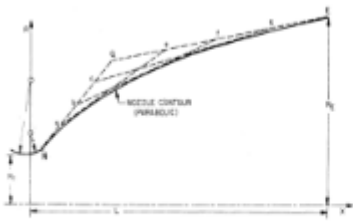


Figure 3.2 Ancient geometrical methods

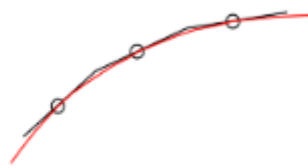


Figure 3.3 Parabolic contour by using spline

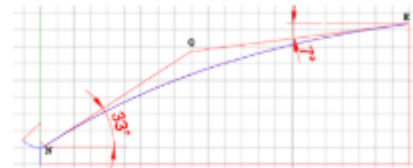


Figure 3.4 Angles of tangent to draw θ_n and θ_e

The bell is a quadratic Bezier curve, which has equations:

$$x(t) = (1-t)^2 N_x + 2(1-t)t Q_x + t^2 E_x \quad 0 \leq t \leq 1 \quad 3.1$$

$$y(t) = (1-t)^2 N_y + 2(1-t)t Q_y + t^2 E_y \quad 0 \leq t \leq 1 \quad 3.2$$

Selecting equally spaced divisions between 0 and 1 produces the points described earlier in the graphical method, for example 0.25, 0.5, and 0.75. Equations 6 are defined by points N, Q, and E (see the graphical method earlier for the locations of these points). Point N is defined by equations 5 setting the angle to $(\theta_n - 90)$. Coordinate E_x is defined by equation 3, and coordinate E_y is defined by equation 2. Point Q is the intersection of the lines:

$$NQ = m_1x + C_1 \quad \text{and} \quad QE = m_2x + C_2 \quad 3.3$$

Where, Gradient $m_1 = \tan n$, Gradient $m_2 = \tan e$

And, intercept $C_1 = N_y - m_1 N_x$, intercept $C_2 = E_y - m_2 E_x$

The intersection of these two lines (at point Q) is given by:

$$Q_x = \frac{c_2 - c_1 m_1 - m_2}{m_1 - m_2} \quad \text{and} \quad Q_y = \frac{(m_1 c_2 - m_2 c_1)}{(m_1 - m_2)} \quad 3.4$$

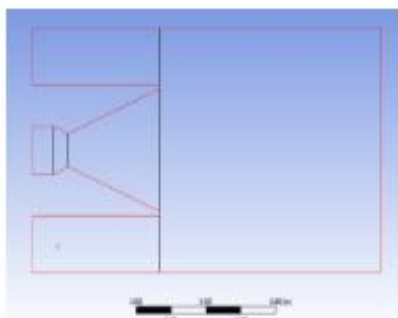


Figure 3.5 wireframe of the conical nozzle

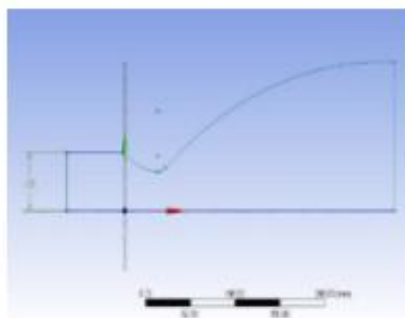


Figure 3.6 Wireframe sketch of parabolic nozzle

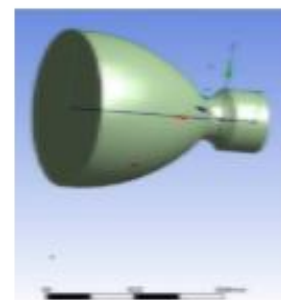


Figure 3.7 Geometry of Parabolic Nozzle in 3D

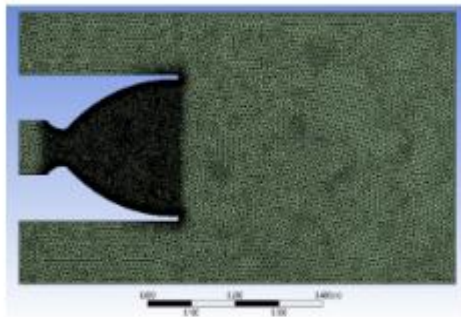


Figure 3.8 Domain discretization

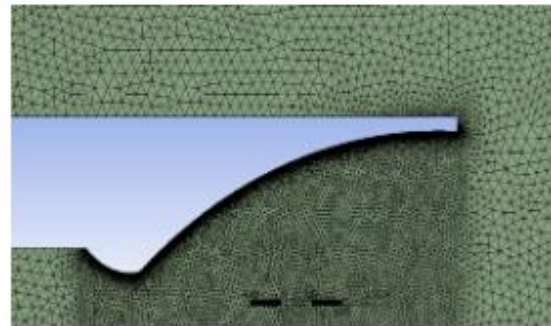


Figure 3.9 Condensed mesh near the nozzle wall

The used mesh is 2D, unstructured and axisymmetric. The region near the axis has smaller cells to capture details of the plume pattern. The throat region is the densest part of the mesh. An inflation layer is applied to the nozzle wall. The boundary layer consists of 20 cells with a total thickness of ~0.6 mm. Total cell number for the baseline mesh used in the validation process is 119685. The modified grids used in mesh and model sensitivity studies vary from that. The mesh generated during the optimization process is similar to the baseline mesh and cell numbers vary less than 5 % from the baseline. The fluent conditions to the design nozzle geometry are energy equation included viscous model- K epsilon.

Boundary Name	Parameters
Pressure-inlet (chamber)	Ptotal = 2100000 Pa, Ttotal = 400K, Air as ideal gas
Pressure-inlet (ambient)	Ptotal = 100000 Pa, Ttotal = 300K, Air as ideal gas
Free-slip wall	Adiabatic, Zero shear
Wall	Adiabatic, No Slip
Pressure-Outlet	Ptotal= 100000 Pa

Table 3.2 Boundary conditions for validation

For conical nozzle

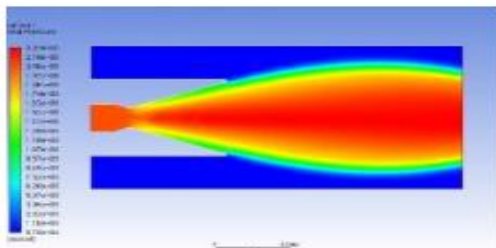


Figure 3.10 Total pressure contour

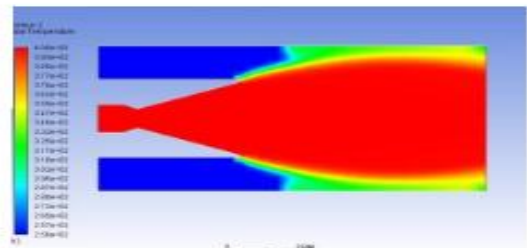


Figure 3.11 Total temperature contour

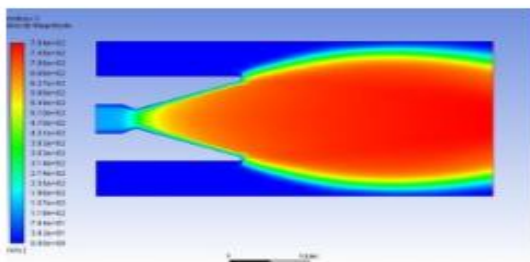


Figure 3.12 Velocity magnitude contour

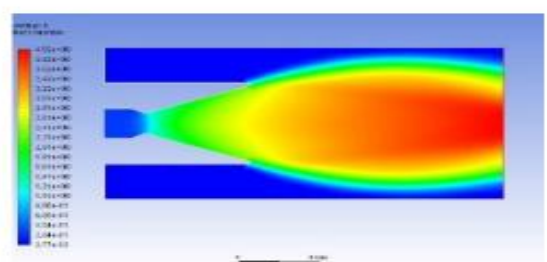


Figure 3.13 Mach number contour

For parabolic nozzle

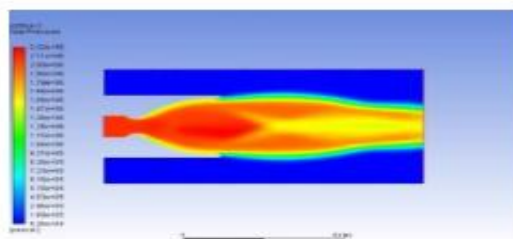


Figure 3.14 Total pressure contour

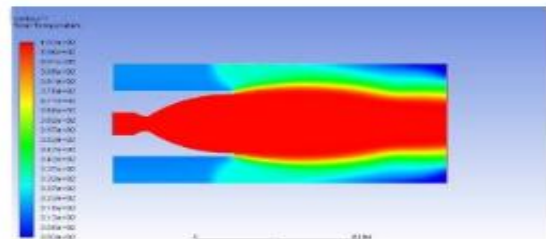


Figure 3.15 Total temperature contour

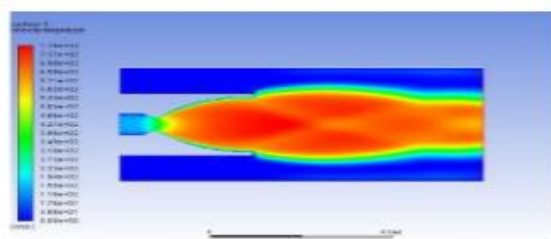


Figure 3.16 Velocity magnitude contour

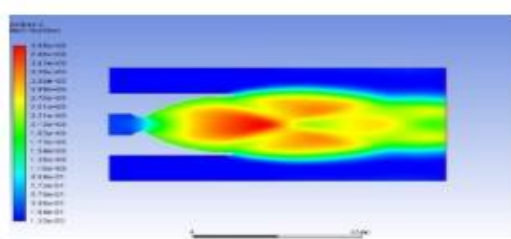


Figure 3.17 Mach number contours

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Experiment Run	Parameter 1	Parameter 2	Parameter 3
1	1	1	1
2	1	2	2
3	1	3	3
4	2	1	2
5	2	2	3
6	2	3	1
7	3	1	3
8	3	2	1
9	3	3	2
Factors	Level 1	Level 2	Level 3
Area Ratio	13.9214	10.4675	17.3807
Angle Ratio	2.8000	2.6638	3.9275
Length Ratio	7.6797	6.2923	8.9205

Table 3.4 Orthogonal Array L9

S. No.	Area Ratio	Angle Ratio	Length Ratio	Thrust	Mach No.
1	13.9214	2.8000	7.6797	16.2493	4.2974
2	13.9214	2.6638	6.2923	16.0976	4.1354
3	13.9214	3.9275	8.9205	16.1734	4.2164
4	10.4675	2.8000	6.2923	15.7823	4.0544
5	10.4675	2.6638	8.9205	15.4670	3.9734
6	10.4675	3.9275	7.6797	15.6246	4.0139
7	17.3807	2.8000	8.9205	17.7392	4.5571
8	17.3807	2.6638	7.6797	17.0442	4.4275
9	17.3807	3.9275	6.2923	17.3917	4.4923

Table 3.5 Trial Experiments and respective Response Value

Taguchi Analysis: Thrust versus Area Ratio, Angle Ratio, Length Ratio

1. Linear Model Analysis: SN ratios versus Area Ratio, Angle Ratio, Length Ratio

Term	Coef	SE Coef	T	P
Constant	24.2859	0.02243	1082.736	0.000
Area Rat 10.4675	-0.4100	0.03172	-12.924	0.006
Area Rat 13.9214	-0.1099	0.03172	-3.464	0.074
Angle Ra 2.6638	-0.1009	0.03172	-3.181	0.086
Angle Ra 2.8000	0.1004	0.03172	3.165	0.087
Length R 6.2923	0.0160	0.03172	0.503	0.665

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Length R 7.6797	-0.0444	0.03172	-1.399	0.297		
Source	DF	Seq SS	Adj SS	Adj MS	F	P
Area Ratio	2	1.35107	1.35107	0.675535	149.19	0.007
Angle Ratio	2	0.06078	0.06078	0.030392	6.71	0.130
Length Ratio	2	0.00910	0.00910	0.004549	1.00	0.499
Residual Error	2	0.00906	0.00906	0.004528		
Total	8	1.43001				

Table 3.6 Estimated Model Coefficients for SN ratios

Table 3.8 Analysis of Variance for SN ratios

S	R-Sq	R-Sq(adj)
0.0673	99.37%	97.47%

Table 3.7 Model Summary

2. Linear Model Analysis: Means versus Area Ratio, Angle Ratio, Length Ratio

Term	Coef	SE Coef	T	P		
Constant	16.3966	0.04645	352.980	0.000		
Area Rat 10.4675	-0.7720	0.06569	-11.751	0.007		
Area Rat 13.9214	-0.2232	0.06569	-3.397	0.077		
Angle Ra 2.6638	-0.1937	0.06569	-2.948	0.098		
Angle Ra 2.8000	0.1937	0.06569	2.948	0.098		
Length R 6.2923	0.0273	0.06569	0.415	0.718		
Length R 7.6797	-0.0906	0.06569	-1.378	0.302		
Source	DF	Seq SS	Adj SS	Adj MS	F	P
Area Ratio	2	4.90788	4.90788	2.45394	126.36	0.008
Angle Ratio	2	0.22504	0.22504	0.11252	5.79	0.147
Length Ratio	2	0.03885	0.03885	0.01942	1.00	0.500
Residual Error	2	0.03884	0.03884	0.01942		
Total	8	5.21061				

Table 3.9 Estimated Model Coefficients for Means

Table 3.11 Analysis of Variance for Means

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S	R-Sq	R-Sq(adj)	
0.1394	99.25%	97.02%	
Level	Area Ratio	Angle Ratio	Length Ratio
1	15.62	16.20	16.42
2	16.17	16.59	16.31
3	17.39	16.40	16.46
Delta	1.77	0.39	0.15
Rank	1	2	3

Table 3.10 Model Summary

Level	Area Ratio	Angle Ratio	Length Ratio
1	23.88	24.18	24.30
2	24.18	24.39	24.24
3	24.81	24.29	24.31
Delta	0.93	0.20	0.07
Rank	1	2	3

Response Table for Signal to Noise Ratios (Table 3.12)

Response Table for Means (Table 3.13)

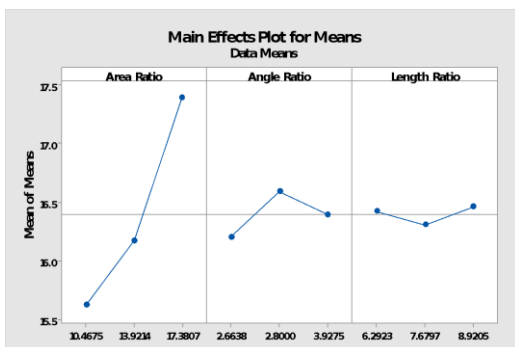


Figure 3.18 Main Effect Plot for Means (Thrust Analysis)

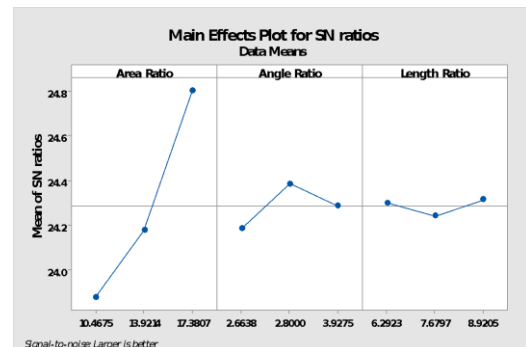


Figure 3.19 Main Effect Plot for SN Ratio

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General Linear Model: Thrust versus Area Ratio, Angle Ratio and Length Ratio: Factor coding (-1, 0, +1)

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Area Ratio	2	4.90788	2.45394	126.36	0.008
Angle Ratio	2	0.22504	0.11252	5.79	0.147
Length Ratio	2	0.03885	0.01942	1.00	0.500
Error	2	0.03884	0.01942		
Total	8	5.21061			

Factor	Type	Levels	Values
Area Ratio	Fixed	3	10.4675, 13.9214, 17.3807
Angle Ratio	Fixed	3	2.6638, 2.8000, 3.9275
Length Ratio	Fixed	3	6.2923, 7.6797, 8.9205

Table 3.14 Factor Information

Table 3.15 Analysis of Variance

S	R-sq	R-sq(adj)	R-sq(pred)		
0.139356	99.25%	97.02%	84.91%		
Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	16.3966	0.0465	352.98	0.000	
Area Ratio					
10.4675	-0.7720	0.0657	-11.75	0.007	1.33
13.9214	-0.2232	0.0657	-3.40	0.077	1.33
Angle Ratio					
2.6638	-0.1937	0.0657	-2.95	0.098	1.33
2.8000	0.1937	0.0657	2.95	0.098	1.33
Length Ratio					
6.2923	0.0273	0.0657	0.42	0.718	1.33
7.6797	-0.0906	0.0657	-1.38	0.302	1.33

Table 3.16 Model Summary

Table 3.17 Coefficients

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Taguchi Analysis: Mach No. versus Area Ratio, Angle Ratio, Length Ratio

1. Linear Model Analysis: SN ratios versus Area Ratio, Angle Ratio, Length Ratio

Term	Coef	SE Coef	T	P		
Constant	12.5392	0.01320	949.762	0.000		
Area Rat 10.4675	-0.4682	0.01867	-25.074	0.002		
Area Rat 13.9214	-0.0414	0.01867	-2.219	0.157		
Angle Ra 2.6638	-0.1269	0.01867	-6.799	0.021		
Angle Ra 2.8000	0.1263	0.01867	6.763	0.021		
Length R 6.2923	-0.0264	0.01867	-1.416	0.292		
Length R 7.6797	0.0137	0.01867	0.733	0.540		
Source	DF	Seq SS	Adj SS	Adj MS	F	P
Area Ratio	2	1.44167	1.44167	0.720837	459.50	0.002
Angle Ratio	2	0.09618	0.09618	0.048090	30.66	0.032
Length Ratio	2	0.00315	0.00315	0.001574	1.00	0.499
Residual Error	2	0.00314	0.00314	0.001569		
Total	8	1.54414				

Table 3.18 Estimated Model Coefficients for SN ratios

S	R-Sq	R-Sq(adj)
0.0396	99.80%	99.19%

Table 3.20 Analysis of Variance for SN ratios

Table 3.19 Model Summary

2. Linear Model Analysis: Means versus Area Ratio, Angle Ratio, Length Ratio

Term	Coef	SE Coef	T	P
Constant	4.24087	0.006795	624.129	0.000
Area Rat 10.4675	-0.22697	0.009609	-23.619	0.002
Area Rat 13.9214	-0.02447	0.009609	-2.546	0.126
Angle Ra 2.6638	-0.06210	0.009609	-6.462	0.023

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Angle Ra 2.8000	0.06210	0.009609	6.462	0.023		
Length R 6.2923	-0.01350	0.009609	-1.405	0.295		
Length R 7.6797	0.00540	0.009609	0.562	0.631		
Source	DF	Seq SS	Adj SS	Adj MS	F	P
Area Ratio	2	0.345994	0.345994	0.172997	416.33	0.002
Angle Ratio	2	0.023138	0.023138	0.011569	27.84	0.035
Length Ratio	2	0.000831	0.000831	0.000416	1.00	0.500
Residual Error	2	0.000831	0.000831	0.000416		
Total	8	0.370794				

Table 3.21 Estimated Model Coefficients for Means

S	R-Sq	R-Sq(adj)
0.0204	99.78%	99.10%

Table 3.22 Model Summary

Table 3.23 Analysis of Variance for Means

Level	Area Ratio	Angle Ratio	Length Ratio
1	12.07	12.41	12.51
2	12.50	12.67	12.55
3	13.05	12.54	12.55
Delta	0.98	0.25	0.04
Rank	1	2	3
Level	Area Ratio	Angle Ratio	Length Ratio
1	4.014	4.179	4.227
2	4.216	4.303	4.246
3	4.492	4.241	4.249
Delta	0.478	0.124	0.022
Rank	1	2	3

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Response Table for Signal to Noise Ratios (Table 3.24)
3.25)

Response Table for Means (Table

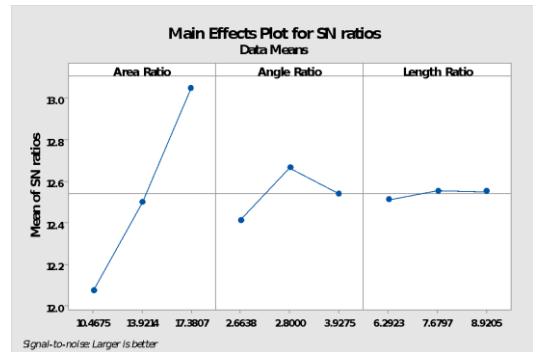
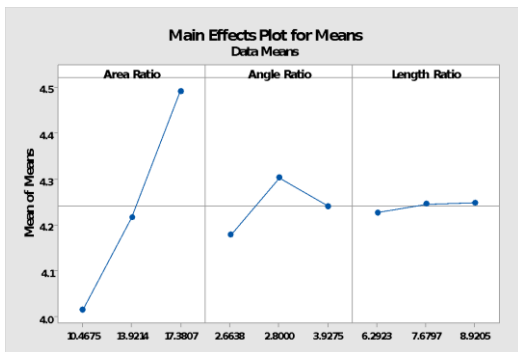


Figure 3.20 Main Effect Plot for Means (Mach No. Analysis)
ratios (Mach No. Analysis)

Figure 3.21 Main Effects Plot for SN

General Linear Model: Mach No. versus Area Ratio, Ratio, Length Ratio: Factor coding (-1, 0, +1)

Source	DF	Adj SS	Adj MS	F-Value	P-Value
Area Ratio	2	0.345994	0.172997	416.33	0.002
Angle Ratio	2	0.023138	0.011569	27.84	0.035
Length Ratio	2	0.000831	0.000416	1.00	0.500
Error	2	0.000831	0.000416		
Total	8	0.370794			

Factor	Type	Levels	Values
Area Ratio	Fixed	3	10.4675, 13.9214, 17.3807
Angle Ratio	Fixed	3	2.6638, 2.8000, 3.9275
Length Ratio	Fixed	3	6.2923, 7.6797, 8.9205

Table 3.26 Factor Information

Table 3.27 Analysis of Variance

Term	Coef	SE Coef	T-Value	P-Value	VIF
Constant	4.24087	0.00679	624.13	0.000	

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Area Ratio					
10.4675	-0.22697	0.00961	-23.62	0.002	1.33
13.9214	-0.02447	0.00961	-2.55	0.126	1.33
Angle Ratio					
2.6638	-0.06210	0.00961	-6.46	0.023	1.33
2.8000	0.06210	0.00961	6.46	0.023	1.33
Length Ratio					
6.2923	-0.01350	0.00961	-1.40	0.295	1.33
7.6797	0.00540	0.00961	0.56	0.631	1.33
S	R-sq	R-sq(adj)	R-sq(pred)		
0.0203846	99.78%	99.10%	95.46%		

Table 3.28 Model Summary

Table 3.29 Coefficients

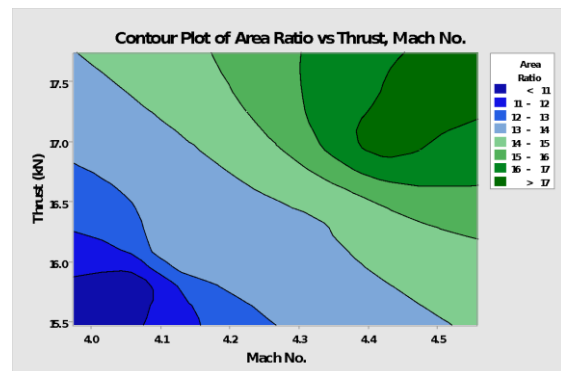
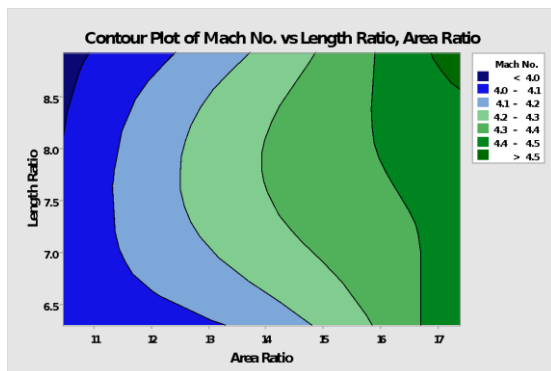


Figure 3.22 Contour Plot of Mach No. vs Length Ratio, Area Ratio Figure 3.23 Contour Plot of Area Ratio vs Thrust, Mach No.

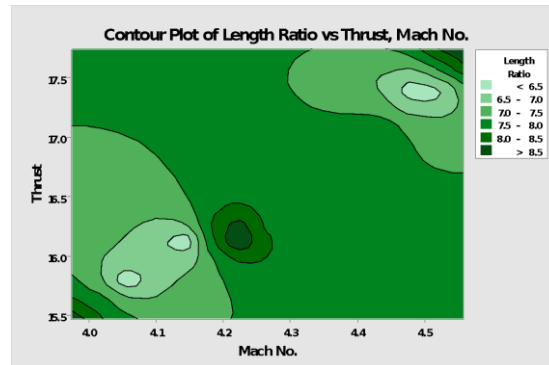


Figure 3.24 Contour Plot of Length Ratio vs Thrust, Mach No.

IV. RESULTS AND CONCLUSION

Phase I

The following figure 4.1 and figure 4.2 shows the comparison of the exit velocity of conical and parabolic nozzle contour throughout the length of the nozzle. It precisely indicates that the exit velocities for conical and parabolic nozzles are 640 m/s and 760 m/s respectively.

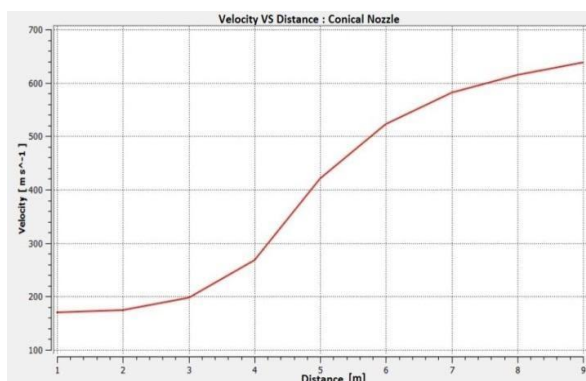


Figure 4.1 Velocity vs distance : Conical Nozzle

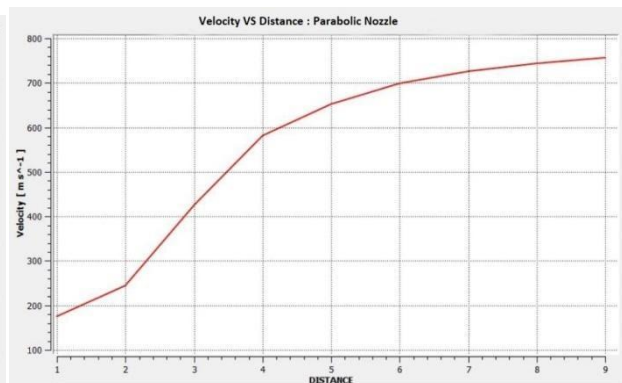


Figure 4.2 Velocity vs distance : Parabolic Nozzle

Phase II

After the optimization of the problem is carried out, using Taguchi Methodology, the three statistically significant parameters were obtained which are as follows:

Parameter	Optimal Value
Area Ratio	17.3807
Angle Ratio	2.80
Length Ratio	8.9205

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The new “Thrust Optimized Parabolic” Nozzle is designed in accordance with the respective optimal value of the constraints. A CFD analysis using ANSYS Fluent is then performed on TOP Nozzle which gives the exit velocity 858 m/s as shown in figure 4.3. The new exit velocity (as per analysis) is found to be increased in comparison to the previous nozzle geometry. Thus, the nozzle geometry is successfully optimized to obtain relatively higher thrust which is in accordance with the desired objective of the project.

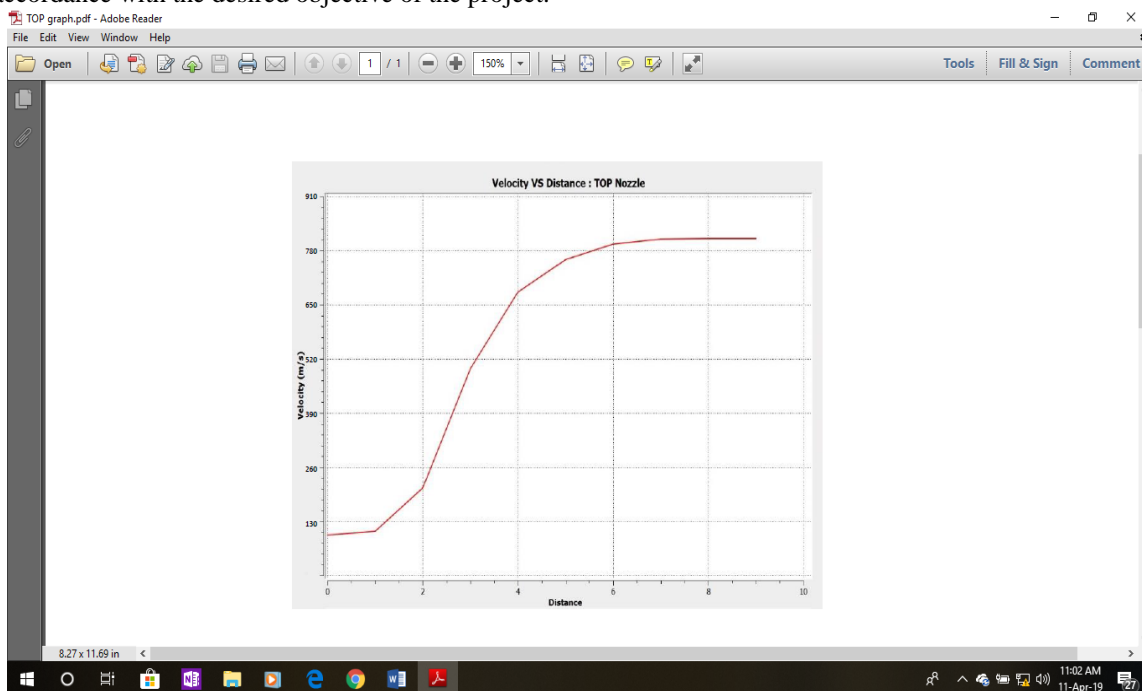


Figure 4.3 Velocity vs distance: TOP Nozzle

Nozzle Contour	Exit Velocity(m/s)
Conical	640
Parabolic	760
TOP	858

Table 4.1 Comparison of exit velocity of conical, parabolic and TOP Nozzle

Conclusion

1. The parabolic nozzle gives better performance than that of conical nozzle. The difference in their respective Mach Numbers is 18.62%.
2. The exit Mach number is increased by 12.97% in Thrust Optimized Parabolic Nozzle.

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