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# Complementary Tree Domination Number of A Graph

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**ABSTRACT:** A set  $D$  of a graph  $G=(V,E)$  is a dominating set if every vertex in  $V-D$  is adjacent to some vertex in  $D$ . The domination number  $\gamma(G)$  of  $G$  is the minimum cardinality of a dominating set. A dominating set  $D$  is called a complementary tree dominating set if the induced sub graph  $\langle V-D \rangle$  is a tree. The minimum cardinality of a complementary tree dominating set is called the complementary tree domination number of  $G$  and is denoted by  $\gamma_{ctd}(G)$ . In this paper, bounds for  $\gamma_{ctd}(G)$  and its exact values for some particular classes of graphs are found. Some results on complementary tree domination number are also established. Mathematics Subject Classification: 05C69

**KEYWORDS:** Domination Number, Graph

## I. INTRODUCTION

A Graph discussed in this paper are undirected and simple graphs. For a graph  $G$ , Let  $V(G)$  denoted vertex set,  $E(G)$  denote its edge set for  $V \in V(G)$ , the neighbourhood  $N(V)$  of  $V$  is the set of all vertices adjacent to  $V$  in  $G$ .  $N[v]=N(V) \cup \{v\}$  is called closed neighbourhood of  $V$ . A vertex  $V \in V(G)$  is called support if it is adjacent to a pendant vertex (that is. a vertex of degree 1) [4].

The graphs considered here are finite, undirected, without loops (or) multiple edges and connected with  $p$  vertices and  $q$  edges. A set  $D \subseteq V(G)$  is said to be a dominating set  $G$ , if every vertex in  $V(G) - D$  is adjacent to some vertex in  $D$ .  $D$  is said to be a Minimal dominating set if  $D - \{u\}$  is not dominating set for any  $u \in D$ . The dominating number  $\gamma(G)$  of  $G$  is the minimum cardinality of a dominating set.  $D$  is a connected dominating set if it is a dominating set and the subgraph induced by  $D$  is connected [5].

The connected domination number  $\gamma_c(G)$  is the minimum cardinality of a connected. A dominating set  $D$  of a connected graph  $G$  is a non - split dominating set, if the induced subgraph  $\langle V(G)-D \rangle$  is connected. The non - split domination number  $\gamma_{ns}(G)$  of  $G$  is the minimum cardinality of a non-split dominating set [5]. We call a set of vertices a  $V$ -set if it is a dominating set with cardinality  $\gamma(g)$ .

### Definition 1

A set  $D \subseteq V(G)$  is said to be complementary tree dominating set (ctd set) if the induced subgraph  $\langle V(G)-D \rangle$  is a tree. The minimum cardinality of a ctd set is called the complementary tree domination number of  $G$  and is denoted by  $\gamma_{ctd}(G)$ .

### Definition 2

A graph is said to be **connected** if there is a path between every pair of vertices. From every vertex to any other vertex, there should be some path to traverse. That is called the connectivity of a graph. A graph with multiple **disconnected** vertices and edges is said to be disconnected.

### Definition 3

A **complete** graph is a simple undirected graph in which every pair of distinct vertices is connected by a unique edge.

### Definition 4

A vertex of a graph is said to be **pendant vertex** if its neighbourhood contains exactly one vertex.



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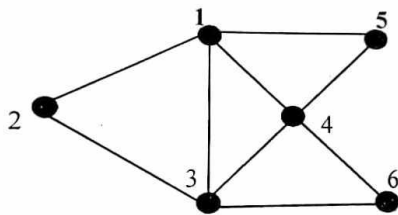
**Definition 5**

A tree is a connected graph without any cycles, or a tree is a connected acyclic graph.

**II. BOUNDS AND SOME EXACT VALUES OF COMPLEMENT TREE DOMINATION NUMBER**

**Observation 3.1 [1]**

For any connected graph G, with  $p \geq 2$ ,  $\gamma_{ctd}(G) \leq p-1$   $p \geq 2$ ,  $p=6$  No. of vertices.



$Ctd(G) = \{2,4\} = D$

$\langle V-D \rangle = \{1,3,5,6\}$  is a tree

$\gamma_{ctd}(G) = 2 \leq p-1 = 2 \leq 6-1 = 2 \leq 5$

$\gamma_{ctd}(G) \leq p-1$

**THEOREM 3.2**

For any connected graph  $(p,q)$  graph G with  $\delta(G) \geq 2$   $\gamma_{ctd}(G) \geq 3p-2q-2$ .

**Proof:**

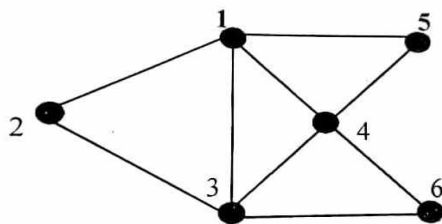
$\Delta(G) \geq 2$ ,

$P=6$ (no of vertices),  $Q=9$ (no of edges)

$\gamma_{ctd}(G) \geq 3p-2q-2$

$2 \geq 3(6) - 2(9) - 2$

$2 \geq 18-18-2$  thus  $2 \geq -2$



Let D be a  $\gamma_{ctd}$  set of G

Let t be the number of edges in G having one vertex in D and the other in  $V(G)-D$ .

Number of vertices and edges in  $\langle V(G)-D \rangle$  is  $p-\gamma_{ctd}(G)-1$  respectively

Hence,  $2(q-p-\gamma_{ctd}(G)-1) = \sum deg(V_i) + t$ .

Since  $|V(G)-D| = p-\gamma_{ctd}(G)$

There are at least  $p-\gamma_{ctd}(G)$  edges from  $V(G)-D$  to D

Also,  $deg(V_i) \geq \delta(G)$

$2[q-p+\gamma_{ctd}(G)-1] \geq 2\gamma_{ctd}(G)+P-\gamma_{ctd}(G)$

But  $\delta(G) \geq 2$

Hence,  $2[q-p+\gamma_{ctd}(G)+1] \geq 2\gamma_{ctd}(G)+P-\gamma_{ctd}(G)$

Thus  $\gamma_{ctd}(G) \geq 3p-2q-2$

This bound is attained if  $G \cong C_n, n \geq 3$  [1].



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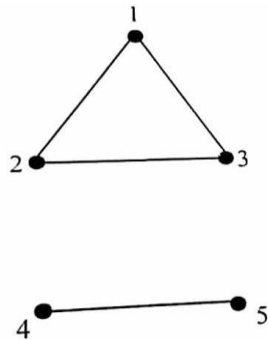
**THEOREM 3.3**

Let G be a connected graph and  $\delta(G)=1$ . Then  $\gamma_{ctd}(G) \geq 3p-2q-m-2$ , where m is the number of pendant vertices.

**Proof:**

$Ctd(G) = \{1,4,5\}$

$P=5$ (no of vertices),  $Q=4$ (no of edges),  $M=2$ (no of pendant vertices)



$$\gamma_{ctd}(G) = 3$$

$$3 \geq 3p - 2q - m - 2$$

$$\geq 3(5) - 2(4) - 2 - 2$$

$$\geq 15 - 8 - 4 \text{ thus } 3 \geq 3$$

Let D be a  $\gamma_{ctd}$  set of G such that  $|D| = \gamma_{ctd}(G)$

And let t be the number of edges in G having one vertex in D and the other in  $V(G) - D$

As in them 3.2,

$$2[q - (p - \gamma_{ctd}(4) - m)] = \sum_{v \in D} \deg(v) \geq t \geq m + 2(\gamma_{ctd}(4) - m) = p - \gamma_{ctd}(G)$$

$$\gamma_{ctd}(G) \geq 3p - 2q - m - 2$$

This bound is attained, when  $G = P_n$ ,  $n \geq 3$  [1].

**COROLLARY 3.4** [1]

If G is a connected K-regular graph  $n(K > 3)$  with p vertices, then  $\gamma_{ctd}(G) \leq (P(k-3) + 2)/(k-3)$

1 = (3,4,5,6), 2 = (5,6,7,8), 3 = (1,6,5,7),

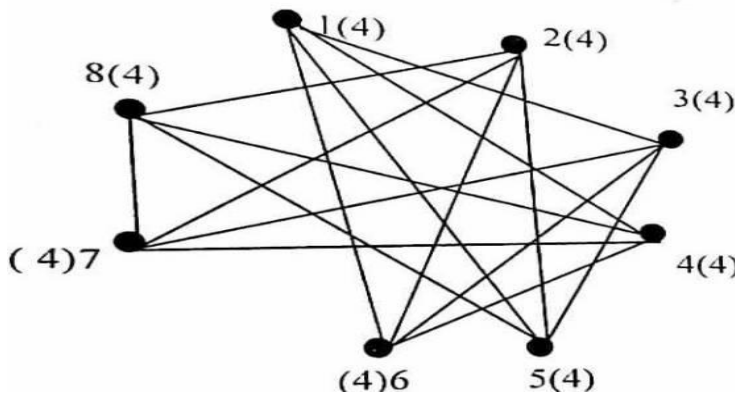
4 = (1,6,7,8), 5 = (1,2,3,8), 6 = (1,2,3,4),

7 = (2,3,4,8), 8 = (2,4,5,7)

$$\gamma_{ctd}(G) \leq (p(k-3) + 2)/(k-3)$$

$$= (8(4-3) + 2)/(4-3)$$

$$= 10$$





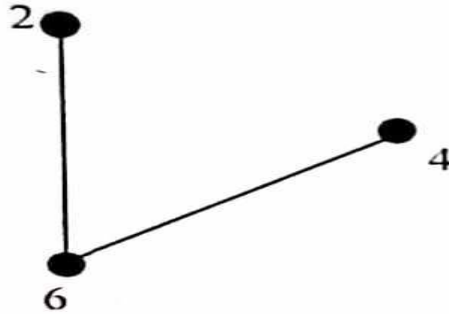
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**COROLLARY 3.5 [1]**

If  $\delta(G) > 3$ , then  $\gamma_{ctd}(G) \leq (2q - 3p + 2) / (\delta(G) - 3)$



- 1-3, 1-4, 1-5, 1-6
- 2-5, 2-6, 2-7, 2-8
- 3-5, 3-6, 3-7
- 4-6, 4-7, 4-8
- 5-8
- 7-8

**III. CONCLUSION**

A set D of a graph  $G = (V, E)$  is a dominating set if every vertex in  $V - D$  is adjacent to some vertex in D. The dominating number  $\gamma(G)$  of G is a minimum cardinality of a dominating set. A dominating set D is called a complementary tree dominating set if the induced subgraph  $\langle V - D \rangle$  is a tree.

In this paper, bounds for  $\gamma_{ctd}(G)$  and its exact values for some particular classes of graphs are found. Some results on complementary tree domination number are also established. We discussed the complementary tree domination number of a graph and its theorems.

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